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ON NORMAL VAGUE ORDERED GAMMA-NEAR RINGS

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ABSTRACT

I*n this paper, we originate and inspect the notion of Normal vague Ordered Gamma- Near rings and it's properties. Also, we develop some relations on normal vague sets like* A^{+}, B^{+} *Also, we prove that for a given vague set we can construct a normal vague set which contains the given vague set.*

Keywords: *Vague set, Vague Ordered* $\Gamma-$*Near ring, Normal Vague Ordered* $\Gamma-$ *Near ring.*

Mathematics Subject Classification: *08A72, 20N25, 03E72.*

1. INTRODUCTION

The theory of fuzzy $\Gamma-$Near rings has been developed by Bh. Satyanarayana [1] and G.L. Booth. Later W.L.Gau and D.J. Buehrer [19] introduced the theory of vague sets as an improvement of the theory of fuzzy sets in approximating the real life situation. Vague sets are higher order fuzzy sets. According to them a vague set A in the universe of discourse U is a pair (t_A, f_A), where t_A *and* f_A are fuzzy subsets of U satisfying $t_A(u)+f_A(u)\le 1, \forall u \in U$. Later several authors had studied and applied fuzzy sets and vague sets on various algebraic structures like semi rings, semi groups,…etc. Further, by taking, the above as origin S.Ragamayi [18] has developed Vague Normal $\Gamma-$ Near rings in her doctorial thesis.

Furthermore K. Balakoteswara rao [2] had introduced and applied the concepts of Fuzzy sets and vague sets on "Ordered $\Gamma-$Near ring". As a sequel of our above work, now we introduce the structure of Normal vague Ordered $\Gamma-$ Near rings. Also, we prove that for a given vague set we can construct a normal vague set which contains the given vague set. Also we prove that a non – constant maximal element in the set of all Normal vague Ordered $\Gamma-$Near rings of an ordered $\Gamma-$Near ring takes only two vague values $[0,0]$ and [0, 1].

2. PRELIMINARIES

Definition 2.1: A zero – symmetric $\Gamma-$Near ring is a trip [*ie* $(M,+,\Gamma)$, where

1. $(M,+)$ is a group
2. $\Gamma-$ is a non – empty set of binary operators on M such that for each $\alpha \in \Gamma, (M,+,\alpha)$ is a near ring.
3. $x \in (y\beta z) = (xay)\beta z,$ for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$.
4. $x\alpha 0 = 0$ for every $x \in M, \alpha \in \Gamma$.

Definition 2.2: Let M be a $\Gamma-$ Near ring and 'A' be a nonempty of M then A is said to be sub $\Gamma-$Near ring if
1. $x - y \in A$
2. $x\alpha y \in A$, for each $\alpha \in \Gamma$ and $x, y \in M$.

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Definition 2.3: Let M be a $\Gamma-$Near ring and μ be a Fuzzy subset of M. We say that μ is a Fuzzy sub $\Gamma-$Near ring of M If

1. $\mu(x-y)\geq\min\{\mu(x),\mu(y)\}$
2. $\mu(x\alpha y)\geq\min\{\mu(x),\mu(y)\}$ for $x,y\in M,\alpha\in\Gamma$

Definition 2.4: A vague set A in the universe of discourse U is a pair (t_A, f_A),
where $t_A:U\to[0,1], f_A:U\to[0,1]$ are mappings such that $t_A(u)+f_A(u)\leq 1,\forall u\in U$. The functions t_A *and* f_A are called true membership function and false membership function respectively.

Definition 2.5: The interval $\left[t_A(\mu),1-f_A(u)\right]$ is called the vague value of u in A and it is denoted by $V_A(\mu)$ i.e., $V_A(\mu)=\left[t_A(u),1-f_A(u)\right]$

Definition 2.6: A vague sets A is contained order vague set $B, A\subseteq B$ if and only if $V_A(u)\leq V_B(u)$ i.e., $t_A(u)\leq t_B(u)$ and $1-f_A(u)\leq 1-f_B(u),\ni u\in U$

Definition 2.7: Two vague sets A and B are equal written as $A=B.$ If and only if $A\subseteq B$ and $B\subseteq A$
i.e., $V_A(u)\leq V_B(u)$ and $V_B(u)\leq V_A(u),\forall u\in U$

Definition 2.8: The union of two vague sets A and B with respect ive truth member ship and false membership function $t_A, f_A : t_B, f_B$ is a vague set S, written as $C=A\cup B$, whose truth membership and false membership functions are related to those of A and B by $t_C=\max\{t_A,t_B\}$;$1-f_C=\max\{1-f_A,1-f_B\}=1-\min\{f_A,f_B\}$.

Definition 2.9: The intersection of two vague sets A and B with respective truth membership and false membership functions $t_A, f_A : t_B, f_B$ is a vague set C, written as $C=A\cap B$, whose truth membership and false membership functions are related to those of A and B by $t_C=\min\{t_A,t_B\}$;$1-f_C=\min\{1-f_A,1-f_B\}=1-\max\{f_A,f_B\}$.

