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THE CONCEPT OF $g^*\beta$ – CLOSED SETS IN TOPOLOGICAL SPACES

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ABSTRACT

The aim of this paper is to introduce and study new class of sets called $g^*\beta$ – closed sets. This new class of sets lies between closed sets and βg-closed sets. Applying these sets, new spaces namely, ${}_{\beta}T^{**}_{1/2}$- space, ${}_{\alpha\beta}T^{*}_{c}$ – space, ${}_{\beta}{}^{*}T^{*}_{1/2}$ – space, ${}_{\beta}{}^{**}T_{1/2}$ – space, ${}_{\beta}T^{*}_{c}$ – spaces are introduced.

Key words*: $g^*\beta$ – irresolute map, ${}_{\beta}T^{**}_{1/2}$- space, ${}_{\alpha\beta}T^{*}_{c}$ – space.*

INTRODUCTION

The Kutatowiski closure axioms or axioms for the closed sets are used to define every topological spaces. Hence we can understand how the important the concept of closed sets in the topological Spaces. In 1970, Levine [11] initiated the study of g – closed sets. Maki. *et.al* [14] defined αg – closed sets and α**g – closed sets in 1994. S.P. Arya and T. Nour [3] defined gs – closed sets in 1990. Dontchev [9], Gnanambal [10] and Palaniappan and Rao [19] introduced gsp – closed sets, gpr – closed sets, and rg- closed sets respectively. M.K.R.S. Veerakumar [20] introduced g* - closed sets in 1991. P.M. Helen [22] introduced g** - closed sets. We introduce a new class of sets called g*β-closed sets which is properly placed in between the class of closed sets and the class of βg- closed sets. Levine [11] Devi. *et.al* [6,8] introduced $T_{1/2}$- spaces, T_b- spaces and ${}_{\alpha}T_b$ spaces respectively. The purpose of this paper is to introduce the concepts of, ${}_{\beta}T^{**}_{1/2}$- space, ${}_{\alpha\beta}T^{*}_{c}$ – space, ${}_{\beta}{}^{*}T^{*}_{1/2}$ – space are introduced and investigated.

PRELIMINARIES

Throughout this paper (X, τ), (Y, σ) and (Z, η) represent non-empty topological spaces of which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X, τ), cl(A) and int(A) denote the closure and the interior of A respectively. The class of all closed subsets of a space (X, τ) is denoted by C(X, τ). The smallest semi closed (resp. pre-closed and α-closed) set containing a subset of a space (X, τ) is called the semi-closure (resp. pre-closure and α-closure) of A is denoted by scl (A)(resp. pcl (A) and αcl (A)).

Definition 2.1: A subset A of a topological space (X, τ) is called
1) a pre-open set [16] if A⊆ int(cl(A))and a preclosed set if cl(int(A)) ⊆ A.
2) a semi-open set [12] if A⊆cl(int(A))and semi-closed set if int(cl(A)) ⊆ A.
3) a semi-preopen set [1] if A⊆cl(int(cl(A)))and a semi preclosed set [1] if int(cl(int(A))) ⊆ A.
4) an α-open set [18] if A⊆int(cl(int(A))) and an α-closed set [18] if cl(int(cl(A))) ⊆ A.
5) a regular-open set [16] if int(cl(A))=A and regular-closed set [16]if A=int(cl(A)).

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Definition 2.2: A subset A of a topological space (X, τ) is called

1) a generalised closed set (briefly g - closed) [11] if cl(A)⊆U whenever A⊆U and U is open in (X, τ).
2) g*- closed if [20] cl(A)⊆U whenever A⊆U and U is g*-open in (X, τ).
3) a generalised semi-closed set (briefly gs - closed) [3] is scl(A)⊆U whenever A⊆U and U is open in (X, τ).
4) an generalised semi pre-closed set (briefly gsp-closed) [9] if spcl(A)⊆U whenever A⊆U and U is open in (X, τ).
5) regular generalised closed set (briefly rg - closed) [19] if cl(A)⊆U whenever A⊆U and regular open in (X, τ).
6) α- generalised closed set (briefly αg - closed) [14] if αcl(A)⊆U whenever A⊆U and U is g*-open in (X, τ).
7) g**-closed [21] if cl(A)⊆U whenever A⊆U and U is open in (X, τ).
8) generalised pre regular-closed set (briefly gpr - closed)[10] if pcl(A)⊆U whenever A⊆U and U is regular open in (X, τ).
9) weakly generalised closed set [18] (briefly wg - closed) if cl(int(A)))⊆U whenever A⊆U and U is open in (X, τ).
10) generalised pre-closed set (briefly gp - closed) [13] if pcl(A) whenever A⊆U and U is open in (X, τ).
11) generalised α - closed (briefly gα - closed) [14] if αcl(A) whenever A⊆U and U is α-open in (X, τ).
12) Semi-generalised closed (briefly sg-closed) [5] if scl(A) whenever A⊆U and U is semiopen in (X, τ).

