

**PART I KALANGI NON-ASSOCIATIVE  $\Gamma$ -SEMI SUB NEAR-FIELD SPACE  
OF A  $\Gamma$ -NEAR-FIELD SPACE OVER NEAR-FIELD**

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**ABSTRACT**

*In this manuscript we introduce new notions on PART I Kalangi non-associated  $\Gamma$ -semi sub near-field space of a  $\Gamma$ -near-field space over near-field, quasi non associative  $\Gamma$ -semi sub near-field space, K-quasi N - $\Gamma$ -semi sub near-field space, quasi ideals, etc and concepts like PART I Kalangi quasi bipotent elements and several analogous properties done in case of  $\Gamma$ -near-field spaces.*

**Keywords:** *Non-associative  $\Gamma$ -semi sub near-field space, Kalangi- $\Gamma$ -semi sub near-field space,  $\Gamma$ -near-field space;  $\Gamma$ -Semi sub near-field space of  $\Gamma$ -near-field space; Semi near-field space of  $\Gamma$ -near-field space, quasi  $\Gamma$ -semi sub near-field space, quasi non-associative  $\Gamma$ -semi near-field space.*

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**SECTION 1: INTRODUCTION AND PRELIMINARIES**

In this paper we together introduced several concepts and new notions in PART I Kalangi non-associative  $\Gamma$ -semi sub near-field space of a  $\Gamma$ -near-field space over near-field like quasi non associative  $\Gamma$ -semi sub near-field space, K-quasi N - $\Gamma$ -semi sub near-field space, quasi ideals, etc and concepts like PART I Kalanga quasi bipotent elements and several analogous properties done in case of  $\Gamma$ -near-field spaces.

**Definition 1.1:** Let N be a K-quasi non-associative  $\Gamma$ -semi sub near-field space of a  $\Gamma$ -near-field space over near-field an element x is said to be quasi central if  $xy = yx$  for all  $y \in M$ ;  $M \subseteq N$  is a  $\Gamma$ -near-field (or  $M \subset N$  and M is a  $\Gamma$ -semi near-field space).

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**Definition 1.2:** Let  $N$  be a  $K$ -quasi non-associative  $\Gamma$ -semi sub near-field space of a  $\Gamma$ -near-field space over near-field we say  $N$  is said to be Kalanga quasi non-associative sub-directly irreducible  $\Gamma$ -semi sub near-field space ( $K$ -quasi sub-directly irreducible non-associative  $\Gamma$ -semi sub near-field space) if the intersection of all non zero  $K$ -quasi ideals of  $N$  is non-zero.

**Definition 1.3:** Let  $N$  be a  $K$ -quasi non-associative  $\Gamma$ -semi sub near-field space of a  $\Gamma$ -near-field space over near-field we say  $N$  is said to have Kalanga quasi intersection of factors property ( $K$ -quasi IFP) if  $a, b \in N$ ,  $ab = 0$  implies  $amb = 0$  where  $m \in M$ ,  $M \subset N$  and  $M$  is a near-field ( or  $m \in M$ ,  $M \subset N$ ,  $M$  is a non-associative  $\Gamma$ -semi sub near-field space).

**Note 1.4:** We can take  $a(nb) = (an)b$  in all cases it should vanish that is  $anb = 0$ .

Now we define the concept of Kalangi quasi divisibility and Kalangi divisibility.

**Definition 1.5:** Let  $N$  be a non-associative  $K$ - $\Gamma$ -semi sub near-field space of a  $\Gamma$ -near-field space over near-field we say  $N$  is Kalangi weakly divisible ( $K$ -weakly divisible) if for all  $x, y \in N$  there exists a  $z \in P$ ;  $P \subset N$  where  $P$  is an associative  $\Gamma$ -semi sub near-field space of a  $\Gamma$ -near-field space over near-field or  $P$  is a near-field such that  $xz = y$  or  $zx = y$ .

