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INTUITIONISTIC FUZZY PARTIAL ISOMETRY OPERATOR

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ABSTRACT

In this paper, we focus our discuss on Intuitionistic Fuzzy Partial Isometry Operator (IF-Partial Isometry operator) on an intuitionistic fuzzy Hilbert space (IFH-space). In this discuss, the definition of IF-Partial Isometry operator acting on an IFH-space is discussed and some important characteristics are examined. AnIntuitionistic fuzzy continuous linear operator $\mathbb{P}$ on an IFH-space $\mathbb{H}$ is said to be IF-Partial Isometry operator if there exists closed subspace $\mathcal{M}$,such that$\mathcal{P}_{\mu,\nu}(\mathbb{P}u, t) = \mathcal{P}_{\mu,\nu}(u, t)$ for any $u \in \mathcal{M}$and $\mathbb{P}u = 0$ for any $u \in \mathcal{M}^{\perp}$, where $\mathcal{M}$ is said to be the initial space of $\mathbb{P}$ and $\mathcal{N} = \mathcal{R}(\mathbb{P})$ is said to be the finial space of $\mathbb{P}$, which are related to IFU-operator.

Keywords: *Intuitionistic Fuzzy Partial Isometry operator (IF-Partial Isometry operator), Intuitionistic Fuzzy Normal operator (IFN-operator), Intuitionistic Fuzzy Self-Adjoint operator (IFSA-operator), Intuitionistic Fuzzy Unitary operator (IFU-operator), Intuitionistic Fuzzy Adjoint operator (IFA-operator), Intuitionistic Fuzzy Projection operator (IF-Projection operator).*

I. INTRODUCTION

In very first, the concept of intuitionistic fuzzy set was introduced by Atanossov [11] in 1986. The notion of intuitionistic fuzzy metric space $(\mathbb{H}, \mathrm{M}, \mathrm{N}, *, \diamond)$ with the use of continuous t-norm $*$ and continuous t-conorm $\diamond$ was introduced by Park [10], in 2004. From this, using the intuitionistic fuzzy metric space in IFH-space was introduced by Saadati and Park [18] in 2005.Majumdar and Samanta [15] in 2007, gave the definition of IFIP-space and some of their properties using $(\mathbb{H}, \mu, \mu^*)$. Goudarzi *et al.* [12] introduced the new idea of the notion of intuitionistic fuzzy normed spaces and introduced the modified definition of intuitionistic fuzzy inner product space (IFIP-space) with the help of continuous t-representable $(\mathcal{T})$ in 2009, as a triplet $(\mathbb{H}, \mathcal{F}_{\mu,\nu}, \mathcal{T})$ where $\mathbb{H}$ is a real Vector Space, $\mathcal{T}$ is a continuous t-representable and $\mathcal{F}_{\mu,\nu}$ is an Intuitionistic Fuzzy set on $\mathbb{H}^2 \times \mathbb{R}$.

The definition of IFH-space first introduced by Radharamani *et al.* [1], [2] in 2018, and also discussed some properties of IFA & IFSA operators in IFH-space. An operator$\mathbb{P} \in IFB(\mathbb{H})$is said to be IFA-operator, if there exists unique $\mathbb{P}^* \in$IFB($\mathbb{H}$) such that $\langle \mathbb{P}x, y\rangle = \langle x, \mathbb{P}^* y\rangle \; \forall x, y \in \mathbb{H}$**,** where $IFB(\mathbb{H})$ denotes the set of all Intuitionistic Fuzzy Bounded (continuous) linear operators on$\mathbb{H}$**.** Also, $\mathbb{P}$ is an IFSA-operator, if $\mathbb{P} = \mathbb{P}^*$.

In 2020, Radharamani et al. [3] introduced the concept of Intuitionistic Fuzzy Normal operator. If $\mathbb{P} \in IFB(\mathbb{H})$ is called IFN-operator, if it commutes with its Intuitionistic fuzzy adjoint. i.e, $\mathbb{P}\mathbb{P}^* = \mathbb{P}^*\mathbb{P}$.In 2020, Radharamani *et al.* [4] introduced the definition of Intuitionistic Fuzzy Unitary operator (IFU-operator) on IFH-space $\mathbb{H}$, if $\mathbb{P}\mathbb{P}^* = I = \mathbb{P}^*\mathbb{P}$ and gave some important properties of IFU-operator in IFH-space and also the relation with isometric isomorphism of $\mathbb{H}$ on to itself.

