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# COMPUTATION OF MULTIPLICATIVE STATUS INDICES OF GRAPHS

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***ABSTRACT***

*A topological index or a graph index is a numerical parameter mathematically derived from the graph structure. In this study, we define the multiplicative vertex status index, multiplicative modified vertex status index, multiplicative F-status index, general multiplicative vertex status index of a graph and compute exact formulas for some standard graphs and friendship graphs.*

***Keywords:*** *Multiplicative vertex status index, multiplicative F-status index, graph.*

***Mathematics Subject Classification:*** *05C05, 05C07, 05C035, 05C90.*

## 1. INTRODUCTION

In this paper, we consider only a finite, simple, connected graph $G$ with vertex set $V(G)$ and edge set $E(G)$. The degree $d_G(v)$ of a vertex $v$ is the number of vertices adjacent to $v$. The distance between any two vertices $u$ and $v$ is the length of the shortest path joining $u$ and $v$ and is denoted by $d(u, v)$. The status of a vertex $u$ is defined as the sum of its distance from every other vertex in $G$ and is denoted by $\sigma(u)$. We refer [1] for undefined term and notation.

Topological indices or graph indices have found some applications in chemical documentation, isomer discrimination, QSAR/QSPR study, see [2, 3].

For more about graph indices one can refer [4]. Some different graph indices can be found in [5, 6, 7, 8, 9, 10, 11].

In [12], Kulli introduced the multiplicative first and second status indices, defined as

$$S_1II(G) = \prod_{uv\in E(G)} [\sigma(u)+\sigma(v)], \qquad S_2II(G) = \prod_{uv\in E(G)} \sigma(u)\sigma(v).$$

We now propose the following multiplicative status indices.

The multiplicative vertex status index of a graph $G$ is defined as $S_vII(G) = \prod_{u\in V(G)} \sigma(u)^2.$

The multiplicative total status index of a graph $G$ is defined as $T_sII(G) = \prod_{u\in V(G)} \sigma(u).$

The multiplicative modified vertex status index of a graph $G$ is defined as ${}^mS_vII(G) = \prod_{u\in V(G)} \frac{1}{\sigma(u)^2}.$

The multiplicative status inverse is defined as $SI\ II(G) = \prod_{u\in V(G)} \frac{1}{\sigma(u)}.$

The multiplicative status zeroth order index of a graph $G$ is defined as $SZII(G) = \prod_{u\in V(G)} \frac{1}{\sqrt{\sigma(u)}}.$

The multiplicative $F$-status index of a graph $G$ is defined as $FSII(G) = \prod_{u\in V(G)} \sigma(u)^3.$

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The general multiplicative vertex status index of $G$ is defined as $S_v^a II(G) = \prod_{u \in V(G)} \sigma(u)^a$, where $a$ is a real number.

Some of the research work on status indices can be found in [13, 14, 15, 16, 17, 18]. Recently, some multiplicative indices were studied, for example, in [19, 20, 21, 22, 23, 24. 25, 26, 27, 28, 29, 30, 31, 32, 33, 34].

In this paper, the multiplicative vertex status index, multiplicative modified vertex status index, multiplicative $F$-status index, general multiplicative vertex status index of some standard graphs are computed.

## 2. RESULTS FOR COMPLETE GRAPHS

**Theorem 1:** If $K_n$ is a complete graph with $n$ vertices, then the general multiplicative vertex status index of $K_n$ is

$$S_v^a II(K_n) = (n-1)^{an}. \tag{1}$$

**Proof:** Let $K_n$ be a complete graph with $n$ vertices. Then $\sigma(u) = n - 1$ for every vertex $u$ of $K_n$. Therefore

$$S_v^a II(K_n) = \prod_{u \in V(K_n)} \sigma(u)^a = (n-1)^{an}.$$

We obtain the following results by using Theorem 1.

**Corollary 1.1:** If $K_n$ is a complete graph with $n$ vertices, then

(i) $S_v II(K_n) = (n-1)^{2n}$. (ii) $T_s II(K_n) = (n-1)^{n}$.

(iii) ${}^m S_v II(K_n) = \left(\frac{1}{(n-1)^2}\right)^n$. (iv) $SI\ II(K_n) = \left(\frac{1}{n-1}\right)^n$.

(v) $SZII(K_n) = \left(\frac{1}{n-1}\right)^{\frac{n}{2}}$. (vi) $FSII(K_n) = (n-1)^{3n}$.