**Definition 1.6:** Let  $N$  be a non-associative  $K$ - $\Gamma$ -semi near-field space of a  $\Gamma$ -near-field space over near-field we say  $N$  is Kalangi weakly divisible ( $K$ -weakly divisible) if for all  $x, y \in N$  there exists a  $z \in P$ ;  $P \subset N$  where  $P$  is an associative  $\Gamma$ -semi near-field space of a  $\Gamma$ -near-field space over near-field or  $P$  is a near-field such that  $xz = y$  or  $zx = y$ .

**Definition 1.7:** Let  $N$  be a  $K$ -quasi  $\Gamma$ -semi sub near-field space ( $K$ -quasi  $\Gamma$ -semi near-field space) of a  $\Gamma$ -near-field space over near-field. We say  $N$  is Kalanga quasi weakly divisible ( $K$ -quasi weakly divisible) if for all  $x, y \in N$  there exists  $z \in M$ ;  $M$  is a  $\Gamma$ -semi near-field space  $\subset N$  ( $M$  is a  $\Gamma$ -semi near-field space  $M \subset N$ ) such that  $xz = y$  or  $yz = x$ .

**Definition 1.8:** Let  $N$  be a  $K$ - $\Gamma$ -semi sub near-field space ( $K$ - $\Gamma$ - semi near-field spaces) we say  $N$  is said to be a Kalanga strongly prime ( $K$ -strongly prime) I if for each  $a \in N \setminus \{0\}$  there exists a finite  $K$ - $\Gamma$ -semi sub near-field space  $F$  such that  $aFx \neq 0$  for all  $x \in P \setminus \{0\}$   $P \subset N$ ;  $P$  is a associative  $\Gamma$ -semi sub near-field space /  $P$  is a associative  $\Gamma$ -semi near-field space.

In case  $N$  is  $K$ - $\Gamma$ -semi sub near-field space II (III or IV) we say  $N$  is a Kalangi strong prime II (III or IV) ( $K$ -strong prime II(III or IV) if for each  $a \in N \setminus \{0\}$  there exists a finite  $K$ - $\Gamma$ - semi near-field space  $F$  such that  $aFx \neq 0$  for all  $x \in P \setminus \{0\}$ ,  $P \subset N$ ;  $P$  is a associative  $\Gamma$ - semi near-field space/  $P$  is associative  $\Gamma$ - semi near-field space.

**Definition 1.9:** Let  $N$  be a non associative right near-field space and  $A$  an  $K$ -ideal or a  $K$ -left ideal of  $N$ . we define three properties as follows

- (i)  $A$  is kalangi equi-prime ( $K$ -equi-prime) if for any  $a, x, y \in N$  such that  $a(nx) - a(ny) \in A \quad \forall n \in N$  or  $(an)x - (an)y \in A$  we have  $a \in A$  or  $x - y \in A$ .
- (ii)  $A$  is Kalangi strongly semi prime ( $K$ -strongly semi prime) if for each a finite subset  $F$  of  $N$  such that if  $x, y \in N$  and  $(af)x - (af)y \in A$  or  $a(fx) - a(fy) \in A$  or  $(af)x - a(fy) \in A$  or  $A(fx) - A(fy) \in A$  for all  $f \in F$  and  $x - y \in A$ .
- (iii)  $A$  is Kalangi completely equi prime ( $K$ -completely equi prime) if  $a \in N \setminus A$  and  $ax - ay \in A$  imply  $x - y \in A$ .

**Definition 1.10:** Let  $Q$  be a non empty subset of a  $K$ -right  $\Gamma$ -semi sub near-field space of a  $\Gamma$ -near-field space over near-field  $N$  which is non-associative. Define left and right Kalangi polar subsets ( $K$ -polar subsets) of  $N$  by

$$SL(Q) = \begin{cases} \{x/x(NQ) = 0 \text{ or} \\ (xN)Q = 0 \text{ for all } q \text{ in } Q\} \end{cases} \text{ and } SR(Q) = \begin{cases} \{y/(qN)y = 0 \text{ or} \\ q(Ny) = 0 \text{ for all } q \text{ in } Q\} \end{cases}$$

Suppose,  $SQ_L(N)$  is the set of  $Q$ -left polar subsets of  $N$  and  $SQ_R(N)$  is the set of  $Q$ -right polar subsets of  $N$  one need to test whether  $SQ_L(N)$  and  $SQ_R(N)$  are complete bounded lattices.