Definition 2.3: A function f: (X, τ)→ (Y, σ) is called

1) g- continuous [4] if f^{-1}(V) is a g- closed set of (X, τ) for every closed set V of (Y, σ).
2) αg- continuous [10] if f^{-1}(V) is a αg- closed set of (X, τ) for every closed set V of (Y, σ).
3) gs- continuous [7] if f^{-1}(V) is a gs- closed set of (X, τ) for every closed set V of (Y, σ).
4) gsp- continuous [9] if f^{-1}(V) is a gsp- closed set of (X, τ) for every closed set V of (Y, σ).
5) rg- continuous [19] if f^{-1}(V) is a rg- closed set of (X, τ) for every closed set V of (Y, σ).
6) gp- continuous [2] if f^{-1}(V) is a gp- closed set of (X, τ) for every closed set V of (Y, σ).
7) gpr- continuous [10] if f^{-1}(V) is a gpr- closed set of (X, τ) for every closed set V of (Y, σ).
8) g*- continuous [20] if f^{-1}(V) is a g- closed set of (X, τ) for every closed set V of (Y, σ).
9) g*-irresolute [20] if f^{-1}(V) is a g*- closed set of (X, τ) for every g*-closed set of (Y, σ).
10) wg- continuous [18] if f^{-1}(V) is a wg- closed set of (X, τ) for every closed set V of (Y, σ).
11) g**- continuous [22] if f^{-1}(V) is a g**- closed set of (X, τ) for every closed set of (Y, σ).
12) g**-irresolute [22] if f^{-1}(V) is a g**- closed set of (X, τ) for every g*-closed set V of (Y, σ).

Definition 2.4:A topological space (X, τ) is said to be

1) a $T_{1/2}$ space [11] if every g- closed set in it is closed.
2) a T_b space [6] if every gs- closed set in it is closed.
3) a T_d space [6] if every gs- closed set in it is g- closed.
4) a ${}_aT_d$ space [4] if every αg- closed set in it is g- closed.
5) a ${}_aT_b$ space [8] if every αg- closed set in it is closed.
6) a*$T_{1/2}$ space [20] if every g- closed set in it is g*- closed set.
7) a $T^*_{1/2}$space [20] if every g*- closed set in it is closed.

3. BASIC PROPERTIES OF g*β-CLOSED SETS

We introduce the following definition

Definition 3.1: A subset A of (X, τ) is said to be g*β closed set if βcl (A)⊆U whenever A⊆U and U is g* open in X. The family of all g*β- closed sets are denoted by G*β-C(X).

Proposition 3.2: Every closed set is g*β- closed.
Proof follows from the definition.

Proposition 3.3: Every β-closed set is g*β- closed set.
Proof follows from the definition.

Proposition 3.4: Every g**-closed set is g*β- closed set.
Proof follows from the definition.

Proposition 3.5: Every g*-closed set is g*β- closed set.
Proof follows from the definition.

Proposition 3.6: Every g-closed set is g*β- closed set.
Proof follows from the definition.
The converse of the above propositions need not be true in general.

Example 3.7: Let X = {a, b, c, d}. τ = {φ, X, {a}, {b}, {a, b}, {a, b, c}}.

Let A = {c} is a g*β-closed set but not a closed set and g**-closed set. So the class of a g*β -closed sets properly contains the class of closed sets and the class of g**-closed sets. Also {c} is not a g-closed set.

Example 3.8: Let X = {a, b, c,}, τ = {φ, X, {a}}. Let A = {a, b} is a g*β -closed set but not a β-closed set and g*- closed set of (X, τ). So the class of g*β -closed sets properly contains the class of β-closed sets and the class of g*-closed sets.

Proposition 3.9: Every g*β -closed set is (1) rg-closed (2) gp-closed (3) gpr- closed (4) gsp-closed (5) wg-closed.
Proof follows from the definition.

The converse of the above propositions need not be true in general as seen in the following examples.

Example 3.10: Let X = {a, b, c,}, τ = {φ, X, {a}, {a, c}}. Let A = {a} is gpr-closed set and a rg-closed set but not g*β -closed set.

Let X = {a, b, c,}, τ = {φ, X, {a, c}}. Let A = {c} is a gsp-closed, wg-closed and a gp-closed set but not a g*β -closed set of (X, τ). Therefore the class of g*β -closed set is properly contained in the class of gpr-closed, rg-closed, gsp-closed, gp-closed and wg-closed.

Remark 3.11: g*β-closedness is independent of pre-closedness, semi pre-closedness, semiclosedness, gβ-closedness, β-closedness and sg-closedness. Let X = {a, b, c,}, τ = {φ, X, {a}, {a, c}}. Let A = {a, b} then A is g*β-closed set. A is neither β-closed nor semi-closed, infact it is not even a semipreclosed. Also it is not a sg-closed and gβ-closed.