In this paper, we consider an Intuitionistic fuzzy self-adjoint operator in IFH- space and introduced the definition of Intuitionistic Fuzzy Partial isometry operator (IF-Partial Isometry operator) and we provided some characteristics of IF-Partial Isometry operator on IFH-space. And also introduce Intuitionistic Fuzzy Projection operator (IF-Projection operator) which is using in IF-Partial Isometry operator and also the relation between them, which all are discuss in detail.

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II. PRELIMINARIES

Definition 2.1: [12] IFIP-space
Let $\mu: \mathbb{H}^2 \times (0, +\infty) \to [0,1]$ and $\nu: \mathbb{H}^2 \times (0, +\infty) \to [0,1]$ be Fuzzy sets, such that $\mu(u, v, t) + \nu(u, v, t) \leq 1$, $\forall\, u, v \in \mathbb{H}\,\&\, t > 0$. An Intuitionistic Fuzzy Inner Product Space (IFIP-Space) is a triplet $(\mathbb{H}, \mathcal{F}_{\mu,\nu}, \mathcal{T})$, where $\mathbb{H}$ is a real Vector Space, $\mathcal{T}$ is a continuous t-representable and $\mathcal{F}_{\mu,\nu}$ is an Intuitionistic Fuzzy set on $\mathbb{H}^2 \times \mathbb{R}$ satisfying the following conditions for all $u, v, w \in \mathbb{H}$ and $s,\ r,\ t \in \mathbb{R}$:
(IFI – 1) $\mathcal{F}_{\mu,\nu}(u,\ v,\ 0) = 0$ and $\mathcal{F}_{\mu,\nu}(u,\ u,\ t) > 0$, for every $t > 0$.
(IFI - 2) $\mathcal{F}_{\mu,\nu}(u,\ v,\ t) = \mathcal{F}_{\mu,\nu}(v,\ u,\ t)$.
(IFI - 3) $\mathcal{F}_{\mu,\nu}(u,\ u,\ t) \neq H(t)$ for some $t \in \mathbb{R}$ iff $u \neq 0$, where $H(t) = \begin{cases} 1, & \text{if} \quad t > 0 \\ 0, & \text{if} \quad t \leq 0 \end{cases}$
(IFI - 4) For any $\alpha \in \mathbb{R}$,

$$\mathcal{F}_{\mu,\nu}(\alpha u, v, t) = \begin{cases} \mathcal{F}_{\mu,\nu}\left(u, v, \dfrac{t}{\alpha}\right), & \alpha > 0 \\ H(t), & \alpha = 0 \\ \mathcal{N}_s\left(\mathcal{F}_{\mu,\nu}\left(u, v, \dfrac{t}{\alpha}\right)\right), & \alpha < 0 \end{cases}$$

(IFI - 5) $\sup\left\{\mathcal{T}\left(\mathcal{F}_{\mu,\nu}(u,\ w,\ s), \mathcal{F}_{\mu,\nu}(v,\ w,\ r)\right)\right\} = \mathcal{F}_{\mu,\nu}(u\ +\ v,\ v,\ t)$.
(IFI - 6) $\mathcal{F}_{\mu,\nu}(u,\ v,\ \cdot): \mathbb{R} \to [0,1]$ is Continuous on $\mathbb{R} \setminus \{0\}$.
(IFI - 7) $\lim_{t\to 0} \mathcal{F}_{\mu,\nu}(u,\ v,\ t) = 1$.

Definition 2.2: [1], [12] IFH-space
Let $(\mathbb{H}, \mathcal{F}_{\mu,\nu}, \mathcal{T})$ be an IFIP-Space with IP: $\langle u, v\rangle = \sup\{t \in \mathbb{R}: \mathcal{F}_{\mu,\nu}(u, v, t) < 1\}, \forall u, v \in \mathbb{H}$. If $(\mathbb{H}, \mathcal{F}_{\mu,\nu}, \mathcal{T})$ is complete in the norm $\mathcal{P}_{\mu,\nu}$, then $\mathbb{H}$ is an Intuitionistic Fuzzy Hilbert Space (IFH-Space).

