

GENERALIZED SEMI STAR b-CLOSED SETS IN TOPOLOGICAL SPACES

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(Received On: 24-01-20; Received & Accepted On: 21-02-20)

ABSTRACT

The aim of this paper is to introduce the notion of generalized semi star b-closed (briefly, gs^*b -closed) set in topological spaces. The properties of this closed set are investigated and they are compared with the existing relevant generalized closed sets in topological spaces. Also, we study the characterizations of gs^*b -closed sets. Further, gs^*b -open sets in topological spaces are discussed with the suitable examples.

Keywords: gs^*b -closed, gs^*b -open.

1. INTRODUCTION

In 1970, Levine introduced the class of generalized closed sets. The notion of generalized closed sets has been extended and studied exclusively in recent years by many topologists. In 1996, Andrijevic gave a new type of generalized closed sets in topological spaces called b-closed sets.

In this paper, a new class of closed set called generalized semi star b-closed set is introduced. The notion of generalized semi star b-closed set and its different characterizations are given in this paper. It has been proved that the class of generalized semi star b-closed set lies between the class of semi*-closed set and sgb-closed set.

2. PRELIMINARIES

Throughout this paper (X, τ) represents a topological space on which no separation axiom is assumed unless otherwise mentioned. For a subset A of a topological space X, $cl(A)$ and $int(A)$ denote the closure of A and the interior of A respectively. (X, τ) will be replaced by X if there is no changes of confusion. We recall the following definitions and results.

Definition 2.1: Let (X, τ) be a topological space. A subset A of the space X is said to be

1. semi-open [9] if $A \subseteq cl(int(A))$ and semi-closed [3] if $int(cl(A)) \subseteq A$.
2. α -open [13] if $A \subseteq int(cl(int(A)))$ and α -closed if $cl(int(cl(A))) \subseteq A$.
3. pre-open [14] if $A \subseteq int(cl(A))$ and pre-closed if $cl(int(A)) \subseteq A$.
4. b-open [16] if $A \subseteq int(cl(A)) \cup cl(int(A))$ and b-closed if $int(cl(A)) \cap cl(int(A)) \subseteq A$.
5. regular open if $int(cl(A)) = A$ and regular closed if $cl(int(A)) = A$.
6. π -open [4] if A is the union of regular open sets and π -closed if A is the intersection of regular closed sets.

Definition 2.2: Let (X, τ) be a topological space and $A \subseteq X$. The b-closure (resp.pre-closure, semi-closure, α -closure) of A, denoted by $bcl(A)$ (resp. $pcl(A)$, $scl(A)$, $\alpha cl(A)$) and is defined by the intersection of all b-closed (resp. pre-closed, semi-closed, α -closed) sets containing A.

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Definition 2.3: Let (X, τ) be a topological space. A subset A of X is said to be

1. generalized closed [8] (briefly g-closed) if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ) .
2. generalized b-closed [2] (briefly gb-closed) if $\text{bcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ) .
3. regular generalized closed [7] (briefly rg-closed) if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in (X, τ) .
4. regular generalized b-closed [17] (briefly rgb-closed) if $\text{bcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in (X, τ) .
5. generalized α b-closed [15] (briefly α gb-closed) if $\text{bcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is α -open in (X, τ) .
6. generalized pre-regular closed [20] (briefly gpr-closed) if $\text{pcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is rg-open in (X, τ) .
7. generalized p-closed [2] (briefly gp-closed) if $\text{pcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ) .
8. α -generalized closed [10] (briefly α g-closed) if $\alpha\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is an open in (X, τ) .
9. π -generalized b-closed [6] (briefly π gb-closed) if $\text{bcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is π -open in (X, τ) .
10. π -generalized pre-closed [6] (briefly π gp-closed) if $\text{pcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is π -open in (X, τ) .
11. π -generalized semi-closed [6] (briefly π gs-closed) if $\text{scl}(A) \subseteq U$ whenever $A \subseteq U$ and U is π -open in (X, τ) .
12. weakly generalized closed [18] (briefly wg-closed) [2] if $\text{cl}(\text{int}(A)) \subseteq U$ whenever $A \subseteq U$ and U is an open in (X, τ) .
13. Semi weakly generalized closed (briefly swg-closed) [18] if $\text{scl}(A) \subseteq U$ whenever $A \subseteq U$ and U is an wg-open in (X, τ) .
14. Semi-generalized closed [2] (briefly sg-closed) if $\text{scl}(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in (X, τ) .
15. Semi generalized b-closed [6] (briefly sgb-closed) if $\text{bcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in (X, τ) .
16. generalized s-closed [2] (briefly gs-closed) if $\text{scl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ) .

