International Journal of Mathematical Archive-11(5), 2020, 17-22
IJMA Available online through www.ijma.info ISSN 2229 – 5046

FUZZY ARITHMETIC DEVELOPED BY TRAPEZOIDAL FUZZY NUMBERS

R. K. DAS*[1] AND MUKHLAL SAH[2]

[1]P. G. Dept. of Mathematics, S. K. M. University, Dumka - (Jharkhand), India.

[2]N. K. High School, Raghunathpur, Godda - (Jharkhand), India.

(Received On: 16-03-20; Revised & Accepted On: 13-04-20)

ABSTRACT

Fuzzy arithmetic is based on properties of fuzzy numbers. Each fuzzy number can uniquely be determined by its α- cut and the α- cut of each fuzzy number ($0 \leq \alpha \leq 1$) is a closed interval of real numbers.

In this paper, fuzzy arithmetic +, –, and ÷ have been developed by trapezoidal fuzzy numbers.

Key words: *Fuzzy number, α- cut, Fuzzy arithmetic, Trapezoidal fuzzy number.*

1. INTRODUCTION

Fuzzy set theory has been introduced by L.A. Zadeh [1] which permits the gradual assessments of the membership of elements in a set described in the interval [0, 1].

Fuzzy numbers and triangular fuzzy numbers have their importance in fuzzy set theory [2], [3]. G.J. Klir [2] introduced the interval arithmetic method of developing fuzzy arithmetic by triangular fuzzy numbers, where as in this paper, we have developed fuzzy arithmetic by trapezoidal fuzzy numbers. Figures have been sketched for justification.

2. PRELIMINARIES

A fuzzy set $\tilde{A}$ on a non empty set X is characterised by its membership function $\mu\tilde{A}$: X ⟶ [0, 1] where $\mu\tilde{A}(x)$ is the degree of membership of the elements of X in fuzzy set $\tilde{A}$ for each x∈X [1].

A fuzzy set $\tilde{A}$ is called normal if $\mu\tilde{A}(x) = 1$ for at least one $x \in X$ and the α- cut of a fuzzy set $\tilde{A}$ on X is a crisp set defined by:- $\widetilde{A_\alpha} = \{ x \in X : \mu\widetilde{A_\alpha}(x) \geq \alpha \}$, $\alpha \in [0, 1]$

Definition 2.1: Fuzzy numbers are close to a given real number or around a given interval of real numbers. To qualify for a fuzzy number, a fuzzy set $\tilde{A}$ on R must be

i) A normal fuzzy set.
ii) The α- cut, $\widetilde{A_\alpha}$ must be a closed interval for every $\alpha \in [0, 1]$
iii) The support set O + $\tilde{A}$ must be bounded.

It is remarkable that a fuzzy number is convex normalised fuzzy set [2].

Definition 2.2: A fuzzy numbers $\tilde{A} = (a_1, a_2, a_3)$ is said to be a triangular fuzzy number [4] if its membership function is given by :-

$$\mu\tilde{A}(x) = \begin{cases} 0, & x \leq a_1 \\ \frac{x - a_1}{a_2 - a_1}, & a_1 \leq x \leq a_2 \\ 1, & x = a_2 \\ \frac{a_3 - x}{a_3 - a_2}, & a_2 \leq x \leq a_3 \\ 0, & x > a_3 \end{cases}$$

Corresponding Author: R. K. Das[1],*
[1]P. G. Dept. of Mathematics, S. K. M. University, Dumka (Jharkhand), India.

3. TRAPEZOIDAL FUZZY NUMBER

On the basis of the definition (2.2), a fuzzy number $\widetilde{A}$ = (a_1, a_2, a_3, a_4) may be called trapezoidal fuzzy number if its membership function is given by

$$\mu \widetilde{A}\,(x) = \begin{cases} 0, & \text{if } x \le a_1 \\ \frac{x-a_1}{a_2-a_1}, & \text{if } a_1 \le x \le a_2 \\ 1, & \text{if } a_2 \le x \le a_3 \\ \frac{a_4-x}{a_4-a_3}, & \text{if } a_3 \le x \le a_4 \\ 0, & \text{if } x > a_4 \end{cases}$$

