COMPUTATION OF MULTIPLICATIVE STATUS GOURAVA INDICES OF GRAPHS

V. R. KULLI*

Department of Mathematics, Gulbarga University, Gulbarga 585 106, India.

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ABSTRACT

The status of a vertex u in G is defined as the sum of the distances between u and all other vertices of a graph. In this paper, we introduce the multiplicative first and second hyper status Gourava indices, multiplicative sum and product connectivity status Gourava indices, general multiplicative first and second status Gourava indices of a graph. We compute these multiplicative status Gourava indices for wheel and friendship graphs.

Keywords: Status, multiplicative hyper status Gourava indices, multiplicative sum and product connectivity status Gourava indices, graph.

Mathematics Subject Classification: 05C05, 05C07, 05C35.

1. INTRODUCTION

A graph index is a numerical parameter mathematically derived from the graph structure. Several graph indices have their applications in various disciplines of Science and Technology, see [1, 2]. Let G be a finite, simple, connected graph. Let V(G) be the vertex set and E(G) be the edge set of G. The degree of a vertex u is the number of vertices adjacent to u and is denoted by $d_G(u)$. The distance between two vertices u and v, denoted by d(u, v), is the length of the shortest path joining u and v. For graph theoretic terminology, we refer [3].

The status [4] of a vertex u in G is the sum of distances of all other vertices from u in G and it is denoted by $\sigma(u)$. Recently, the harmonic status index [5], first and second status connectivity indices [6], first and second status connectivity coindices [7], geometric-arithmetic status index [8], first and second hyper status indices [9], multiplicative first and second status indices [10], atom bond connectivity status index [11], multiplicative geometric-arithmetic status index [12], multiplicative (a, b)-status index [13], general vertex status index [14], Status Gourava indices [15] were introduced and studied.

In [16], Kulli introduced the multiplicative first and second Gourava indices of a graph, defined as

$$GO_{1}II(G) = \prod_{uv \in E(G)} \left[d_{G}(u) + d_{G}(v) + d_{G}(u) d_{G}(v) \right].$$

$$GO_{2}II(G) = \prod_{uv \in E(G)} \left(d_{G}(u) + d_{G}(v) \right) d_{G}(u) d_{G}(v).$$

Motivated by these multiplicative Gourava indices, we propose the multiplicative first and second status Gourava indices of a graph, defined as

$$SG_{1}II(G) = \prod_{uv \in E(G)} [\sigma(u) + \sigma(v) + \sigma(u)\sigma(v)].$$

$$SG_{2}II(G) = \prod_{uv \in E(G)} (\sigma(u) + \sigma(v))\sigma(u)\sigma(v).$$

Corresponding Author: V. R. Kulli*
Department of Mathematics, Gulbarga University, Gulbarga 585 106, India.

Furthermore, we introduce the following multiplicative status Gourava indices of a graph as follows:

The multiplicative first and second hyper status Gourava indices of a graph G are defined as

$$HSG_{1}II(G) = \prod_{uv \in E(G)} \left[\sigma(u) + \sigma(v) + \sigma(u)\sigma(v)\right]^{2},$$

$$HSG_{2}II(G) = \prod_{uv \in E(G)} \left[\left(\sigma(u) + \sigma(v)\right)\sigma(u)\sigma(v)\right]^{2}.$$

The multiplicative sum connectivity status Gourava index of a graph G is defined as

$$SSGII(G) = \prod_{uv \in E(G)} \frac{1}{\sqrt{\sigma(u) + \sigma(v) + \sigma(u)\sigma(v)}}.$$

The multiplicative product connectivity status Gourava index of a graph G is defined as

$$PSGII(G) = \prod_{uv \in E(G)} \frac{1}{\sqrt{(\sigma(u) + \sigma(v))\sigma(u)\sigma(v)}}.$$

The multiplicative reciprocal product connectivity status Gourava index of a graph G is defined as

$$RPSGII(G) = \prod_{uv \in E(G)} \sqrt{(\sigma(u) + \sigma(v))\sigma(u)\sigma(v)}$$

Finally, we propose the general multiplicative first and second status Gourava indices of a graph G, defined as

$$\begin{split} &SG_1^a II(G) = \prod_{uv \in E(G)} \left[\sigma(u) + \sigma(v) + \sigma(u)\sigma(v)\right]^a, \\ &SG_2^a II(G) = \prod_{uv \in E(G)} \left[\left(\sigma(u) + \sigma(v)\right)\sigma(u)\sigma(v)\right]^a, \end{split}$$

where a is a real number

Recently, some new multiplicative indices were studied, for example, in [17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33].

In this paper, some newly proposed multiplicative status Gourava indices of wheel and friendship graphs were computed.

2. RESULTS FOR WHEEL GRAPHS

A wheel graph W_n is the join of C_n and K_1 . A graph W_4 is presented in Figure 1.