**Definition 1.11: I,II and III three levels of Kalangi  $\Gamma$ -semi sub pseudo near-field space( $K$ - $\Gamma$ -SSPNFS).** Let  $Q$  be a  $\Gamma$ -semi sub pseudo near-field space ( $\Gamma$ -SSPNFS) of a  $\Gamma$ -near-field space over near-field  $N$  we say  $Q$  is a Kalangi  $\Gamma$ -SSPNFS I ( $K$ - $\Gamma$ -SSPNFS I) if  $Q$  has a proper subset  $T \subset Q$  such that  $T$  is a  $\Gamma$ -semi sub near-field space. Kalangi  $\Gamma$ -semi sub pseudo near-field space II ( $K$ - $\Gamma$ -SSPNFS II) if  $Q$  has proper subset  $M \subset Q$  such that  $M$  is a  $\Gamma$ -semi sub near-field space. Kalangi  $\Gamma$ -semi sub pseudo near-field space III ( $K$ - $\Gamma$ -SSPNFS III) if  $Q$  has a proper  $W \subset Q$

such  $(W, \oplus, \otimes)$  is a  $\Gamma$ -semi near-field space. Thus we have three levels (I, II and III levels) of  $K$ - $\Gamma$ -SSPNFS near-field spaces over a near-field  $N$ . A Kalangi  $\Gamma$ -SSPNFS  $\Gamma$ -semi near-field space ( $K$ - $\Gamma$ -SSPNFS) is defined as a proper subset  $U$  of  $Q$  such that  $(U, \oplus, \otimes)$  is a  $K$ - $\Gamma$ -SSPNFS  $\Gamma$ -semi near-field space.

**Definition 1.12:** Let  $(Q, \oplus, \otimes)$  be a  $\Gamma$ -semi sub pseudo near-field space ( $\Gamma$ -SSPNFS) of a  $\Gamma$ -near-field space over near-field. A proper subset  $I$  of  $Q$  is called a Kalangi  $\Gamma$ -semi sub pseudo near-field space ideal ( $K$ - $\Gamma$ -SSPNFS-ideal) if

- a. for all  $p, q \in I$ ,  $p \oplus q \in I$
- b.  $0 \in I$
- c. for all  $p \in I$  and  $r \in P$  we have  $p \otimes r$  or  $r \otimes p \in I$ .
- d.  $I$  is a  $K$ - $\Gamma$ -SSPNFS,  $\Gamma$ -semi near-field space.

**Definition 1.13:** Let  $(N, \oplus, \otimes)$  be a quasi  $\Gamma$ -semi sub pseudo near-field space ( $\Gamma$ -SSPNFS) of a  $\Gamma$ -near-field space over near-field.  $M$  is said to be a Kalangi quasi  $\Gamma$ -semi sub pseudo near-field space ( $K$ - $\Gamma$ -SSPNFS) if and only if  $M$  is a  $K$ - $\Gamma$ -SSPNFS  $\Gamma$ -semi near-field space.