Definition 2.3: [2] IFA-operator
Let $(\mathbb{H}, \mathcal{F}_{\mu,\nu}, \mathcal{T})$ be an IFH-Space and let $\mathbb{P} \in$ IFB($\mathbb{H}$). Then there exists unique
$\mathbb{P}^* \in$IFB($\mathbb{H}$)$\ni \langle \mathbb{P}u, v\rangle = \langle u, \mathbb{P}^* v\rangle\ \forall\, u, v \in \mathbb{H}$.

Definition 2.4: [2] IFSA-operator
Let $(\mathbb{H}, \mathcal{F}_{\mu,\nu}, \mathcal{T})$ be an IFH-Space with IP:$\langle u, v\rangle = \sup\{\, t \in \mathbb{R}: \mathcal{F}_{\mu,\nu}(\,u,\ v,\ t) < 1\}, \forall\, u, v \in \mathbb{H}$ and let $\mathbb{P} \in$ IFB($\mathbb{H}$). Then $\mathbb{P}$ is Intuitionistic Fuzzy Self-Adjoint Operator, if $\mathbb{P} = \mathbb{P}^*$, where $\mathbb{P}^*$ is Intuitionistic Fuzzy Self-Adjoint of $\mathbb{P}$.

Definition 2.5: [3] IFN-operator
Let $(\mathbb{H}, \mathcal{F}_{\mu,\nu}, \mathcal{T})$ be an IFH-space with an IP:$\langle u, v\rangle = \sup\{t \in \mathbb{R}: \mathcal{F}_{\mu,\nu}(u,\ v,\ t) < 1\}, \forall\, u,\ v\ \in\ \mathbb{H}$ and let $\mathbb{P} \in$IFB($\mathbb{H}$). Then $\mathbb{P}$ is an Intuitionistic Fuzzy Normal Operator if it commutes with its IF-Adjoint. i.e.$\mathbb{P}\mathbb{P}^* = \mathbb{P}^*\mathbb{P}$.

Definition 2.6: [3] IFU-operator
Let $(\mathbb{H}, \mathcal{F}_{\mu,\nu}, \mathcal{T})$ be a IFH-space with IP:$\langle u, v\rangle = \sup\{t \in \mathbb{R}: \mathcal{F}_{\mu,\nu}(u, v, t) < 1\}\ \forall u, v \in \mathbb{H}$ and let$\mathbb{P} \in IFB(\mathbb{H})$. Then $\mathbb{P}$ is an Intuitionistic fuzzy unitary operator if it satisfies $\mathbb{P}\mathbb{P}^* = I = \mathbb{P}^*\mathbb{P}$.

Definition 2.7: [3]Intuitionistic Fuzzy Isometric Isomorphism
Let X and Y be intuitionistic fuzzy normed linear spaces. AnIntuitionistic Fuzzy isometric isomorphism of X into Y is a one to one linear transformation $\mathbb{P}$ of X into Y such that $\mathcal{P}_{\mu,\nu}(\mathbb{P}u, t) = \mathcal{P}_{\mu,\nu}(u, t)$ for every $u \in X$.

Theorem 2.8: [3] Let $(\mathbb{H}, \mathcal{F}_{\mu,\nu}, \mathcal{T})$ be an IFH-space with IP: $\langle u, v\rangle = \sup\{t \in \mathbb{R}: \mathcal{F}_{\mu,\nu}(u, v, t) < 1\}\ \forall u, v \in \mathbb{H}$ and let $\mathbb{P} \in IFB(\mathbb{H})$. If $\mathbb{P}$ is Intuitionistic Fuzzy Unitary operator if and only if it is an isometric isomorphism of $\mathbb{H}$ onto itself.