**Proof:** Put $a$ = 2, 1, –2, –1, –½, 3 in equation (1), we get the desired results.

## 3. RESULTS FOR CYCLES

**Theorem 2:** Let $C_n$ be a cycle with $n$ vertices. Then the general multiplicative vertex status index of $C_n$ is

$$S_v^a II(C_n) = \left(\frac{n^2}{4}\right)^{an}, \quad \text{for } n \text{ is even,} \tag{2}$$

$$= \left(\frac{n^2-1}{4}\right)^{an}, \quad \text{for } n \text{ is odd.} \tag{3}$$

**Proof:** Let $C_n$ be a cycle with $n$ vertices.

**Case-1:** Suppose $n$ is even. For every vertex $u$ in $C_n$, $\sigma(u) = \frac{n^2}{4}$. Thus

$$S_v^a II(C_n) = \prod_{u \in V(C_n)} \sigma(u)^a = \left(\frac{n^2}{4}\right)^{an}.$$

**Case-2:** Suppose $n$ is odd. Then $\sigma(u) = \frac{n^2-1}{4}$ for every vertex $u$ in $C_n$. Thus

$$S_v^a II(C_n) = \prod_{u \in V(C_n)} \sigma(u)^a = \left(\frac{n^2-1}{4}\right)^{an}.$$

We establish the following results by Theorem 2.

 

**Corollary 2.1:** Let $C_n$ be a cycle with $n$ vertices. Then

(i) $S_v II(C_n) = \left(\frac{n^2}{4}\right)^{2n}$, if $n$ is even,

$= \left(\frac{n^2-1}{4}\right)^{2n}$, if $n$ is odd.

(ii) $T_v II(C_n) = \left(\frac{n^2}{4}\right)^{n}$, if $n$ is even,

$= \left(\frac{n^2-1}{4}\right)^{n}$, if $n$ is odd.

(iii) ${}^m S_v II(C_n) = \left(\frac{n^2}{4}\right)^{-2n}$, if $n$ is even,

$= \left(\frac{n^2-1}{4}\right)^{-2n}$, If $n$ is odd.

(iv) $SI\ II(C_n) = \left(\frac{n^2}{4}\right)^{-n}$, if $n$ is even,

$= \left(\frac{n^2-1}{4}\right)^{-n}$, if $n$ is odd.

(v) $SZII(C_n) = \left(\frac{n^2}{4}\right)^{-\frac{n}{2}}$, if $n$ is even,

$= \left(\frac{n^2-1}{4}\right)^{-\frac{n}{2}}$, if $n$ is odd.

(vi) $FSII(C_n) = \left(\frac{n^2}{4}\right)^{3n}$, if $n$ is even,

$= \left(\frac{n^2-1}{4}\right)^{3n}$, if $n$ is odd.

**Proof:** Put $a$ = 2, 1, –2, –1, –½, 3 in equations (2) and (3), we get the desired results.

## 4. RESULTS FOR COMPLETE BIPARTITE GRAPHS

Let $K_{p,q}$ be a complete bipartite graph. Then it has $p+q$ vertices and $pq$ edges. In $K_{p,q}$, there are two types of status vertices as given in Table 1.

| $\sigma(u) \setminus u \in E(K_{p,q})$ | $p + 2(q-1)$ | $q + 2(p-1)$ |
|---|---|---|
| Number of vertices | $q$ | $p$ |

**Table-1:** Status vertex partition of $K_{p,q}$

**Theorem 3:** The general multiplicative vertex status index of $K_{p,q}$ is

$$S_v^a II(K_{p,q}) = [p + 2(q-1)]^{aq} \times [q + 2(p-1)]^{ap}. \tag{4}$$

**Proof:** Let $K_{p,q}$ be a complete bipartite graph. By definition, we have

$$S_v^a II(K_{p,q}) = \prod_{u \in V(K_{p,q})} \sigma(u)^a.$$

By using Table 1, we deduce

$$S_v^a II(K_{p,q}) = [p + 2(q-1)]^{aq} \times [q + 2(p-1)]^{ap}.$$

From Theorem 3, we establish the following results.

**Corollary 3.1:** Let $K_{p,q}$ be a complete bipartite graph. Then

(i) $S_vII(K_{p,q}) = [p+2(q-1)]^{2q} \times [q+2(p-1)]^{2p}$.

(ii) $T_sII(K_{p,q}) = [p+2(q-1)]^{q} \times [q+2(p-1)]^{p}$.

(iii) ${}^mS_vII(K_{p,q}) = \dfrac{1}{[p+2(q-1)]^{2q}} \times \dfrac{1}{[q+2(p-1)]^{2p}}$.

(iv) $SIII(K_{p,q}) = \dfrac{1}{[p+2(q-1)]^{q}} \times \dfrac{1}{[q+2(p-1)]^{p}}$.

(v) $SZII(K_{p,q}) = \dfrac{1}{[p+2(q-1)]^{\frac{q}{2}}} \times \dfrac{1}{[q+2(p-1)]^{\frac{p}{2}}}$.