**Definition 1.14:** Let  $(N, \oplus, \otimes)$  and  $(N_1, \oplus, \otimes)$  be any two  $K$ - $\Gamma$ -semi sub pseudo near-field spaces ( $\Gamma$ -SSPNFS) of a  $\Gamma$ -near-field space over near-field. We say a map  $\phi$  is a Kalangi  $\Gamma$ -semi sub pseudo near-field space-homomorphism I (II or III) ( $K$ - $\Gamma$ -SSPNFS homomorphism I, II or III) if  $\phi : L \rightarrow L_1$  where  $L \subset N$  and  $L_1 \subset N_1$  are  $\Gamma$ -semi sub pseudo near-field spaces (or  $\Gamma$ -semi near-field space or  $\Gamma$ -semi near-field space) respectively and  $\phi$  is a  $\Gamma$ -semi near-field space homomorphism from  $L$  to  $L_1$  (or near-field homomorphism from  $L$  to  $L_1$  or semi near-field space homomorphism from  $L$  to  $L_1$ ).  $\phi$  need not be defined on the entire set  $N$  and  $N^1$  it is sufficient if it is well defined on  $L$  to  $L_1$ .

## SECTION 2: MAIN RESULT ON KALANGI -QUASI GAMMA SEMI PSEUDO SUB NEAR-FIELD SPACES OF A GAMMA NEAR-FIELD SPACE OVER A NEAR-FIELD.

In this section, author present theorem as main result on Kalangi quasi Gamma semi pseudo sub near-field spaces of a Gamma near-field space over a near-field.

Now we proceed on to define Kalangi right quasi regular element. We just recall that an element  $x \in N$ ,  $N$  is a Gamma semi pseudo sub near-field space said to be the right quasi regular if there exist  $y \in N$  such that  $x \circ y = x + y - xy = 0$  and left quasi regular if there exist  $y^l \in N$  such that  $y^l \circ x = 0 = y^l + x - y^l x$ .

The study of the quasi regular concept happens to be an interesting study in case of near-field spaces and semi near-field spaces.

Quasi regular if it is right and left quasi regular simultaneously. We say an element  $x \in N$  is Kalangi right quasi regular ( $K$ -right quasi regular) if there exist  $y$  and  $z \in N$  such that  $x \circ y = x + y - xy$ ,  $x \circ z = x + z - xz = 0$  but  $y \circ z = y + z - yz \neq 0$  and  $z \circ y = y + z - zy \neq 0$ .

Similarly we define Kalangi left quasi regular ( $K$ -left quasi regular) and  $x$  will be Kalangi quasi regular ( $K$ -quasi regular) if it is simultaneously  $K$ -right quasi regular and  $K$ -left quasi regular, that is of Kalangi quasi Gamma semi pseudo sub near-field spaces of a Gamma near-field space over a near-field.

If we define  $K$ -non-associative  $\Gamma$ -semi pseudo sub near-field space of a  $\Gamma$ -near-field space over near-field ( $K$ -quasi  $\Gamma$ -semi pseudo sub near-field space)  $N$  then we have main interesting result out of several results below.

**Theorem 2.1:** Let  $N$  be a Kalangi quasi  $\Gamma$ -semi pseudo sub near-field space of a  $\Gamma$ -near-field space over near-field ( $K$ -quasi  $\Gamma$ -semi pseudo sub near-field space) having a proper subset  $P$  of  $N$  to be a commutative near-field space with unit and of a characteristic 0.  $L$  any loop of finite order. Then the near loop near-field space  $NL$  has a right quasi regular element  $x = \sum \alpha_i m_i$  ( $m_i \in L$ )  $\alpha_i \in P \subset N$  is right quasi regular then  $\sum \alpha_i \neq 1$ .

**Proof:** Let  $y = \sum \beta_i h_j$  where  $\beta_i \in P$  and  $h_j \in L$  be the right quasi inverse of  $x$  then  $x + y - xy = 0$  i.e.,  $\sum \alpha_i m_i + \sum \beta_i h_j - (xy) = 0$ .

Equating the coefficients of the like terms and adding these coefficients we get,

$\sum \alpha_i + \sum \beta_i - \sum \alpha_i \sum \beta_i = 0$ . Or  $\sum \alpha_i = \sum \beta_i - \sum \alpha_i \sum \beta_i = \sum \beta_i (\sum \alpha_i - 1)$ . Now if  $\sum \alpha_i = 1$  then  $\sum \alpha_i = 0$  a contradiction. Hence  $\sum \alpha_i \neq 0$ . This completes the proof of the theorem.