Proposition 3.12: If A and B are g*β -closed sets, then A∪B is also a g*β -closed set.
Proof follows from the fact that βcl (A∪B) = βcl (A) ∪ βcl (B)

Proposition 3.13: If A is both g*-open and g*β -closed then A is β-closed.
Proof follows from the definition of g*β -closed sets.

Proposition 3.14:A is g*β -closed set of (X, τ) if βcl (A)\A does not contain any non-empty g*-closed set.

Proof: Let F be a g*-closed set of (X, τ) such that F⊆βcl (A)\A. Then A ⊆ X\f. Since A is g*β -closed and X\F is g*-open, βcl (A) ⊆ X\F. This implies F ⊆ X\, βcl (A). So F ⊆ (*X*\ βcl (A)) ∩ (βcl (A)\A) ⊆ (X\ βcl (A)) ∩ (βcl (A)) = φ. Hence F = φ.

Proposition 3.15: If A is g*β -closed set of (X, τ) such that A ⊆ B ⊆ βcl (A) then B is also a g*β -closed set of (X, τ).

Proof: Let U be a g*-open set of (X, τ) such that B ⊆ U. Then A ⊆ U. Since A is g*β -closed, then βcl (A) ⊆ U. Now βcl (B) ⊆ βcl (βcl (A)) = βcl (A) ⊆ U. Therefore B is also a g*β -closed set of (X, τ).

Proposition 3.16: If A and B are two g*β-closed sets in a topological space X such that either A ⊂ B or B ⊂ A then both intersection and union of two g*β-closed set is g*β-closed set.

Proof: If A is contained in B or B is contained in A then A ∪ B = B or A ∪ B = A respectively. This shows that A ∪ B is g*β-closed as A and B are g*β-closed sets.

Similarly, A ∩ B is also a g*β-closed set.

Remark 3.17: Difference of two g*β-closed sets is not a g*β-closed set.
Let X = {a, b, c, d}, τ = {φ, X, {a}, {b}, {a, b}, {a, b, c}}. Here A = {a, b, d} and B = {b, d} are g*β-closed sets but A-B = {a} is not.

Proposition 3.18: Let (X, τ) be a topological space then for each x ϵ X, the set X\{x} is g*β-closed or g*-open.

Proof: If X\{x} is g*β-closed or g*-open then we are done. Now suppose X\{x} is not g*-open then X is the only g*-open set containing X\{x} and also βcl (X\{x}) is contained in X, as it is the biggest set containing all its subsets. Hence X\{x} is a g*β -closed in X.

4. g*β– CONTINUOUS AND g*β– IRRESOLUTE MAPS

We introduce the following definitions

Definition 4.1: A function f: (X, τ)→ (Y, σ) is called g*β – continuous if f^{-1} (V) is g*β -closed set of (X, τ) for every closed set of (Y, σ).

Theorem 4.2: Every continuous map is g*β – continuous.

Proof: Let f: (X, τ)→ (Y, σ) be continuous and let F be any closed set of Y, then f^{-1} (F) is closed in X. Since every closed set is g*β-closed set, f^{-1} (F) is g*β -closed set. Therefore f is g*β– continuous.

The following example supports that the converse of the above theorem need not be true in general.

Example 4.3: Let X = Y = {a, b, c}, τ = {φ, X, {a}}, σ = {φ, Y, {a, c}}, f: (X, τ→ (Y, σ) is defined as the identity map. The inverse image of all the closed sets of (Y, σ) is g*β -closed set in (X, τ) but not closed. Therefore f is g*β – continuous but not continuous.

Theorem 4.4: Every g*β – continuous map is (1) rg- continuous (2) gp- continuous (3) gpr- continuous (4) gsp- continuous (5) wg- continuous.

Proof: Let f: (X, τ)→ (Y, σ) be g*β – continuous map. Let V be a closed set of (Y, σ). Since f is g*β– continuous by prop (3.9) f^{-1}(V) is (1) rg-closed (2) gp-closed (3) gpr-closed (4) gsp-closed (5) wg-closed of (X, τ). Therefore f is rg- continuous, gp- continuous, gpr- continuous, gsp- continuous, wg- continuous.

Example 4.5: Let X = Y = {a, b, c}, τ = {φ, X, {a}}, σ = {φ, Y, {b, c}}, f: (X, τ→ (Y, σ) is defined as the identity map. Then f^{-1}({a}) = {a} is not g*β-closed set in (X, τ). But {a} is rg-closed and gpr-closed. Therefore f is rg- continuous and gpr- continuous.

Example 4.6: Let X = Y = {a, b, c}, τ = {φ, X, {a}, {b}, {a, b}}, σ = {φ, Y, {b, c}}, f: (X, τ) → (Y, σ) is defined as the identity map. Then f^{-1}({a}) = {a} is not g*β-closed set in (X, τ). But {a} is gsp-closed. Therefore f is gsp- continuous.