Definition 2.10: The union and intersection of a family $\{A_i / i\in\Delta\}$ of vague sets of a set U are defined by

$$V\bigcup_{i\in\Delta}A_i(u)=\sup_{i\in\Delta}V_{A_i}(u),\forall u\in U$$

$$V\bigcap_{i\in\Delta}A_i(u)=\inf_{i\in\Delta}V_{A_i}(u),\forall u\in U$$

Definition 2.11: A vague set A of a set U with $t_A(u)=0$ and $f_A(u)=1,\forall u\in U$ is called zero vague set of U.

Definition 2.12: A vague set A of set U with $t_A(u)=1$ and $f_A(u)=0,\forall u\in U$ is called unit vague set of U.

Definition 2.13: Let A be a vague set of a universe U with true membership function t_A and false membership function f_A. For $\alpha,\beta\in[0.1]$ with $\alpha\leq\beta$, the (α,β) cut or vague cut of a vague set A is the crisp subset of U is given by

$$A_{(\alpha,\beta)}=\{x\in U / V_A(x)\geq[\alpha,\beta]\}$$

i.e., $$A_{(\alpha,\beta)}=\{x\in U / t_A(x)\geq\alpha \text{ and } 1-f_A(x)\geq\beta\}$$

Definition 2.14: The $\alpha-$ cut, A_α of the vague set A is the (α,α) - cut of A and hence given by $A_\alpha=\{x\in U / t_A(x)\geq\alpha\}$.

Definition 2.15: A vague set $A=(t_A, f_A)$ of M is said to be vague $\Gamma-$ Near ring if the following conditions are true, for all

$p,q\in M;\ \alpha\in\Gamma$

$V_A(p-q)\geq\min\{V_A(p),V_A(q)\}$ *and*

$V_A(p\alpha q)\geq\min\{V_A(p),V_A(q)\}$

i.e.,

$(i).\ t_A(p-q) \geq \min\{t_A(p), t_A(q)\}$

$1-f_A(p-q) \geq \min\{1-f_A(p), 1-f_A(q)\}$ *and*

$1-f_A(p\alpha q) \geq \min\{1-f_A(p), f_A(q)\}.$

Definition 2.16: A $\Gamma-$ Near ring M is called an ordered $\Gamma-$ Near ring if it admits a compatible relation $"\leq": i.e.\ "\lesssim"$ is a partial ordering on M satisfies the following conditions. If $a \leq b$ and $c \leq d$ then

$(i)\ a+c \leq b+d$

$(ii)\ a\gamma c \leq b\gamma d$

$(iii)\ c\gamma a \leq d\gamma b;\ \text{for all } a,b,c,d \in M;\ \gamma \in \Gamma$

Definition 2.17: Let M be a partially Ordered $\Gamma-$ Near ring and A be a nonempty subset of M then A is said to be sub Ordered $\Gamma-$ Near ring if

$i)\ x-y \in A$

$ii)\ x\gamma y \in A\ \text{ for each } \gamma \in \Gamma \text{ and } x, y \in M$

$iii)\ x \leq y \text{ then } x+g \leq y+g, \forall x, y, g \in M$

$iv)\ x \leq y \text{ and } c \geq 0 \text{ then } y\gamma c \text{ and } c\gamma c \text{ and } c\gamma x \leq c\gamma y,\ \forall x, y, c \in M$

Definition 2.18: Let M be an Ordered Near ring and μ be a Fuzzy subset of M. We say that μ is an Fuzzy sub Ordered Near ring Of M if

1) $\mu(x-y) \geq \min\{\mu(x), \mu(y)\}$

2) $\mu(x\gamma y) \geq \min\{\mu(x), \mu(y)\}$

3) $x \leq y \Rightarrow \mu(x) \geq \mu(y)\ \text{ for all } x, y \in M, \gamma \in \Gamma$

Notations: Throught out the following section, we use the following notations.

1. **OGNR** stands for Ordered $\Gamma-$ Near ring.
2. **M** stands for Zero – Symmetric Ordered $\Gamma-$ Near ring.
3. **GNR** stands for $\Gamma-$ Near ring.

3. NORMAL VAGUE ORDERED $\Gamma-$NEAR RING

In this section, we introduce and study the concept of normal vague OGNRs and prove that for a given vague set we can construct a normal vague set which contains the given vague set. Also we prove that a non – constant maximal element in the $[1,1]$.

Now, we introduce the following

Definition 3.1: A vague set $A = (t_A, f_A)$ of M is said to be normal, if

$$V_A(0) = [1,1]\ i.e., t_A(0) = 1 \text{ and } 1-f_A(0) = 1.$$

The following theorem, gives necessary condition for a vague set to be normal vague set

Theorem 3.2: *Let* $A = (t_A, f_A)$ be a vague set of M such that

$t_A(p) + f_A(p) \leq t_A(0) + f_A(0), \forall p \in M$. Define $A^+ = (t_{A^+}, f_{A^+})$, where

$t_A + (p) = t_A(p) + 1 - t_A(0)$ *and*

$f_A + (p) = f_A(p) - f_A(0), \forall p \in M$. *Then* A^+ *is a normal vague set.*

Proof: First we show that A^+ is vague set.