**Example 2.2:** Let  $L$  be any finite loop.  $N = Z_9 \times Z_7$  be  $K$ -mixed direct product of the Kalangi quasi  $\Gamma$ -semi pseudo sub near-field space of a  $\Gamma$ -near-field space over near-field ( $K$ -quasi  $\Gamma$ -semi pseudo sub near-field space)  $Z_9$  and the prime field of characterize 7,  $Z_7$ ,  $N$  is  $K$ -quasi  $\Gamma$ -semi pseudo sub near-field space.  $NL$  is the near loop near-field of the loop  $L$  over the near-field space  $N$ . If  $x \in S$  ( $J(Z_7L)$ ).

**Definition 2.3:** Let  $N = N_1 \times N_2$  where  $N_1$  is a Kalangi quasi  $-\Gamma$ -semi pseudo sub near-field space of a  $\Gamma$ -near-field space over near-field (K-quasi  $\Gamma$ -semi pseudo sub near-field space) of characterize 0 and  $N_2$  is any quasi  $-\Gamma$ -semi pseudo sub near-field space of a  $\Gamma$ -near-field space over near-field.  $NL$  be the near loop quasi  $-\Gamma$ -semi pseudo sub near-field space of a  $\Gamma$ -near-field space over near-field of the loop  $L$  over the quasi  $-\Gamma$ -semi pseudo sub near-field space of a  $\Gamma$ -near-field space  $N$ .

**Definition 2.4:**  $QJ(Q)$  said to be the Kalangi Jacobson radical (K-Jacobson radical) of  $NL$  if  $Q \subset NL$  is a non associative quasi  $-\Gamma$ -semi pseudo sub near-field space of a  $\Gamma$ -near-field space and  $J(Q)$  denoted the usual Jacobson radical of the non-associative quasi  $-\Gamma$ -semi pseudo sub near-field space of a  $\Gamma$ -near-field space  $Q$ .

**Example 2.5:** Let  $N = Z \times Z_{18}$  be the mixed direct product of the Kalangi quasi  $-\Gamma$ -semi pseudo sub near-field space of a  $\Gamma$ -near-field space over near-field (K-quasi  $\Gamma$ -semi pseudo sub near-field space)  $Z$  and the  $-\Gamma$ -semi pseudo sub near-field space  $Z_{18}$   $L$  any finite loop,  $NL$  the near loop of the loop  $L$  over the Kalangi quasi  $-\Gamma$ -semi pseudo sub near-field space  $N$ . clearly  $ZL \subset NL$  and  $ZL$  is a non-associative Kalangi quasi  $-\Gamma$ -semi pseudo sub near-field space  $N$ . If  $x = \sum \alpha_i h_i \in ZL$  such that  $\sum \alpha_i \neq 0$  then  $x \notin QJ(ZL)$ . It is left for the scholar or reader to verify, as the conclusion derived is straightforward.

**Theorem 2.6:** Let  $N = Z_2 \times Z_{15}$  where  $Z_2$  is the prime Kalangi quasi  $-\Gamma$ -semi pseudo sub near-field space of a  $\Gamma$ -near-field space over near-field (K-quasi  $\Gamma$ -semi pseudo sub near-field space) of characterize two and  $Z_{15}$  is a  $-\Gamma$ -semi pseudo sub near-field space of a  $\Gamma$ -near-field space over near-field. Let  $L$  be any loop.  $NL$  be the near loop Kalangi quasi  $-\Gamma$ -semi pseudo sub near-field space. If  $x \in Z_2 L \times \{0\} \subset (Z_2 \times Z_{15})$ ;  $L$  is right quasi regular Kalangi quasi  $-\Gamma$ -semi pseudo sub near-field space then  $|\text{supp } x|$  is an even number.

**Proof:** The proof is obvious and easily obtained by simple calculations.

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