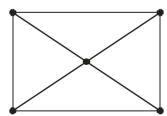


Figure-1: Wheel graph W_{Δ}

A graph W_n has n + 1 vertices and 2n edges. There are two types of edges in W_n as follows:

$$E_1 = \{uv \in E(W_n) \mid d(u) = d(v) = 3\},$$
 $|E_1| = n.$
 $E_2 = \{uv \in E(W_n) \mid d(u) = 3, d(v) = n\},$ $|E_1| = n.$

Therefore, in W_n , there are two types of status edges as follows.

$\sigma(u), \sigma(v) \setminus uv \in E(W_n)$	(2n-3, 2n-3)	(n, 2n - 3)
Number of vertices	n	n

Table-1: Status edge partition of W_n

Theorem 1: The general multiplicative first status Gourava index of a wheel graph W_n is given by

$$SG_1^a H(W_n) = (4n^2 - 8n + 3)^{an} \times (2n^2 - 3)^{an}.$$
 (1)

Proof: Let W_n be a wheel graph with n+1 vertices and 2n edges. By definition, we have

$$SG_1^a II(W_n) = \prod_{uv \in E(W_n)} \left[\sigma(u) + \sigma(v) + \sigma(u)\sigma(v)\right]^a.$$

Therefore by using Table 1, we deduce

$$SG_1^a II(W_n) = [2n-3+2n-3+(2n-3)(2n-3)]^{an} \times [n+2n-3+n(2n-3)]^{an}$$
$$= (4n^2-8n+3)^{an} \times (2n^2-3)^{an}.$$

From Theorem 1, we establish the following results.

Corollary 1.1: The multiplicative first status Gourava index of W_n is

$$SG_1II(W_n) = (4n^2 - 8n + 3)^n \times (2n^2 - 3)^n$$
.

Corollary 1.2: The multiplicative first hyper status Gourava index of W_n is

$$HSG_1H(W_n) = (4n^2 - 8n + 3)^{2n} \times (2n^2 - 3)^{2n}$$
.

Corollary 1.3: The multiplicative sum connectivity status Gourava index of W_n is

$$SSGII(W_n) = \left(\frac{1}{\sqrt{4n^2 - 8n + 3}}\right)^n \times \left(\frac{1}{\sqrt{2n^2 - 3}}\right)^n.$$

Proof: Put $a = 1, 2, -\frac{1}{2}$ in equation (1), we obtain the desired results.

Theorem 2: The general multiplicative second status Gourava index of a wheel graph W_n is given by

$$SG_1^a H(W_n) = 2^{an} \times 3^{an} \times (2n-3)^{3an} \times (2n^3 - 5n^2 + 3n)^{an}.$$
 (2)

Proof: Let W_n be a wheel graph with n+1 vertices and 2n edges. By using definition and Table 1, we derive

$$SG_1^a H(W_n) = \prod_{uv \in E(W_n)} \left[(\sigma(u) + \sigma(v)) \sigma(u) \sigma(v) \right]^a$$

$$= \left[(2n - 3 + 2n - 3)(2n - 3)(2n - 3) \right]^{an} \times \left[(n + 2n - 3)n(2n - 3) \right]^{an}$$

$$= 2^{an} \times 3^{an} \times (2n - 3)^{3an} \times \left(2n^3 - 5n^2 + 3n \right)^{an}.$$

We obtain the following results by using Theorem 2.

Corollary 2.1: The multiplicative second status Gourava index of W_n is

$$SG_2II(W_n) = 2^n \times 3^n \times (2n-3)^{3n} \times (2n^3 - 5n^2 + 3n)^n$$
.

Corollary 2.2: The multiplicative second hyper status Gourava index of W_n is

$$HSG_2H(W_n) = 2^{2n} \times 3^{2n} \times (2n-3)^{6n} \times (2n^3 - 5n^2 + 3n)^{2n}$$
.

Corollary 2.3: The multiplicative product connectivity status Gourava index of W_n is

$$PSG_2H\left(W_n\right) = \left(\frac{1}{\sqrt{2}}\right)^n \times \left(\frac{1}{\sqrt{3}}\right)^n \times \left(\frac{1}{\sqrt{2n-3}}\right)^{3n} \times \left(\frac{1}{\sqrt{2n^3-5n^2+3n}}\right)^n.$$

Corollary 2.4: The multiplicative reciprocal product connectivity status Gourava index of W_n is

$$RPSGII(W_n) = 2^{\frac{n}{2}} \times 3^{\frac{n}{2}} \times (2n-3)^{\frac{3n}{2}} \times (2n^3-5n^2+3n)^{\frac{n}{2}}.$$

Proof: Put $a = 1, 2, -\frac{1}{2}$, $\frac{1}{2}$ in equation (2), we get the desired results.

3. RESULTS FOR FRIENDSHIP GRAPHS

A friendship graph is the graph obtained by taking $n \ge 2$ copies of C_3 with a vertex in common, and it is denoted by F_n . A friendship graph F_4 is depicted in Figure 2.