Example:

$$\mu \widetilde{A}\,(x) = \begin{cases} 0, & \text{for } x \le -1, x > 4 \\ \frac{x+1}{2} & \text{for } -1 < x \le 1 \\ 1 & \text{for } 1 \le x \le 2 \\ \frac{4-x}{2} & \text{for } 2 < x \le 4 \end{cases}$$

is a trapezoidal fuzzy number $\tilde{A}$ = (-1, 1, 2, 4) fig. (a)

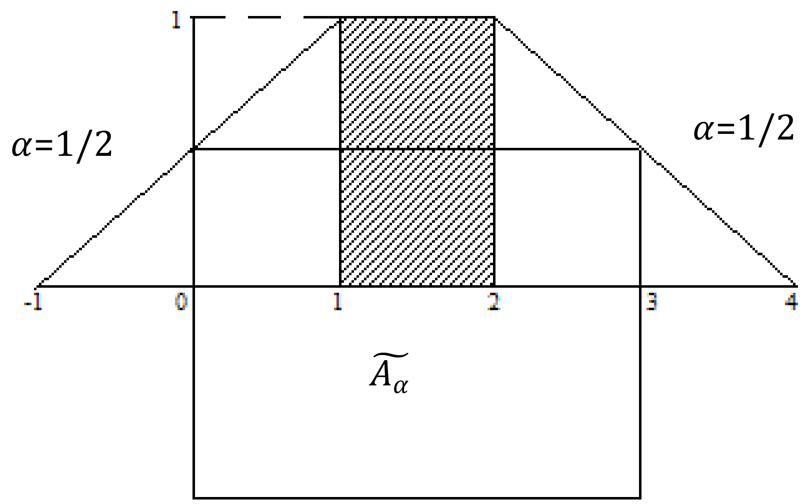


Figure: (a)

Excluding 0 and 1 in the definition of $\mu \widetilde{A}$ (x) of the above example, The α-cut of such trapezoidal fuzzy number is obtained by putting $\frac{x+1}{2} = \alpha$ and $\frac{4-x}{2} = \alpha$, Thus getting two values of x as $2\alpha - 1$, $4 - 2\alpha$, the lower and upper bound.

$\therefore\ \widetilde{A}\alpha = [2\alpha - 1, 4 - 2\alpha]$ is α–cut of above trapezoidal fuzzy number.

4. FUZZY ARITHMETIC

Arithmetic operations on fuzzy numbers mean the arithmetic operations on α – cuts which are closed intervals

Let O denote any one of the basic arithmetic operations +, –, . and ÷.

Then [a, b] O [d, e] = {fog / a≤f ≤ b, d ≤ g ≤ e}

We define [a, b] + [d, e] = [a + d, b + e]
[a, b] – [d, e] = [a – e, b – d]
[a, b] . [d, e] = [min (ad, ae, bd, be), max (ad, ae, bd, be)]
[a, b] ÷ [d, e] = [a, b] . $[\frac{1}{d}, \frac{1}{e}]$ provide [d, e]≠0 [2]

Let $\widetilde{A}$ and $\widetilde{B}$ denote fuzzy number and O denotes any of the basic four operations. Then we define a fuzzy set $\widetilde{A}\text{O}\widetilde{B}$ on R by introducing its α–cut as

$$(\tilde{A}\text{O}\widetilde{B})_\alpha=\tilde{A}_\alpha\text{O }\widetilde{B}_\alpha \quad \text{for all } \alpha \in (0, 1] \tag{1}$$

Obviously when O stend's for division, we require that O $\notin \widetilde{B}_\alpha$ for all $\alpha \in (0, 1]$

By first decomposition theorem

$$\widetilde{A}\,\text{O}\widetilde{B} = \cup\alpha\,. \quad (\widetilde{A}\ \text{O}\ \widetilde{B})_\alpha \quad - \tag{2}$$
$$\alpha \in [0, 1]$$

Since $\mu\,(\widetilde{A}\ \text{O}\ \widetilde{B})_\alpha$ is a closed interval and $\widetilde{A}$, $\widetilde{B}$ are fuzzy numbers, So $\widetilde{A}$ O $\widetilde{B}$ is also a fuzzy number.