Definition 2.9: [12] IF-orthogonal
Let $(\mathbb{H}, \mathcal{F}_{\mu,\nu}, \mathcal{T})$ be an IFH-space. $u, v \in \mathbb{H}$is saidto be IF-orthogonal to each other if $\mathcal{F}_{\mu,\nu}(u, v, t) = H(t)$, for each $t \in \mathbb{R}$ and it is denoted by $u \perp v$.

Theorem 2.10: [12] Let $(\mathbb{H}, \mathcal{F}_{\mu,\nu}, \mathcal{T})$be an IFH-space. The orthogonality has the following properties:
(1) $0 \perp u, \forall\, u \in \mathbb{H}$.
(2) If $u \perp v$ then $v \perp u$.
(3) If $u \perp v$ then $u = 0$.
(4) If $u \perp u_i$ $(i = 1,2, \ldots, n)$ then $u \perp (\sum_{i=1}^{n} u_i)$.
(5) If $u \perp v$ then for any $a \in \mathbb{R}, u \perp av$.
(6) Let $\mathcal{F}_{\mu,\nu}$ be IF-continuous. If $u_n \overset{\tau_F}{\to} u, v \perp u_n$ $(n = 1,2, \ldots)$ then $v \perp u$.

Definition 2.11: [12] Let $(\mathbb{H}, \mathcal{F}_{\mu,\nu}, \mathcal{T})$ be an IFH-space and $\mathcal{M} \subset \mathbb{H}$. $\mathcal{M}^{\perp}$ is the set of all $v \in \mathbb{H}$that are orthogonal to every $u \in \mathcal{M}$.

Theorem 2.12: [12] Let $(\mathbb{H}, \mathcal{F}_{\mu,\nu}, \mathcal{T})$ be an IFH-space, $\mathcal{F}_{\mu,\nu}$ be IF-continuous and $\mathcal{M}$ be a subset of $\mathbb{H}$. Then $\mathcal{M}^{\perp}$ is a closed subspace of $\mathbb{H}$ and $\mathcal{M} \cap \mathcal{M}^{\perp} = \{0\}$.

Theorem 2.13: [12] The Pythagorean Theorem
Let $(\mathbb{H}, \mathcal{F}_{\mu,\nu}, \mathcal{T})$ be an IFH-space and let $u \perp v$. Then $\mathcal{P}_{\mu,\nu}(u+v,t) = \mathcal{T}(\mathcal{P}_{\mu,\nu}(u,t), \mathcal{P}_{\mu,\nu}(v,t))$.

III. MAIN RESULTS

In this section, we introduce the definition of Intuitionistic Fuzzy Partial Isometry operator in IFH-space as well as some elementary properties of Intuitionistic Fuzzy Partial Isometry operator in IFH-space are presented. First, we will give the definition of Intuitionistic Fuzzy projection (IF-projection) operator.

Definition 3.1: Intuitionistic Fuzzy Projection operator
Let $(\mathbb{H}, \mathcal{F}_{\mu,\nu}, \mathcal{T})$ be an IFH-space. $\mathbb{H}$ can be decomposed into $\mathbb{H} = \mathcal{M} \oplus \mathcal{M}^{\perp}$, i.e. for any $u \in \mathbb{H}$, $u = v \oplus w$ where $v \in \mathcal{M}$&$w \in \mathcal{M}^{\perp}$. An operator $\mathbb{P}$from $\mathbb{H}$ onto $\mathcal{M}$ is said to be IF-projection if $\mathbb{P}u = v$. It is denoted by $\mathbb{P}_{\mathcal{M}}$.

Note: Let $(\mathbb{H}, \mathcal{F}_{\mu,\nu}, \mathcal{T})$ be an IFH-space and $\mathcal{M} \subset \mathbb{H}$ be a closed subspace. The IF-orthogonal projection (IF-Projection operator) of $\mathbb{H}$ onto $\mathcal{M}$ is an operator from $\mathbb{H}$onto itself such that for $u \in \mathbb{H}$, $\mathbb{P}_{\mathcal{M}}u$ is the unique element in$\mathcal{M}$, i.e. $\mathbb{P}_{\mathcal{M}}u = v, v \in \mathcal{M}$.