(vi) $FSII(K_{p,q}) = [p+2(q-1)]^{3q} \times [q+2(p-1)]^{3p}$.

**Proof:** Put $a$ = 2, 1, –2, –1, –½, 3 in equation (4), we obtain the desired results.

## 5. RESULTS FOR WHEEL GRAPHS

A wheel graph, denoted by $W_n$, is the join of $C_n$ and $K_1$. A wheel graph $W_4$ is shown in Figure 1.

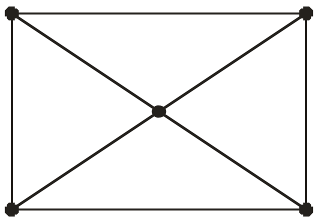

**Figure-1:** Wheel graph $W_4$

Clearly, a wheel graph $W_n$ has $n$+1 vertices and 2$n$ edges. This graph has two types of status vertices as given in Table 2.

| $\sigma(u) \setminus u \in V(W_n)$ | $n$ | $2n-3$ |
|---|---|---|
| Number of vertices | 1 | $n$ |

**Table-2:** Status vertex partition of $W_n$

**Theorem 4:** The general multiplicative vertex status index of a wheel graph $W_n$ is

$$S_v^aII(W_n) = n^a \times (2n-3)^{an}. \tag{5}$$

**Proof:** By definition and by using Table 2, we deduce

$$S_v^aII(W_n) = \prod_{u\in V(W_n)} \sigma(u)^a = (n^a)^1 \times [(2n-3)^a]^n = n^a \times (2n-3)^{an}$$

From Theorem 4, we obtain the following results.

**Corollary 4.1:** Let $W_n$ be a wheel graph with $n$+1 vertices and 2$n$ edges. Then

(i) $S_vII(W_n) = n^2(2n-3)^{2n}$.

(ii) $T_SII(W_n) = n(2n-3)^n$.

(iii) ${}^mS_vII(W_n) = \dfrac{1}{n^2(2n-3)^{2n}}$.

(iv) $SIII(W_n) = \dfrac{1}{n(2n-3)^n}$.

(v) $SZII(W_n) = \dfrac{1}{\sqrt{n(2n-3)^n}}$.

(vi) $FSII(W_n) = n^3(2n-3)^{3n}$.

**Proof:** Put a = 2, 1, –2, –1, –½, 3 in equation (5), we get the desired results.

## 6. RESULTS FOR FRIENDSHIP GRAPHS

A friendship graph, denoted by $F_n$, is the graph obtained by taking $n \geq 2$ copies of $C_3$ with vertex in common. The friendship graph $F_4$ is shown in Figure 2.

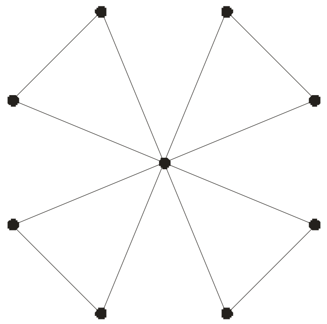

**Figure-2:** Friendship graph $F_4$

Clearly, a friendship graph $F_n$ has $2n + 1$ vertices and $3n$ edges. This graph has two types of status vertices as given in Table 3.

| $\sigma(u) \setminus u \in V(F_n)$ | $2n$ | $4n - 3$ |
|---|---|---|
| Number of vertices | 1 | 2n |

**Table-3:** Status vertex partition of $F_n$

**Theorem 5:** The general multiplicative vertex status index of a friendship graph $F_n$ is

$$S_v^a II(F_n) = (2n)^a \times (4n-3)^{2an}. \tag{6}$$

**Proof:** By definition and using Table 3, we derive

$$S_v^a II(F_n) = \prod_{u \in V(F_n)} \sigma(u)^a = \left((2n)^a\right)^1 \times \left[(4n-3)^a\right]^{2n} = (2n)^a \times (4n-3)^{2an}.$$

We establish the following results from Theorem 5.

**Corollary 5.1:** Let $F_n$ be a friendship graph with $2n$+1 vertices and $3n$ edges. Then

(i) $S_v II(F_n) = 4n^2 (4n-3)^{4n}$.

(ii) $T_S II(F_n) = 2n(4n-3)^{2n}$.

(iii) ${}^m S_v II(F_n) = \dfrac{1}{4n^2 (4n-3)^{4n}}$.

(iv) $SI\ II(F_n) = \dfrac{1}{2n(4n-3)^{2n}}$.

(v) $SZII(F_n) = \dfrac{1}{(2n-3)^n \sqrt{2n}}$.

(vi) $FSII(F_n) = 8n^3 (4n-3)^{6n}$.

**Proof:** Put $a$ = 2, 1, –2, –1, –½, 3 in equation (6), we obtain the required results.

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