The complements of the above mentioned closed sets are their respective open sets.

Definition 2.4: [22] If A is a subset of X ,

- (i) The generalized closure of A is defined as the intersection of all g-closed sets in X containing A and is denoted by $\text{cl}^*(A)$.
- (ii) The generalized interior of A is defined as the union of all g-open sets in X that are contained in A and is denoted by $\text{int}^*(A)$.

Definition 2.5: [22] A subset A of a topological space (X, τ) is said to be semi*-open if $A \subseteq \text{cl}^*(\text{int}(A))$ and semi*-closed if $\text{int}^*(\text{cl}(A)) \subseteq A$.

Definition 2.6: [22] The semi*-closure of A (briefly $s^*\text{cl}(A)$) is defined as the intersection of all semi*-closed sets in X containing A and the semi*-interior of A (briefly $s^*\text{int}(A)$) is defined as the union of all semi*-open sets contained in A .

Theorem 2.7: For a topological space (X, τ) ,

- (i) Every open set is b-open.
- (ii) Every α -open set is b-open.
- (iii) Every semi-open set is b-open.

Theorem 2.8: [21] For any subset A of a topological space (X, τ) ,

- (i) $\text{sint}(A) = A \cap \text{cl}(\text{int}(A))$
- (ii) $\text{pint}(A) = A \cap \text{int}(\text{cl}(A))$
- (iii) $\text{scl}(A) = A \cup \text{int}(\text{cl}(A))$
- (iv) $\text{pcl}(A) = A \cup \text{cl}(\text{int}(A))$.

Remark 2.9: [21] Jankovic and Reilly pointed out that every singleton $\{x\}$ of a space X is either nowhere dense or pre-open. This provides another decomposition $X = X_1 \cup X_2$, where $X_1 = \{x \in X / \{x\} \text{ is nowhere dense}\}$ and $X_2 = \{x \in X / \{x\} \text{ is pre-open}\}$.

Definition 2.10: [21] The intersection of all gb-open sets containing A is called the gb-kernel of A and it is denoted by $\text{gb-ker}(A)$.

Lemma 2.11: [21] For any subset A of X , $X_2 \cap \text{cl}(A) \subseteq \text{gb-ker}(A)$.

Remark 2.12: [21] $\text{cl}(X \setminus A) = X \setminus \text{int}(A)$.

3. GENERALIZED s*b -CLOSED SET

Definition 3.1: A subset A of a topological space (X, τ) is called a generalized semi star b-closed set (briefly, gs*b-closed) if $s^*\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is b-open in (X, τ) .

Theorem 3.2: For a topological space (X, τ) ,

- (i) Every closed set is gs*b-closed.
- (ii) Every semi*-closed set is gs*b-closed.
- (iii) Every regular closed set is gs*b-closed.
- (iv) Every π -closed set is gs*b-closed.

Proof:

- (i) Let A be a closed set. Let $A \subseteq U$, U is b-open in X. Since A is closed, then $\text{cl}(A)=A \subseteq U$. But $s^*\text{cl}(A) \subseteq \text{cl}(A)$. Thus we have $s^*\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is b-open. Therefore, A is a gs*b-closed set.
- (ii) (ii),(iii) and (iv) are similar to (i)

Theorem 3.3: For a topological space (X, τ) ,

- (i) Every gs*b-closed set is gb-closed.
- (ii) Every gs*b-closed set is gs-closed.
- (iii) Every gs*b-closed set is sg-closed.
- (iv) Every gs*b-closed set is rgb-closed.

Proof:

- (i) Let A be a gs*b-closed set. Let $A \subseteq U$, U is open. Since open set is b-open, then U is b-open. Since A is gs*b-closed, $s^*\text{cl}(A) \subseteq U$. But $bcl(A) \subseteq s^*\text{cl}(A)$. Thus, we have $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open. Therefore, A is gb-closed set.
- (ii) (ii),(iii) and (iv) are similar to (i)

Theorem 3.4: For a topological space (X, τ) ,

- (i) Every gs*b-closed set is $g\alpha b$ -closed set.
- (ii) Every gs*b-closed set is πgb -closed set.
- (iii) Every gs*b-closed set is πgs -closed set.
- (iv) Every gs*b-closed set is sgb-closed set.