Example 4.7: Let X = Y = {a, b, c}, τ = {φ, X, {a}, {b}, {a, c}}, σ = {φ, Y, {b, c}}, f: (X, τ) → (Y, σ) is defined as the identity map. Then f^{-1}({a}) = {a} is not g*β-closed set in (X, τ). But {a} is wg-closed and gp-closed. Therefore f is wg- continuous and gp- continuous. Thus the class of g*β-continuous maps is properly contained in the class of rg- continuous, gp- continuous, gsp- continuous and wg- continuous.

Theorem 4.8: Every g*-continuous map is g*β -continuous.

Proof: Let f: (X, τ) → (Y, σ) be g*-continuous map. Let V be a closed set of (Y, σ). Then f^{-1}(V) is g*-closed and hence by prop (3.5) it is g*β-closed set. Hence f is g*β-continuous map.

Example 4.9: Let X = Y = {a, b, c}, τ = {φ, X, {a}}, σ = {φ, Y, {b}}, f: (X, τ) → (Y, σ) is defined as the identity map. Here A = {a, c} is closed in (Y, σ). Then f^{-1}({a, c}) = {a, c} is g*β -closed set in (X, τ) but not g*-closed in (X, τ). Therefore f is g*β-continuous but not g*-continuous.

Theorem 4.10: Every g-continuous map is g*β-continuous.

Proof: Let f: (X, τ)→ (Y, σ) be g -continuous map. Let V be a closed set of (Y, σ). Then f^{-1}(V) is g-closed and hence by prop (3.6) it is g*β-closed set. Hence f is g*β-continuous map.

Example 4.11: Let X = Y = {a, b, c}, τ = {φ, X, {a}, {a, b}}, σ = {φ, Y, {a, c}}, f: (X, τ) → (Y, σ) be the identity map. Here A = {b} is closed in (Y, σ). Then f^{-1}({b}) = {b} is g^{*}β-closed set in (X, τ) but not g-closed in (X, τ). Therefore f is g*β-continuous but not g-continuous.

Theorem 4.12: Every g**-continuous map is g*β-continuous.

Proof: Let f: (X, τ)→ (Y, σ) be g** -continuous map. Let V be a closed set of (Y, σ). Then f^{-1}(V) is g**-closed and hence by prop (3.4) it is g*β-closed set. Hence f is g*β-continuous map.

The following example helps that the converse of the above theorem need not be true in general.

Example 4.13: X = Y = {a, b, c}, τ = {φ, X, {a}, {a, b}}, σ = {φ, Y, {a, c}}, f: (X, τ) (Y, σ) be the identity map. Here A = {b} is closed in (Y, σ). Then f^{-1}({b}) = {b} is g*β-closed set in (X, τ) but not g**-closed in (X, τ). Therefore f is g*β-continuous but not g**-continuous.

Definition 4.14: A function f: (X, τ)→ (Y, σ) is called g*β -irresolute if f^{-1}(V) is g*β-closed set of (X, τ) for every g*β-closed set of (Y, σ).

Definition 4.15: A function f: (X, τ)→ (Y, σ) is called g*β -resolute if f (U) is g*β-open in Y whenever U is g*β-open in X.

Definition 4.16: A function f: (X, τ) → (Y, σ) is called g*β-homeomorphism if

i. f is one and onto
ii. f is g*β-irresolute and g*β-resolute.

Theorem 4.17: Every g*β-irresolute function is g*β-continuous.
Proof follows from the definition.

Theorem 4.18: Every g-irresolute function is g*β-continuous.
Proof follows from the definition.

Theorem 4.19: Every g*-irresolute function is g*β-continuous.
Proof follows from the definition.

Theorem 4.20: Every g**-irresolute function is g*β-continuous.
Proof follows from the definition.

Example 4.21: Let X = {a, b, c, d}, τ = {φ, X, {a}, {b}, {a, b}, {a, b, c}}, σ = {φ, Y, {a, b, d}}, f: (X, τ)→ (Y, σ) be defined by f (a) = b, f (b) = a and f (c) = c. {c} is the only closed set of Y. f^{-1}({c}) = {c} is g*β-closed set in (X, τ). Hence f is g*β-continuous map. But f^{-1}({c}) = {c} is not g-closed, g*-closed and g**-closed in X. Therefore f is not g-irresolute, g*-irresolute, g**-irresolute. Therefore f is g*β-continuous but not g-irresolute, g*-irresolute and g**-irresolute.

Example 4.22: Let X = {a, b, c}, τ = {φ, X, {a, c}}, σ = {φ, Y, {b}}, let f: (X, τ) (Y, σ) be defined by f (a) = b, f (b) = a and f (c) = c. {a, c} is the only closed set of Y. f^{-1}({a, c}) = {b, c} is g*β-closed set in (X, τ). Hence f is g*β-continuous map. But {c} is $g^*\beta$-closed set in Y, f^{-1}({c}) = {c} is not g*β-closed set in (X, τ). Hence g*β-continuous but not g*β-irresolute.

Theorem 4.23: Let f: (X, τ) → (Y, σ) and g: (Y, σ) → (Z, σ) be any two functions. Then

i. g∘f is g*β-continuous if g is continuous and f is g*β-continuous.
ii. g∘f is g*β-irresolute if both f and g are g*β-irresolute.
iii. g∘f is g*β-continuous if g is g*β-continuous and f is g*β-irresolute.