Definition 3.1: Intuitionistic Fuzzy Partial isometry operator
An operator $\mathbb{P} \in IFB(\mathbb{H})$ is said to be Intuitionistic Fuzzy (IF) partial isometry operator if there exists a closed subspace $\mathcal{M}$ such that $\mathcal{P}_{\mu,\nu}(\mathbb{P}u,t) = \mathcal{P}_{\mu,\nu}(u,t)$ for any $u \in \mathcal{M}$ and $\mathbb{P}u = 0$, for any $u \in \mathcal{M}^{\perp}$, here $\mathcal{M}$ is said to be the initial space of $\mathbb{P}$ and $\mathcal{N} = \mathcal{R}(\mathbb{P})$ is said to be the final space of $\mathbb{P}$.

Note:
(i) The Intuitionistic Fuzzy projection on to the initial space and the final space are said to be the initial intuitionistic fuzzy projection and final intuitionistic fuzzy projection of $\mathbb{P}$.
(ii) $\mathbb{P}$ is Intuitionistic Fuzzy isometry if and only if $\mathbb{P}$ is Intuitionistic Fuzzy partial isometry and $\mathcal{M} = \mathbb{H}$.
(iii) $\mathbb{P}$ is Intuitionistic Fuzzy Unitary if and only if $\mathbb{P}$ is Intuitionistic Fuzzy partial isometry and $\mathcal{M} = \mathcal{N} = \mathbb{H}$.

Theorem 3.2: $\mathbb{P} \in IFB(\mathbb{H})$ is an IF-isometry operator if and only if $\mathbb{P}^*\mathbb{P} = I$.

Proof: Let $\mathbb{P} \in IFB(\mathbb{H})$ be IF-isometry. Then

$$\begin{aligned}\langle \mathbb{P}^*\mathbb{P}u, v\rangle &= \sup\{t \in \mathbb{R}: \mathcal{F}_{\mu,\nu}(\mathbb{P}^*\mathbb{P}u, v, t) < 1\}\\ &= \sup\{t \in \mathbb{R}: \mathcal{F}_{\mu,\nu}(\mathbb{P}u, \mathbb{P}v, t) < 1\}\\ &= \sup\{t \in \mathbb{R}: \mathcal{F}_{\mu,\nu}(u, v, t) < 1\}\end{aligned}$$

$$\therefore \langle \mathbb{P}^*\mathbb{P}u, v\rangle = \langle u, v\rangle \;\forall\; u.v \in \mathbb{H}$$

$$\Longrightarrow \mathbb{P}^*\mathbb{P} = I.$$

Conversely, suppose that $\mathbb{P}^*\mathbb{P} = I$.

$$\begin{aligned}{\mathcal{P}_{\mu,\nu}}^2(\mathbb{P}u,t) &= \langle \mathbb{P}u, \mathbb{P}u\rangle\\ &= \sup\{t \in \mathbb{R}: \mathcal{F}_{\mu,\nu}(\mathbb{P}u, \mathbb{P}u, t) < 1\}\\ &= \sup\{t \in \mathbb{R}: \mathcal{F}_{\mu,\nu}(\mathbb{P}^*\mathbb{P}u, u, t) < 1\}\\ &= \langle u, u\rangle\end{aligned}$$

$$\therefore {\mathcal{P}_{\mu,\nu}}^2(\mathbb{P}u,t) = {\mathcal{P}_{\mu,\nu}}^2(u,t)$$

$$\Longrightarrow \mathcal{P}_{\mu,\nu}(\mathbb{P}u,t) = \mathcal{P}_{\mu,\nu}(u,t)$$

Hence $\mathbb{P}$ is an IF-isometry operator.

Theorem 3.3: Let $\mathbb{P} \in IFB(\mathbb{H})$. $\mathbb{P}$ is an IFU-operator iff $\mathbb{P}^*\mathbb{P} = \mathbb{P}\mathbb{P}^* = I$.

Proof: By theorem 2.8, it is enough to prove$\mathbb{P}$ is Intuitionistic Fuzzy Unitary iff $\mathbb{P}$ is an Intuitionistic Fuzzy isometry on $\mathbb{H}$.