Let us now consider two trapezoidal fuzzy numbers $\widetilde{A}$ = (–1, 1, 2, 4) and $\widetilde{B}$ = (1, 3, 4, 6) whose membership functions are given by

$$\mu\widetilde{A}\,(x) = \begin{cases} 0, & \text{for } x \le -1,\ x > 4 \\ \frac{x+1}{2} & \text{for } -1 < x \le 1 \\ 1 & \text{for } 1 \le x \le 2 \\ \frac{4-x}{2} & \text{for } 2 < x \le 4 \end{cases}$$

$$\mu\widetilde{B}\,(x) = \begin{cases} 0, & \text{for } x \le 1,\ x > 6 \\ \frac{x-1}{2} & \text{for } 1 < x \le 3 \\ 1 & \text{for } 3 \le x \le 4 \\ \frac{6-x}{2} & \text{for } 4 < x \le 6 \end{cases}$$

Their α – cut are $\widetilde{A}_{\boldsymbol{\alpha}}$= [2$\alpha$– 1, 4 – 2$\alpha$], $\widetilde{B}_{\boldsymbol{\alpha}}$= [2$\alpha$+ 1, 6 – 2$\alpha$]

I. if O stands for + in equation (1) Then $(\widetilde{A} + \widetilde{B})_\alpha = \widetilde{A}_{\boldsymbol{\alpha}} + \widetilde{B}_{\boldsymbol{\alpha}}$
$= [2\alpha - 1,\ 4 - 2\alpha] + [2\alpha + 1,\ 6 - 2\alpha] = [4\alpha, 10 - 4\alpha]$

When α= 0, [0, 10] is the range. Hence the resulting trapezoidal fuzzy number is

$$\mu\widetilde{A}+\widetilde{B}(x) = \begin{cases} 0, & \text{for } x \le 0,\ x > 10 \\ \frac{x}{4} & \text{for } 0 < x \le 4 \\ 1 & \text{for } 4 \le x \le 6 \\ \frac{10-x}{4} & \text{for } 6 < x \le 10 \end{cases}$$

So $\widetilde{A} + \widetilde{B}$ = (0, 4, 6, 10) fig. (b)

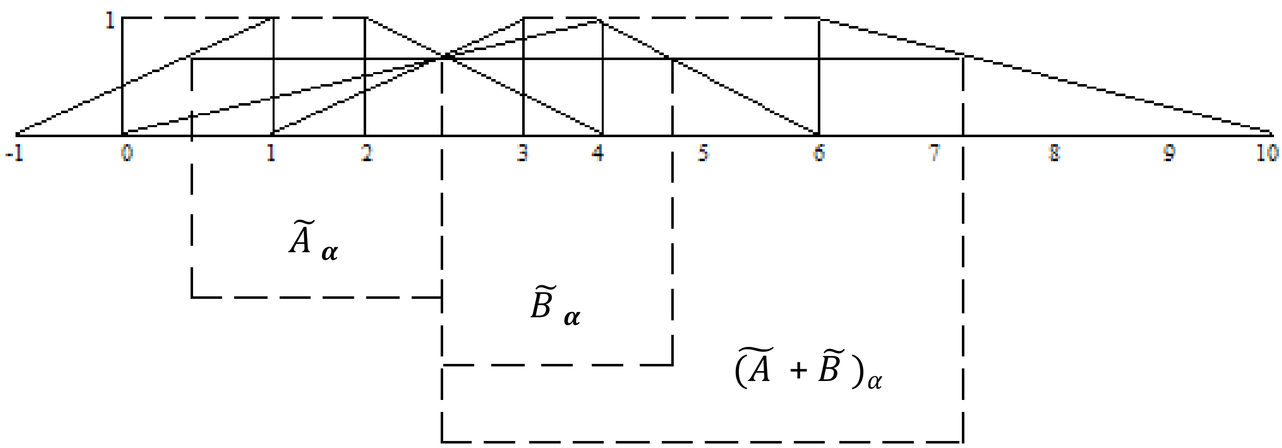


Figure: (b)

By equation (2) $(\widetilde{A} + \widetilde{B})_{\alpha}$ is the union of $\widetilde{A}_{\boldsymbol{\alpha}}$ and $\widetilde{B}_{\boldsymbol{\alpha}}$