5. APPLICATIONS OF g*β-CLOSED SETS

As applications of g*β-closed sets, new spaces namely new spaces namely ${}_{\beta}T^{**}_{1/2}$- space, ${}_{\alpha\beta}T^{*}_{c}$ – space, ${}_{\beta}{}^{*}T^{*}_{1/2}$ – space, are introduced.

Definition 5.1: A space (X, τ) is called ${}_{\beta}T^{**}_{1/2}$- space if every g*β-closed set is closed.

Theorem 5.2: Every ${}_{\beta}T^{**}_{1/2}$- space is a $T_{1/2}$-space.
Proof follows from the definition.

Theorem 5.3: Every ${}_{\beta}T^{**}_{1/2}$- space is a $T^{*}_{1/2}$-space.

Proof follows from the definition.
The converse need not be true in general as seen in the following example.

Example 5.4: Let X = {a, b, c}, τ = {φ, X, {a}}. G*βCl(x, τ) = {φ, X, {b, c}} = C (X, τ). Therefore (X, τ) is a $T^{*}_{1/2}$-space but not a ${}_{\beta}T^{**}_{1/2}$- space. Since {a, c} is g*β-closed set but not closed in (X, τ).

Theorem 5.5: Every T_b- space is a ${}_{\beta}T^{**}_{1/2}$- space.

Proof follows from the definition.
The converse need not be true in general as seen in the following example.

Example 5.6: Let X = {a, b, c}, τ = {φ, X, {a}}, {b}, {a, b}}. (X, τ) is a ${}_{\beta}T^{**}_{1/2}$- space but not a T_b- space. Since A = {b} is gs-closed set but not closed in (X, τ).

Remark 5.7: T_d-ness is independent of ${}_{\beta}T^{**}_{1/2}$-ness as it can be seen from the following examples.

Example 5.8: Let X = {a, b, c}, τ = {φ, X, {a}, {b},{a, b}}. (X, τ) is a ${}_{\beta}T^{**}_{1/2}$- space but not a T_d- space. Since A = {a} is gs-closed set but not g-closed in (X, τ).

Example 5.9: Let X = {a, b, c}, τ = {φ, X, {a}, {b, c}}. (X, τ) is a T_d-space but not a ${}_{\beta}T^{**}_{1/2}$-space. Since A = {c} is g*β-closed but not closed.

Theorem 5.10: The following conditions are equivalent in topological space (X, τ).

i. (X, τ) is a ${}_{\beta}T^{**}_{1/2}$- space.
ii. Every singleton set of X is either g*-closed or open.

Proof:
(i)⇒(ii): Let (X, τ) be a ${}_{\beta}T^{**}_{1/2}$- space. Let x∊X and suppose {x} is not g*-closed. Then X\{x} is not g*-open. This implies that X is the only g*-open set containing X\{x}. Therefore X\{x} is closed since (X, τ) is a ${}_{\beta}T^{**}_{1/2}$. Therefore {x} is open in (X, τ).

(ii)⇒(i): Let A be a g*β-closed of (X, τ). A ⊆ βcl (A) ⊆ cl (A) and let x ϵ βcl (A) this implies x ϵ cl (A). By (ii) {x} is g*-closed or open.

Case-(i): Let {x} be g*-closed. If x does not belong to A then βcl (A)\A contains a nonempty g*-closed set {x}. But it is not possible by proposition (3.14). Therefore x ϵ A.

Case-(ii): Let {x} be open. Now x ϵ cl (A), then {x} ∩A = φ. Therefore x ϵ A and so cl (A) ⊆ A and hence A = cl (A) or A is closed. Therefore (X, τ) is a ${}_{\beta}T^{**}{}_{1/2}$- space.

We introduce the following definition.

Definition 5.11: A space (X, τ) is called ${}_{\alpha\beta}T_c^{*}$– space if every βg-closed set is g*β-closed.

Theorem 5.12: Every ${}_{\beta}T_b$-space is a ${}_{\alpha\beta}T_c^{*}$– space but not conversely.

Example 5.13: Let X = {a, b, c}, τ = {φ, X, {a}} is ${}_{\alpha\beta}T_c^{*}$– space but not a ${}_{\beta}T_b$-space but not a ${}_{\beta}T_b$-space since {a, c} is βg-closed but not closed.

Definition 5.14: A subset A of (X, τ) is called g*β-open set if its compliment is g*β-closed set of (X, τ).

Theorem 5.15: If (X, τ) is a ${}_{\alpha\beta}T_c^{*}$– space for each x ϵ X, {x} is either βg-closed or g*β-open.

Proof: Let x ϵ X suppose that {x} is not βg-closed of (X, τ). Then {x} is not closed set since every closed set is a βg-closed set. Therefore, X\{x} is not open. Therefore X\{x} is a βg-closed set since X is the only open set which contains X\{x}. Since (X, τ) is a ${}_{\alpha\beta}T^{*}{}_{c}$ – space, X\{x} is g*β-closed

or {x} is g*β-open.