So $\mathbb{P}^*\mathbb{P} = I$ and for any $u \in \mathbb{H}$, there exists $v \in \mathbb{H}$, such that $\mathbb{P}v = u$.

Now, $\mathbb{P}^*u = \mathbb{P}^*\mathbb{P}v = Iv = v$.

So that $\mathcal{P}_{\mu,v}(\mathbb{P}^*u,t) = \mathcal{P}_{\mu,v}(v,t)$
$= \mathcal{P}_{\mu,v}(\mathbb{P}v,t)$
$= \mathcal{P}_{\mu,v}(u,t)$
$\therefore \mathcal{P}_{\mu,v}(\mathbb{P}^*u,t) = \mathcal{P}_{\mu,v}(u,t)$

Thus $\mathbb{P}^*$is Intuitionistic Fuzzy Isometry and $\mathbb{P}\mathbb{P}^* = (\mathbb{P}^*)^*\mathbb{P}^* = I$.

Conversely, assume that $\mathbb{P}^*\mathbb{P} = \mathbb{P}\mathbb{P}^* = I$.

Then $\mathbb{P}$ is Intuitionistic Fuzzy isometry and for any $u \in \mathbb{H}, u = \mathbb{P}\mathbb{P}^*, u \in \mathcal{R}(\mathbb{P})$, where $\mathcal{R}(\mathbb{P})$ is the range of $\mathbb{P}$.

Thus, $\mathbb{P}$ is intuitionistic Fuzzy isometry operator on $\mathbb{H}$.

Theorem 3.4: Let $\mathbb{P}$ be an Intuitionistic Fuzzy Partial isometry operator on an IFH-space with the initial space $\mathcal{M}$ and the final space $\mathcal{N}$. Then the following hold:

1) $\mathbb{P}\mathbb{P}_{\mathcal{M}} = \mathbb{P}$ and $\mathbb{P}^*\mathbb{P} = \mathbb{P}_{\mathcal{M}}$
2) $\mathcal{N}$ is a closed subspace of $\mathbb{H}$.
3) $\mathbb{P}^*$is an Intuitionistic Fuzzy Partial isometry with the initial space $\mathcal{N}$ and final space $\mathcal{M}$,
   i.e. $\mathbb{P}\mathbb{P}_{\mathcal{N}} = \mathbb{P}^*$&$\mathbb{P}\mathbb{P}^* = \mathbb{P}_{\mathcal{N}}$.

Proof: Given that $\mathbb{P}$ is an intuitionistic Fuzzy partial isometry operator on IFH-space $\mathbb{H}$.