Proof:

- (i) Let A be a gs*b-closed set. Let $A \subseteq U$, U is α -open. Since every α -open set is b-open, then U is b-open. Since A is gs*b-closed, $s^*\text{cl}(A) \subseteq U$. But $bcl(A) \subseteq s^*\text{cl}(A)$. Thus, we have $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is α -open. Therefore, A is $g\alpha b$ -closed.
- (ii) (ii),(iii) and (iv) are similar to (i)

Remark 3.5: The reverse implications of the above theorems need not be true which is shown in the following examples.

Example 3.6: Let $X= \{a, b, c, d\}$ with $\tau= \{\phi, X, \{a\}, \{c\}, \{a, c\}, \{c, d\}, \{a, c, d\}, \{a, b, c\}\}$. Here the gs*b-closed sets in (X, τ) are $\phi, X, \{a\}, \{b\}, \{d\}, \{a, b\}, \{a, d\}, \{b, d\}, \{c, d\}, \{b, c, d\}, \{a, b, d\}$.

- (1) $\{b, d\}$ is gs*b-closed but not regular closed and π -closed. Also $\{a\}$ is gs*b-closed but not πgp -closed, g-closed, rg-closed and gp-closed.
- (2) $\{c\}$ is swg-closed but not gs*b-closed.

Example 3.7: Let $X= \{a, b, c, d\}$ with $\tau= \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{b, c, d\}, \{a, b, c\}\}$. Here the gs*b-closed sets in (X, τ) are $\phi, X, \{a\}, \{c\}, \{d\}, \{a, c\}, \{a, d\}, \{c, d\}, \{b, c, d\}, \{a, c, d\}$.

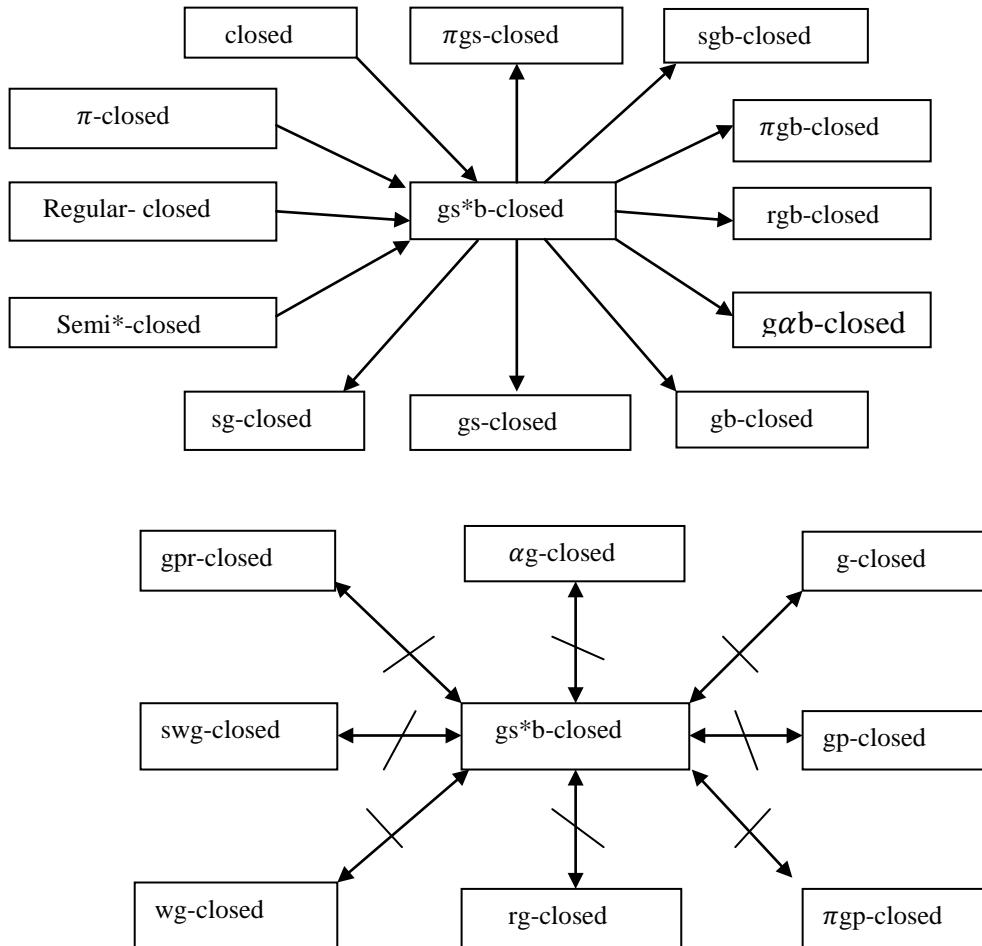
- (1) $\{b\}$ is rgb-closed, πgs -closed and πgb -closed but not gs*b-closed.
- (2) $\{b, d\}$ is gp-closed, g-closed, rg-closed, πgp -closed, αg -closed, gb-closed and gs-closed and wg-closed but not gs*b-closed.