We introduce the following definition.

Definition 5.16: A space (X, τ) is called ${}^{**}_{\beta}T_{1/2}$ – space, if every g*β-closed set is g*-closed.

Theorem 5.17: Every ${}_{\beta}T^{**}{}_{1/2}$- space is a ${}^{**}_{\beta}T_{1/2}$– space.

Proof: Let (X, τ) be a ${}_{\beta}T^{**}_{1/2}$- space. Let A be a g*β-closed set of (X, τ). Since (X, τ) is a ${}_{\beta}T^{**}{}_{1/2}$- space, A is closed. Since every closed set is g*-closed, (X, τ) is a ${}^{**}_{\beta}T_{1/2}$– space.

Theorem 5.18: Every T_b-space is a${}^{**}_{\beta}T_{1/2}$– space.

Proof: Let (X, τ) be a T_b-space. Then by theorem 5.5, it is ${}_{\beta}T^{**}_{1/2}$-space. Therefore by theorem 5.19, it is${}^{**}_{\beta}T_{1/2}$– space. The converse need not be true in general as seen in the following example.

Example 5.19: Let X = {a, b, c}, τ = {φ, X, {a}, {b}, {a, b}}. (X, τ) is a ${}^{**}_{\beta}T_{1/2}$– space but not a T_b-space. Since A= {a} is gs-closed set but not closed in (X, τ).

Theorem 5.20: Every ${}^{**}_{\beta}T_{1/2}$– space is a ${}^{*}T_{1/2}$-space.

Proof: Let (X, τ) be a ${}^{**}_{\beta}T_{1/2}$– space. Let A be a g-closed set of (X, τ). Then by prop (3.6) A is g*β-closed. Since (X, τ) is a${}^{**}_{\beta}T_{1/2}$– space, A is g*-closed. Therefore it is a ${}^{*}T_{1/2}$-space.

The converse of the above theorem need not be true as seen in the following example.

Example 5.21: Let X = Y = {a, b, c}, τ = {φ, X, {a}, {a, c}}. (X, τ) is a ${}^{*}T_{1/2}$-space but not a ${}^{**}_{\beta}T_{1/2}$– space. Since A = {c} is g*β-closed but not g*-closed.

Theorem 5.22: Every ${}_{\beta}{}^{**}T_{1/2}$ – space is a ${}^{**}T_{1/2}$-space.

Proof: Let (X, τ) be a ${}^{**}_{\beta}T_{1/2}$– space. Let A be a g**-closed set of (X, τ). Then by prop (3.4) A is g*β-closed. Since (X, τ) is a${}^{**}_{\beta}T_{1/2}$– space, A is g*-closed. Therefore it is a ${}^{**}T_{1/2}$-space.

The converse of the above theorem need not be true as seen in the following example.

Example 5.23: Let X = {a, b, c, d}, τ = {φ, X, {a}, {b}, {a, b}, {a, b, c}}. Here (X, τ) is a ${}^{**}T_{1/2}$-space but not a ${}^{**}_{\beta}T_{1/2}$– space. Since A = {c} is g*β-closed but not g*-closed.

Theorem 5.24: If (X, τ) is a${}^{**}_{\beta}T_{1/2}$– space for each x ϵ X, {x} is either closed or g*-open.

Proof: Suppose (X, τ) be a${}^{**}_{\beta}T_{1/2}$– space. Let x ϵ X and let {x} not be closed set. Then x\{x} is not open set. Therefore X\{x} is a${}^{**}_{\beta}T_{1/2}$– space, X\{x} is g*-closed set. Therefore {x} is g*-open.

Definition 5.25: A space (X, τ) is called${}^{*}_{\beta}T^{*}_{1/2}$– space if every g*β-closed set is g-closed.

Theorem 5.26: Every ${}_{\beta}T^{**}_{1/2}$- space is a${}^{*}_{\beta}T^{*}_{1/2}$– space.

Proof: Let (X, τ) be a ${}_{\beta}T^{**}_{1/2}$- space. Let A be a g*β-closed set of (X, τ). Then A is closed. Since (X, τ) is a ${}_{\beta}T^{**}_{1/2}$- space. But every closed set is a g-closed set, therefore A is g-closed. Therefore (X, τ) is a ${}^{*}_{\beta}T^{*}_{1/2}$– space. The converse of the above theorem need not be true as seen in the following example.

Example 5.27: Let X = {a, b, c}, τ = {φ, X, {a}}. (X, τ) is a${}^{*}_{\beta}T^{*}_{1/2}$– space but not a ${}_{\beta}T^{**}_{1/2}$- space. Since A = {a, b} is g*β-closed set but not closed in (X, τ).

Theorem 5.28: The space(X, τ) is a ${}_{\beta}T^{**}_{1/2}$- space iff it is a ${}^{*}_{\beta}T^{*}_{1/2}$– space and a $T_{1/2}$– space.