1) To prove $\mathbb{P}\mathbb{P}_{\mathcal{M}} = \mathbb{P}$ and $\mathbb{P}^*\mathbb{P} = \mathbb{P}_{\mathcal{M}}$
For $u \in \mathbb{H}$,$u = \mathbb{P}_{\mathcal{M}}u \oplus w, \forall\, w \in \mathcal{M}^{\perp}$
And $\mathbb{P}u = \mathbb{P}\mathbb{P}_{\mathcal{M}}u \oplus \mathbb{P}w = \mathbb{P}\mathbb{P}_{\mathcal{M}}u$
Hence, $\mathbb{P} = \mathbb{P}\mathbb{P}_{\mathcal{M}}$, since $\mathbb{P}w = 0$.
Now since $\langle \mathbb{P}u, \mathbb{P}v\rangle = \langle u, v\rangle$ for $u, v \in \mathcal{M}$ and $\mathbb{P}_{\mathcal{M}}u$, $\mathbb{P}_{\mathcal{M}}v \in \mathcal{M}$ for any $u, v \in \mathbb{H}$,
$\langle \mathbb{P}^*\mathbb{P}u, v\rangle = \sup\{t \in \mathbb{R}: \mathcal{F}_{\mu,v}(\mathbb{P}^*\mathbb{P}u, v, t) < 1\}$
$= \sup\{t \in \mathbb{R}: \mathcal{F}_{\mu,v}(\mathbb{P}u, \mathbb{P}v, t) < 1\}$
$= \sup\{t \in \mathbb{R}: \mathcal{F}_{\mu,v}(\mathbb{P}\mathbb{P}_{\mathcal{M}}u, \mathbb{P}\mathbb{P}_{\mathcal{M}}v, t) < 1\}$
$= \sup\{t \in \mathbb{R}: \mathcal{F}_{\mu,v}(\mathbb{P}^*\mathbb{P}\mathbb{P}_{\mathcal{M}}u, \mathbb{P}_{\mathcal{M}}v, t) < 1\}$
$= \sup\{t \in \mathbb{R}: \mathcal{F}_{\mu,v}(\mathbb{P}_{\mathcal{M}}u, \mathbb{P}_{\mathcal{M}}v, t) < 1\}$
$= \sup\{t \in \mathbb{R}: \mathcal{F}_{\mu,v}(\mathbb{P}_{\mathcal{M}}u, v, t) < 1\}$
$= \langle \mathbb{P}_{\mathcal{M}}u, v\rangle$
$\Longrightarrow \mathbb{P}^*\mathbb{P} = \mathbb{P}_{\mathcal{M}}$
2) To prove $\mathcal{N}$ is a closed subspace of $\mathbb{H}$.
Since $\mathcal{N} = \mathcal{R}(\mathbb{P}) = \mathbb{P}\mathcal{R}(\mathbb{P}_{\mathcal{M}}) = \mathbb{P}\mathcal{M}$, for any $u \in \bar{\mathcal{N}}$, there exists a sequence $\{v_n\} \subset \mathcal{M}, \ni \mathbb{P}v_n \to u$ and $\mathcal{P}_{\mu,v}(v_m - v_n, t) = \mathcal{P}_{\mu,v}(\mathbb{P}v_m - \mathbb{P}v_n, t) \to 0$ as $m, n \to \infty$.
Thus, by the completeness of $\mathbb{H}$, there exists $v \in \mathbb{H}$, such that $v_n \to v$ and $\mathbb{P}v_n \to \mathbb{P}v \Longrightarrow u = \mathbb{P}v \in N$,
Hence, $\mathcal{N} = \bar{\mathcal{N}}$.
3) To prove that $\mathbb{P}^*$ is Intuitionistic Fuzzy partial isometry with the initial space $\mathcal{N}$ and final space $\mathcal{M}$.

For any $u \in \mathcal{N}$, there exists $v \in \mathcal{M}$, such that $\mathbb{P}v = u$ and $\mathcal{P}_{\mu,v}(u,t) = \mathcal{P}_{\mu,v}(v,t)$.
And $\mathbb{P}^*u = \mathbb{P}^*\mathbb{P}v = \mathbb{P}_{\mathcal{M}}v = v$.

So that $\mathcal{P}_{\mu,v}(\mathbb{P}^*u,t) = \mathcal{P}_{\mu,v}(u,t)$, …(a)

For any $u \in \mathcal{N}^{\perp}$, since $\mathbb{P}v \in \mathcal{N}$ for any $v \in \mathbb{H}$,
$\Longrightarrow \langle \mathbb{P}^*\mathbb{P}u, v\rangle = \langle u, \mathbb{P}v\rangle = 0$
$\Longrightarrow \mathbb{P}^*u = 0$ …(b)

Therefore $\mathbb{P}^*$ is Intuitionistic Fuzzy partial isometry with the initial space $\mathcal{N}$ and final space $\mathcal{M}$, because
$$\mathcal{R}(\mathbb{P}^*) = \mathbb{P}^*\mathcal{N} = \mathbb{P}^*\mathcal{R}(\mathbb{P}) = \mathbb{P}^*\mathbb{P}\mathbb{H} = \mathbb{P}_{\mathcal{M}}\mathbb{H} = \mathcal{M}.$$

From (a) by replacing $\mathbb{P}$ by $\mathbb{P}^*$ and $\mathcal{M}$ by $\mathcal{N}$.

Theorem 3.5: Let $\mathbb{P}$ be an operator on an IFH-space $\mathbb{H}$. Then the following statements are equivalent to one another.