Let $p \in M$

Now, $t_A + (p) + f_A + (p) = t_A(p) + 1 - t_A(0) + f_A(p) - f_A(0) \leq 1$

Thus A^{+} *is a vague set*

Also $t_A+(0)=1$ *and* $f_A+(0)=0$

Hence A^{+} is a normal vague set.

Now, we prove the following theorem.

Theorem 3.3: Let $A=(t_A, f_A)$ be a vague OGNR of M. Then the vague set A^{+} is a normal vague OGNR of M, containing A.

Proof: Let $p,q \in M$; $\gamma_1 \in \Gamma$

Now,

1) $V_A+(p-q)=V_A(p-q)+[1,1]-V_A(0)$

$\geq \min\{V_A(p),V_A(q)\}+[1,1]-V_A(0)$

$=\min\{V_A(p)+[1,1]-V_A(0),V_A(q)+[1,1]-V_A(0)\}$

$=\min\{V_A+(p),V_A+(q)\}$

2) $V_A+(p\gamma_1 q)=V_A(p\gamma_1 q)+[1,1]-V_A(0)$

$\geq V_A(p)+[1,1]-V_A(0)$

$=V_A+(p)$

For every $p,q \in M$, if $p \leq q$ then

$V_A^{+}(p)=V_A(p)\vee V_A(0)'$

$\geq V_A(q)\vee V_A(0)'$

$=V_A^{+}(q)$

Also $V_{A^{+}}(0)=V_A(0)+[1,1]-V_A(0)=[1,1]$

Thus A^{+} is a normal vague OGNR of M.

Clearly $A \subset A^{+}$

Corollary 3.4: If A is a vague OGNR of M satisfying $V_A+(p)=[0,0]$, for some $p \in M$. then $V_A(p)=[0,0].\sqrt{a^2+b^2}$

Theorem 3.5: Let $A \in N(M)$ be a non-constant maximal element of $(N(M),\subseteq)$ (where $N(M)$ is a set of normal vague OGNRs of M) . Then A takes only two vague values [0, 0], [1,1].

Proof: Let $A=(t_A, f_A)$ be a normal vague OGNR of M. Then $V_A(0)$ = [1,1].

Let p belongs to M.

Suppose that $V_A(p)\neq[1,1]$.

We have to show that $V_A(p)=[0,0]$.

Assume that there exists $p_0 \in M$ such that $[0,0]<V_A(p_0)<[1,1]$

Define a vague set $B=(t_B, f_B)$ *on M* by $V_B p=\dfrac{V_A(p)+V_A(p_0)}{2}, \forall p \in M$.

i.e., $t_B(p)=\dfrac{t_A(p)+t_A(p_0)}{2}$ *and* $f_B(p)=\dfrac{f_A(p)+f_A(p_0)}{2}, p \in M$

Let $p,q \in M;\gamma_1 \in \Gamma$

Then

$$1.V_B(p-q)=\frac{V_A(p-q)+V_A(p_0)}{2}$$
$$\geq\frac{\min\{V_A(p).V_A(q)\}+V_A(p_0)}{2}$$
$$=\min\left\{\frac{V_A(p)+V_A(p_0)}{2},\frac{V_A(q)+V_A(p_0)}{2}\right\}$$
$$=\min\{V_B(p),V_A(q)\}$$

$$2.V_B(p\gamma_1 q)=\frac{V_A(p)+V_A(p_0)}{2}$$
$$\geq\frac{V_A(p)+V_A(p_0)}{2}$$
$$\geq V_B(p)$$

$$3.\textit{For } p\leq q \textit{ then } V_B(p)=\frac{V_A(p)+V_A(p_0)}{2}$$
$$\geq\frac{V_A(q)+V_A(p_0)}{2}$$
$$=V_B(q)$$

Thus B is a vague OGNR of M.

Now,

$$V_B+(p)=V_B(p)+[1,1]-V_B(0)$$
$$=\frac{V_A(p)+V_A(p_0)}{2}+[1,1]-\frac{V_A(0)+V_A(p_0)}{2}$$
$$=\frac{V_A(p)+[1,1]}{2}$$

$$\textit{That implies } V_B+(0)=\frac{V_A(0)+[1,1]}{2}=[1,1]$$

Thus B^+ is a normal vague OGNR of M.

Now, $V_{B^+}(0)=[1,1]>V_A(p_0)$

So, B^+ is a non-constant normal vague Ordered Gamma – Near ring of M and hence $B\in N(M)$.

Further, we have $V_B(p_0)>V_A(p_0)$ it gives contradiction for A is maximal.

Hence $V_A(p)=[0,0]$.

Thus A takes only two vague values $[0,0]$ and $[1,1]$

4. CONCLUSION

In this research article, we inspected the idea of Normal vague Ordered GNR.

Also, we proved some interesting results on it. We proved that a non – constant maximal element in the set of all Normal vague Ordered $\Gamma-$Near rings of an ordered $\Gamma-$Near ring takes only two vague values $[0,0]$ and [0,1].

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