Proof: Necessity: Let (X, τ) be a ${}_{\beta}T^{**}_{1/2}$- space. Let A be a g-closed set of (X, τ). Then by prop (3.6) A is g*β-closed. Also since (X, τ) is a ${}_{\beta}T^{**}_{1/2}$- space, A is closed set. Therefore (X, τ) is a $T_{1/2}$– space. By theorem (5.26) (X, τ) is a${}^{*}_{\beta}T^{*}_{1/2}$– space.

Sufficiency: Let (X, τ) be a${}^{*}_{\beta}T^{*}_{1/2}$– space and a $T_{1/2}$– space. Let A be a g*β-closed set. Then A is g-closed. Since (X, τ) is a$T_{1/2}$– space, A is a closed set. Therefore (X, τ) is a ${}_{\beta}T^{**}_{1/2}$- space.

Theorem 5.29: Every ${}^{**}_{\beta}T_{1/2}$- spaceis a ${}^{*}_{\beta}T^{*}_{1/2}$- space.

Proof: Let(X, τ) be a ${}^{**}_{\beta}T_{1/2}$- space. Let A be a g*β- closed set. Then A is g*-closed since (X, τ) is a ${}^{**}_{\beta}T_{1/2}$- space. But every g*-closed is g-closed and hence A is a g-closed set. Therefore (X, τ) be a ${}^{*}_{\beta}T^{*}_{1/2}$- space.

Example 5.30: Let X = {a, b, c}, $\tau = \{\varphi, X, \{a\}\}$.(X, τ) is a ${}_{\beta}^{*}T_{1/2}^{*}$- space but not a ${}_{\beta}^{**}T_{1/2}$- space.
Since A = {b} is g*β - closed but not g*-closed.

We introduce the following definition

Definition 5.31:A space (X, τ) is called ${}_{\beta}^{*}T_{c}^{*}$- Space if every gs-closed of (X, τ) is a g*β- closed.

Theorem 5.32: A T_c- space is a ${}_{\beta}T_c^*$- Space.

Proof: Let (X, τ) be a T_c- space. Let A be a gs-closed set of(X, τ)Then A is g*- closed.

Since (X, τ) be a T_c- space, by proposition (3.5), then A is g*β- closed set. Therefore (X, τ) is a ${}_{\beta}T_c^*$-Space.

Example 5.33: Let X = {a, b, c}, $\tau = \{\varphi, X, \{c\}\}$.(X, τ) is a ${}_{\beta}^{*}T_{c}^{*}$- space but not a T_c- space.
Since A = {b} is gs-closed but not g*-closed.

Theorem 5.34: A T_b- space is a ${}_{\beta}T_c^*$- Space.

Proof: Let (X, τ) be a T_b- space. Let A be a gs-closed set of (X, τ).Then A is closed.
Since (X, τ) be a T_b- space, by proposition (3.2), A is g*β- closed set. Therefore (X, τ) is a ${}_{\beta}T_c^*$- Space.

Example 5.35: Let X = {a, b, c}, $\tau = \{\varphi, X, \{c\}\}$.(X, τ)is a ${}_{\beta}^{*}T_{c}^{*}$- space but not a T_b- space.
Since A = {b} is gs-closed but not a closed set.

Theorem 5.36: If(X, τ) is a ${}_{\beta}T_c^*$- Space and ${}_{\beta}^{*}T_{1/2}^{*}$- Space then it is a ${}_{\beta}T_d$ Space.

Proof: Let (X, τ) is a ${}_{\beta}T_c^*$-Space and ${}_{\beta}^{*}T_{1/2}^{*}$- Space. Let A be a βg-closed set of (X, τ). Then A is also gs-closed. Since (X, τ) is a ${}_{\beta}T_c^*$-Space, A is a g*β- closed set. Also since (X, τ) is a ${}_{\beta}^{*}T_{1/2}^{*}$- Space, A is g-closed set. Therefore (X, τ) is a ${}_{\beta}T_d$-Space.

The following example helps that the converse of the above theorem need not be true in general.

Example 5.37: Let X = {a, b, c}, $\tau = \{\varphi, X, \{a\}, \{b\}, \{a, b\}\}$. (X, τ) is a ${}_{\beta}T_d$- space but not a ${}_{\beta}T_c^*$-space.
Since A = {b} is gs- closed but not a g*β-closed set.

Theorem 5.38: If(X, τ) is a ${}_{\beta}T_c^*$- Space and ${}_{\beta}T_{1/2}^{**}$- Space then it is a ${}_{\beta}T_b$- Space.

Proof: Let (X, τ) is a ${}_{\beta}T_c^*$- Space and ${}_{\beta}T_{1/2}^{**}$- Space. Let A be a βg-closed set of (X, τ). Then A is also gs-closed.
Since (X, τ) is a ${}_{\beta}T_c^*$- Space, A is a g*β- closed set. But every g*β- closed set is closed. Also since (X, τ) is a ${}_{\beta}T_{1/2}^{**}$- Space, A is closed set. Therefore (X, τ) is a ${}_{\beta}T_b$- Space.