II. If O stands for ' – ' in equation (1), Then $(\widetilde{A} - \widetilde{B})_{\alpha} = \widetilde{A}_{\boldsymbol{\alpha}} - \widetilde{B}_{\boldsymbol{\alpha}} = [4\alpha - 7,\ 3 - 4\alpha]$,

When $\alpha = 0$, [-7, 3] is the range, So the resulting trapezoidal fuzzy number is given by

$$\mu_{\widetilde{A} - \widetilde{B}}(x) = \begin{cases} 0, & \text{for } x \le -7,\ x > 3 \\ \frac{x+7}{4} & \text{for } -7 < x \le -3 \\ 1 & \text{for } -3 \le x \le -1 \\ \frac{3-x}{4} & \text{for } -1 < x \le 3 \end{cases}$$

So $\widetilde{A} - \widetilde{B} = (-7,\ -3,\ -1,\ 3)$ and $(\widetilde{A} - \widetilde{B})_{\alpha}$ is the union of $\widetilde{A}_{\boldsymbol{\alpha}}$ and $\widetilde{B}_{\boldsymbol{\alpha}}$

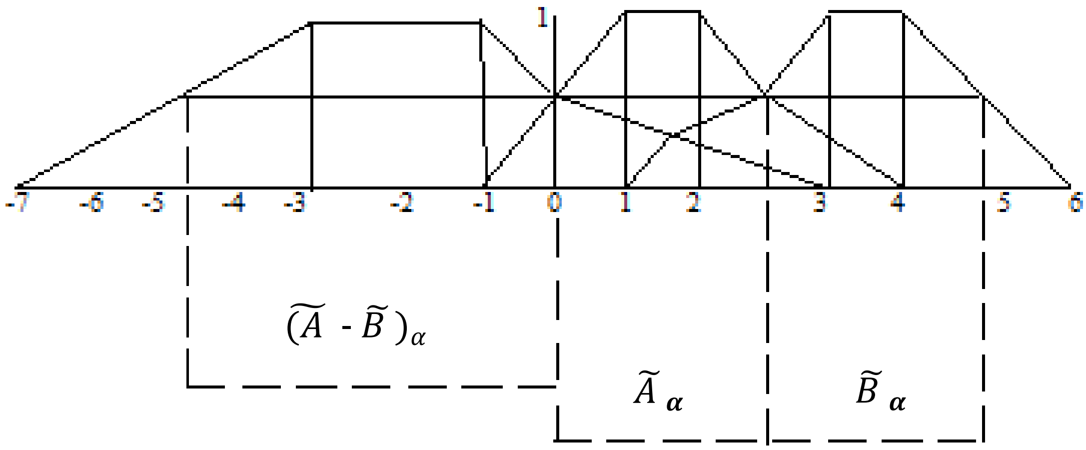


Figure: (c)

III. If O stands for .in (1), we have

$$\begin{aligned} (\widetilde{A} . \widetilde{B})_{\alpha} = \widetilde{A}_{\alpha} . \widetilde{B}_{\alpha} &= [2\alpha - 1,\ 4 - 2\alpha]\,[2\alpha + 1,\ 6 - 2\alpha] \\ &= [\min(4\alpha^2 - 1, -4\alpha^2 + 14\alpha - 6, -4\alpha^2 + 6\alpha + 4, 4\alpha^2 - 20\alpha + 24), \\ &\quad \max(4\alpha^2 - 1, -4\alpha^2 + 14\alpha - 6, -4\alpha^2 + 6\alpha + 4, 4\alpha^2 - 20\alpha + 24)] \\ &= [-4\alpha^2 + 14\alpha - 6,\ 4\alpha^2 - 20\alpha + 24] \text{ for } \alpha \in (0, .5] \end{aligned}$$

and $\qquad = [4\alpha^2 - 1,\ 4\alpha^2 - 20\alpha + 24]$ for $\alpha \in (.5,\ 1]$

when $\alpha = 0$, $[-6, 24]$ and $[-1,\ 24]$ are ranges of $(\widetilde{A} . \widetilde{B})_{\alpha}$

But $[-1, 24] \subset [-6,\ 24]$ So we shall take $[-6,\ 24]$ as range and hence
$(\widetilde{A} . \widetilde{B})_{\alpha} = [-4\alpha^2 + 14\alpha - 6,\ \ 4\alpha^2 - 20\alpha + 24]$