(a) $\mathbb{P}$ is an intuitionistic Fuzzy partial isometry operator.
 (a1) $\mathbb{P}^*$ is an Intuitionistic Fuzzy partial isometry operator.
(b) $\mathbb{P}\mathbb{P}^*\mathbb{P} = \mathbb{P}$.
(b1) $\mathbb{P}^*\mathbb{P}\mathbb{P}^* = \mathbb{P}^*$.
(c) $\mathbb{P}^*\mathbb{P}$ is an Intuitionistic Fuzzy projection operator.
(c1) $\mathbb{P}\mathbb{P}^*$ is an Intuitionistic Fuzzy projection operator.

Proof: Given $\mathbb{P}$ is an operator on IFH-space $\mathbb{H}$.
(a) $\Longrightarrow$ (b):
Assume $\mathbb{P}$ is intuitionistic Fuzzy partial isometry. Then by theorem 3.3 (1),

$$\mathbb{P}\mathbb{P}^*\mathbb{P} = \mathbb{P}\mathbb{P}_{\mathcal{M}} = \mathbb{P} \qquad [\because\ \mathbb{P}\mathbb{P}_{\mathcal{M}} = \mathbb{P} \,\&\, \mathbb{P}^*\mathbb{P} = \mathbb{P}_{\mathcal{M}}]$$
$$\Longrightarrow \mathbb{P}\mathbb{P}^*\mathbb{P} = \mathbb{P}$$

Hence, (a) $\Longrightarrow$ (b).
(b) $\Longrightarrow$ (c): To prove that $\mathbb{P}^*\mathbb{P}$ is Intuitionistic Fuzzy projection.
Let $\mathbb{P}\mathbb{P}^*\mathbb{P} = \mathbb{P}$.
Then, $\mathbb{P}^*\mathbb{P}\mathbb{P}^*\mathbb{P} = \mathbb{P}^*\mathbb{P}$.
i.e. $\mathbb{P}^*\mathbb{P}$ is idempotent and intuitionistic fuzzy self-adjoint (IFSA), so that $\mathbb{P}^*\mathbb{P}$ is an intuitionistic Fuzzy projection operator.
Hence, (b) $\Longrightarrow$ (c)
(c) $\Longrightarrow$ (a):

Let $\mathbb{P}^*$ be an Intuitionistic Fuzzy projection operator. Put $\mathbb{P}^*\mathbb{P} = \mathbb{P}_{\mathcal{M}}$.

Now for any $u \in \mathbb{H}$,

$$\begin{aligned}
{\mathcal{P}_{\mu,v}}^2(\mathbb{P}u, t) &= \langle \mathbb{P}u, \mathbb{P}u \rangle \\
&= \sup\{t \in \mathbb{R}: \mathcal{F}_{\mu,v}(\mathbb{P}u, \mathbb{P}u, t) < 1\} \\
&= \sup\{t \in \mathbb{R}: \mathcal{F}_{\mu,v}(\mathbb{P}^*\mathbb{P}u, u, t) < 1\} \\
&= \sup\{t \in \mathbb{R}: \mathcal{F}_{\mu,v}(\mathbb{P}_{\mathcal{M}}u, u, t) < 1\} \\
&= {\mathcal{P}_{\mu,v}}^2(\mathbb{P}_{\mathcal{M}}u, t)
\end{aligned}$$

So that $\mathcal{P}_{\mu,v}(\mathbb{P}u, t) = \mathcal{P}_{\mu,v}(u, t)$ for any $u \in \mathcal{M}$ and $\mathbb{P}u = 0$, for any $u \in \mathcal{M}^{\perp}$.

Hence, the equivalence relation among **(a), (b) & (c)** is proved.

Similarly, the equivalence relation among **(a1), (b1) & (c1)** can be proved easily and **(b)** $\Leftrightarrow$ **(b1)** is obtained by taking adjoint of both sides.

IV.CONCLUSION

The new idea of Intuitionistic Fuzzy Partial isometry operator (IFPI- operator) on IFH-space is introduced. And also discuss different dimensions of relation between IFU-operator and IFP-operator. These relations are very new and helpful for the further study of functional analysis on intuitionistic fuzzy concept. Some characteristics of IFPI- operator have been investigated. Some results and theorems will be useful for the further research in functional analysis.

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