Example 5.39: Let X = {a, b, c}, $\tau = \{\varphi, X, \{a\}, \{b\}, \{a, b\}\}$.(X, τ) is a ${}_{\beta}T_b$- space but not a ${}_{\beta}T_c^*$-space. Since A = {b} is gs- closed but not a g*β-closed set.

Theorem 5.40: If (X, τ) is a ${}_{\beta}T_c^*$-Space and ${}_{\beta}^{*}T_{1/2}^{*}$- Space then it is a T_d- Space.

Proof: Let (X, τ) is a ${}_{\beta}T_c^*$- Space and ${}_{\beta}^{*}T_{1/2}^{*}$- Space. Let A be a gs-closed set of (X, τ).

Since (X, τ) is a ${}_{\beta}T_c^*$- Space, A is a $g^*\beta$- closed set. Also since (X, τ) is a ${}_{\beta}^{*}T_{1/2}^{*}$- Space, A is g-closed set. Therefore (X, τ) is a T_d- Space.

Theorem 5.41: If (X,τ) is a ${}_{\beta}T_c^*$- Space, then for each $x \epsilon X$, $\{x\}$is either semi-closed or g*β-open.

Proof: Suppose (X,τ) be a ${}_{\beta}T_c^*$- Space. Let $x \epsilon X$and let $\{x\}$not be semiclosed. Then X\$\{x\}$ is sg-closed. Also X\$\{x\}$ is gs-closed. Since (X,τ) is a g*β-closed, $\{x\}$ is g*β-open.

Theorem 5.42: Let f: (X,τ) → (Y, σ) be a g*β-continuous map. If (X,τ) is a ${}_{\beta}T_{1/2}^{**}$- space, then f is continuous.

Theorem 5.43: Let f: (X,τ) → (Y, σ) be a g*β- continuous map. If (X,τ) is a ${}_{\beta}^{**}T_{1/2}$- space, then f is g*-continuous.

Theorem 5.44: Let f: (X,τ) → (Y,σ) be a g*β- continuous map. If (X,τ) is a ${}_{\beta}^{*}T_{1/2}^{*}$- space, then f is g-continuous.

Theorem 5.45: Let f: (X,τ) → (Y,σ) be a gs- continuous map. If (X,τ) is a ${}_{\beta}T_c^{*}$- space, then f is g*β-continuous.

Theorem 5.46: Let f: (X,τ) → (Y,σ) be a g*- irresolute map and a β- closed map. Then f (A) is a g*β-closed set of (Y,σ) for every g*β- closed set A of (X,τ).

Proof: Let A be a g*β- closed set of (X,τ). Let U be a g*- open set of (Y,σ) such that $f(A) \subseteq U.$ Since f is g*- irresolute, $f^{-1}(U)$ is g*- open in (X,τ). Now $f^{-1}(U)$ is g*- open and A is g*β- closed set of (X,τ), then $\beta cl(A) \subseteq f^{-1}(U)$. Then $f(\beta cl(A)) = \beta cl[f(\beta cl(A))]$. Therefore $\beta cl[f(A)] \subseteq \beta cl[f(\beta cl(A)] = f(\beta cl(A)) \subseteq U$. Therefore f (A) is a g*β-closed set of (Y,σ) for every g*β- closed set A of (X,τ).

Theorem 5.47: Let f: (X,τ) → (Y,σ) be a g*β- irresolute and closed. If (X,τ) is a ${}_{\beta}T_{1/2}^{**}$-space, then (Y,σ) is also a ${}_{\beta}T_{1/2}^{**}$-space.

Theorem 5.48: Let f: (X,τ) → (Y,σ) be a g*β- closed map if f(A) is $g^*\beta$- closed set of (Y,σ) for every g*β- closed set of (X,τ) .

Theorem 5.49: Let f: (X,τ) → (Y,σ) be onto g*β- irresolute and pre-g*closed. If (X,τ) is a ${}_{\beta}^{**}T_{1/2}$- space, then (Y,σ) is also a ${}_{\beta}^{**}T_{1/2}$- space.
Proof follows from the definition of g*β- irresolute and pre-g*closed.

Theorem 5.50: Let f: (X,τ) → (Y,σ) be onto gs- irresolute and g*β-closed map. If (X,τ) is a ${}_{\beta}T_c^{*}$- space, then (Y,σ) is also a ${}_{\beta}T_c^{*}$- space.
Proof follows from the definition of gs- irresolute and g*β closed map.

Theorem 5.51: Let f: (X,τ) → (Y,σ) be onto g*β- irresolute and g-closed. If (X,τ) is a ${}_{\beta}^{*}T_{1/2}^{*}$- space, then (Y,σ) is also a ${}_{\beta}^{*}T_{1/2}^{*}$- space.
Proof follows from the definition of g*β- irresolute and g closed map.

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