So the resulting trapezoidal fuzzy number has the membership function.

$$\mu_{\widetilde{A} . \widetilde{B}}(x) = \begin{cases} 0, & \text{for } x \le -6,\ x > 24 \\ \frac{\sqrt{1+x}}{2} & \text{for } -6 < x \le 3 \\ 1 & \text{for } 3 \le x \le 4 \\ \frac{7 - \sqrt{25 - 4x}}{4} & \text{for } 4 < x \le 24 \end{cases}$$

This is found by obtaining value of α when both, $4\alpha^2 - 1$ and $-4\alpha^2 + 14\alpha - 6$ are put equal to x

So $\widetilde{A} . \widetilde{B} = (-6, 3, 4, 24)$ is the trapezoidal fuzzy number figure (d)

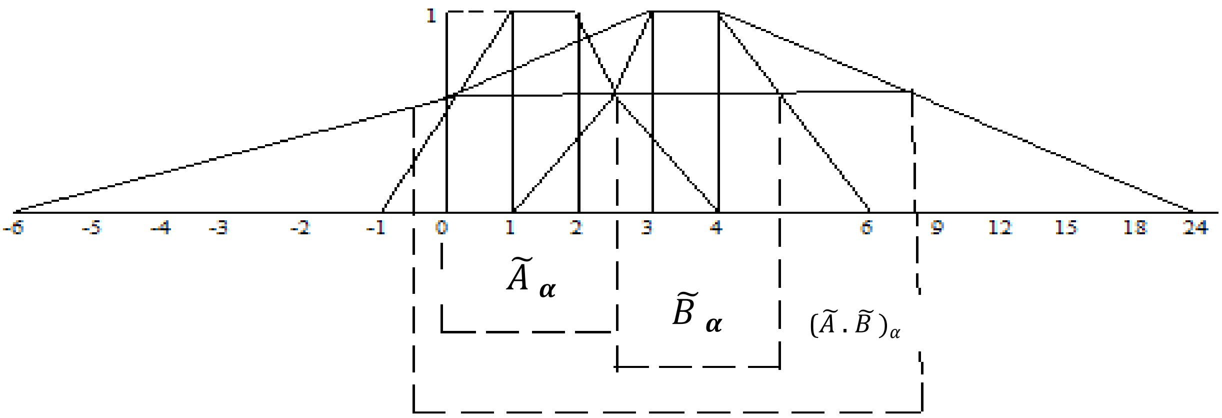


Figure: (d)

$(\widetilde{A}\,.\,\widetilde{B}\,)_{\alpha}$ is the union of $\widetilde{A}_{\alpha}$ and $\widetilde{B}_{\alpha}$