Figure-2: Friendship graph F_4

A friendship graph F_n has 2n+1 vertices and 3n edges. Then there are two types of edges in F_n as given below:

$$E_1 = \{uv \in E(F_n) \mid d(u) = d(v) = 2 \},$$
 $|E_1| = n.$
 $E_2 = \{uv \in E(F_n) \mid d(u) = 2, d(v) = 2n\},$ $|E_2| = 2n$

Thus there are two types of status edges in F_n as given in Table 2.

$$\sigma(u), \sigma(v) \setminus uv \in E(F_n)$$
 (4n - 2, 4n - 2) (2n, 4n - 2)
Number of edges n 2n

Table-2: Status edge partition of F_n

Theorem 3: The general multiplicative first status Gourava index of a friendship graph F_n is given by

$$SG_1^a H(F_n) = 8^{an} \times 2^{2an} \times (2n^2 - n)^{an} \times (4n^2 + n - 1)^{2an}.$$
 (3)

Proof: By definition and by using Table 2, we derive

$$\begin{split} SG_1^a H\left(F_n\right) &= \prod_{uv \in E(F_n)} \left[\sigma(u) + \sigma(v) + \sigma(u)\sigma(v)\right]^{2an} \\ &= \left[4n - 2 + 4n - 2 + (4n - 2)(4n - 2)\right]^{an} \times \left[2n + 4n - 2 + 2n(2n - 2)\right]^{2an} \\ &= 8^{an} \times 2^{2an} \times \left(2n^2 - n\right)^{an} \times \left(4n^2 + n - 1\right)^{2an}. \end{split}$$

We establish the following results by using Theorem 3.

Corollary 3.1: The multiplicative first status Gourava index of F_n is

$$SG_1H(F_n) = 8^n \times 2^{2n} \times (2n^2 - n)^n \times (4n^2 + n - 1)^{2n}$$
.

Corollary 3.2: The multiplicative first hyper status Gourava index of F_n is

$$HSG_1H(F_n) = 8^{2n} \times 2^{4n} \times (2n^2 - n)^{2n} \times (4n^2 + n - 1)^{4n}$$
.

Corollary 3.3: The multiplicative sum connectivity status Gourava index of F_n is

$$SSGII(F_n) = \left(\frac{1}{\sqrt{8}}\right)^n \times \left(\frac{1}{2}\right)^n \times \left(\frac{1}{\sqrt{2n^2 - n}}\right)^n \times \left(\frac{1}{4n^2 + n - 1}\right)^n.$$

Proof: Put $a = 1, 2, -\frac{1}{2}$ in equation (3), we get the desired results.

Theorem 4: The general multiplicative second status Gourava index of a friendship graph F_n is

$$SG_2^a II(F_n) = 2^{an} \times 8^{2an} \times (4n - 2)^{3an} \times (6n^3 - 5n^2 + n)^{2an}.$$
 (4)

Proof: From definition and by using Table 2, we deduce

$$SG_2^a H(F_n) = \prod_{uv \in E(F_n)} \left[\left(\sigma(u) + \sigma(v) \right) \sigma(u) \sigma(v) \right]^a$$

$$= \left[(4n - 2 + 4n - 2)(4n - 2)(4n - 2) \right]^{an} \times \left[(2n + 4n - 2)2n(2n - 2) \right]^{2an}$$

$$= 2^{an} \times 8^{2an} \times (4n - 2)^{3an} \times \left(6n^3 - 5n^2 + n \right)^{2an}.$$

We obtain the following results by using Theorem 4.

Corollary 4.1: The multiplicative second status Gourava index of F_n is

$$SG_2II(F_n) = 2^n \times 8^{2n} \times (4n-2)^{3n} \times (6n^3 - 5n^2 + n)^{2n}$$
.

Corollary 4.2: The multiplicative second hyper status Gourava index of F_n is

$$HSG_2H(F_n) = 2^{2n} \times 8^{4n} \times (4n-2)^{6n} \times (6n^3 - 5n^2 + n)^{4n}$$

Corollary 4.3: The multiplicative product connectivity status Gourava index of F_n is

$$PSGII(F_n) = \left(\frac{1}{2}\right)^{n/2} \times \left(\frac{1}{8}\right)^n \times \left(\frac{1}{4n-2}\right)^{3n/2} \times \left(\frac{1}{6n^3 - 5n^2 + n}\right)^n.$$

Corollary 4.4: The multiplicative reciprocal product connectivity status Gourava index of F_n is

$$RPSGII(F_n) = 2^{n/2} \times 8^n \times (4n-2)^{3n/2} \times (6n^3 - 5n^2 + n)^n$$
.

Proof: Put $a = 1, 2, -\frac{1}{2}, \frac{1}{2}$ equation (4), we get the desired results.

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