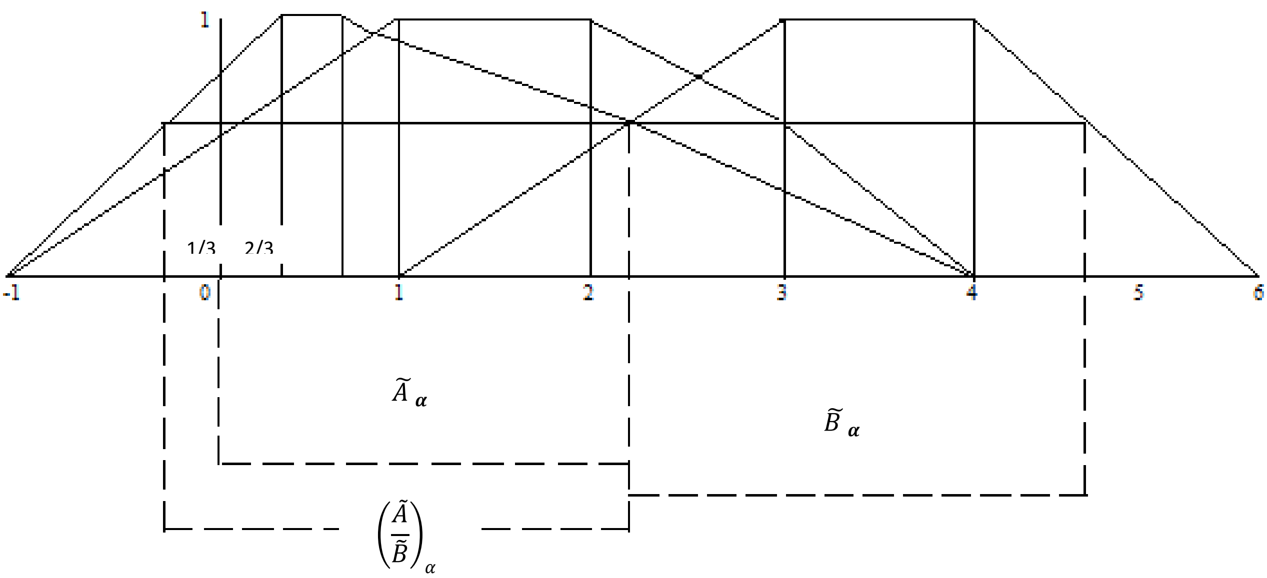
IV. If O stand s for ÷ in (1), we have

$$\left(\frac{A}{B}\right)_{\alpha} = \frac{\tilde{A}_{\alpha}}{\tilde{B}_{\alpha}} = [2\alpha - 1, 4 - 2\alpha]\,.[\frac{1}{2\alpha+1}, \frac{1}{6-2\alpha}]$$
$$= [\min(\frac{2\alpha-1}{2\alpha+1}, \frac{2\alpha-1}{6-2\alpha}, \frac{4-2\alpha}{2\alpha+1}, \frac{4-2\alpha}{6-2\alpha}), \max(\text{same})]$$
$$= [\frac{2\alpha-1}{2\alpha+1}, \frac{4-2\alpha}{2\alpha+1}] \text{ for } \alpha \in (0, .5]$$
and $$= [\frac{2\alpha-1}{6-2\alpha}, \frac{4-2\alpha}{2\alpha+1}] \text{ for } \alpha \in (.5, 1]$$

When $\alpha = 0$, $[-1, 4]$ and $[-\frac{1}{6}, 4]$ are ranges we take $[-1, 4]$ as it is super, So the resulting trapezoidal fuzzy numbers has the grade.

$$\mu\,\frac{\tilde{A}}{\tilde{B}}(x) = \begin{cases} 0, & \text{for } x \le -1, x > 4 \\ \frac{1+x}{2-2x} & \text{for } -1 < x \le \frac{1}{3} \\ 1 & \text{for } \frac{1}{3} \le x \le \frac{2}{3} \\ \frac{4-x}{2+2x} & \text{for } \frac{2}{3} < x \le 4 \end{cases}$$

So $\frac{\tilde{A}}{\tilde{B}} = (-1, \frac{1}{3}, \frac{2}{3}, 4)$ figure (e)



$\left(\frac{\tilde{A}}{\tilde{B}}\right)_{\alpha}$ is the union of $\widetilde{A}_{\alpha}$ and $\widetilde{B}_{\alpha}$

Justification: From the figure (a), (b), (c), (d), (e) we observe that each fuzzy number is a convex normalised fuzzy set.

REFERENCES

1. L.A. zadeh (1965) : ‘ Fuzzy Sets’, Information and Control 8(3) : pp 338-353
2. G.J. Klir and Bo Yuan : ‘ Fuzzy Sets and fuzzy logic, Theory and Information ’ Prentice Hall of India Private Limited, New Delhi (2006)
3. C. Jaisankar and S. Arunavasan: ‘Adjoint of Triangular fuzzy number matrices’ International J of Mathematics Archive 9(2) – 2018 pp 72-78
4. R.K. Das and SanjeevKumar: ‘On intuitionistic triangular fuzzy numbers’ South East Asian J of Mathematics and Mathematical Sciences, Vol. – 15 No. – 1 (2019) pp 63-70
5. H.J. Zimmermann: ‘Fuzzy Set theory and its application (Second Edition) ’ Kluwer Academic Publishers, New Delhi (1996)
6. Shiv Raj Singh : ‘Fuzzy Set Theory ’ Third Edition (2014), Krishna Prakashan Media, Private limited, Meerut (India) pp 35-56
7. S. Pundir and R. Pundir: ‘Fuzzy Sets and their application, Pragati Prakashan, Fourth Edition (2012) ’

Source of support: Nil, Conflict of interest: None Declared.