Example 3.8: Let $X= \{a, b, c, d\}$ with $\tau= \{\phi, X, \{b\}, \{c\}, \{d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{b, c, d\}\}$. Here the gs*b-closed sets in (X, τ) are $\phi, X, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}$.

Also $\{b, c, d\}$ is gs*b-closed but not closed, αg -closed, wg-closed, swg-closed and gpr-closed.

Remark 3.9: The gs*b-closed sets are independent from αg -closed set, g-closed set, rg-closed set, wg-closed set, swg-closed set, gpr-closed set, πgp -closed set, gp-closed set.

Remark 3.10: From the above results, we have the following implications diagrams.



4. CHARACTERIZATION

Theorem 4.1: If a set A is gs*b-closed in (X, τ) , then $s^*\text{cl}(A) \setminus A$ contains no non-empty b-closed sets in (X, τ) .

Proof: Let F be a b-closed subset of X such that $F \subseteq s^*\text{cl}(A) \setminus A$. Then $F \subseteq s^*\text{cl}(A) \cap (X \setminus A)$. That implies, $F \subseteq s^*\text{cl}(A)$ and $F \subseteq (X \setminus A)$. Then $A \subseteq X \setminus F$ and $X \setminus F$ is b-open in (X, τ) . Since A is gs*b-closed in X, $s^*\text{cl}(A) \subseteq X \setminus F$, $F \subseteq X \setminus s^*\text{cl}(A)$. Thus $F \subseteq s^*\text{cl}(A) \cap (X \setminus s^*\text{cl}(A)) = \emptyset$. Hence $s^*\text{cl}(A) \setminus A$ does not contain any non-empty b-closed sets.

Theorem 4.2: If a subset A is gs*b-closed set in (X, τ) and $A \subseteq B \subseteq s^*\text{cl}(A)$, then B is also a gs*b-closed set.

Proof: Let A be a gs*b-closed set and B be any subset of X such that $A \subseteq B \subseteq s^*\text{cl}(A)$. Let U be b-open in (X, τ) such that $B \subseteq U$. Then $A \subseteq U$. Also since A is gs*b-closed, $s^*\text{cl}(A) \subseteq U$. Since $B \subseteq s^*\text{cl}(A)$, $s^*\text{cl}(B) \subseteq s^*\text{cl}(s^*\text{cl}(A)) = s^*\text{cl}(A) \subseteq U$. This implies, $s^*\text{cl}(B) \subseteq U$. Thus B is a gs*b-closed set.

Definition 4.3: Let (X, τ) be a topological space and Y be a subspace of X. Then the subset A of Y is b-open in Y if $A = G \cap Y$, where G is b-open in X.

Theorem 4.4: Let $A \subseteq Y \subseteq X$ and suppose that A is gs*b-closed in X then A is gs*b-closed relative to Y.

Proof: Given that $A \subseteq Y \subseteq X$ and A is a gs*b-closed set in X. To prove that A is gs*b-closed set relative to Y. Let us assume that $A \subseteq Y \cap U$, where U is b-open in X. Since A is gs*b-closed set in X, then $s^*\text{cl}(A) \subseteq U$. That implies $Y \cap s^*\text{cl}(A) \subseteq Y \cap U$, where $Y \cap s^*\text{cl}(A)$ is the semi*-closure of A in Y and $Y \cap U$ is b-open in Y. Therefore $s^*\text{cl}(A) \subseteq Y \cap U$ in Y. Hence, A is gs*b-closed set relative to Y.

Theorem 4.5: Let A be any gs*b-closed set in (X, τ) . Then A is semi*-closed in (X, τ) iff $s^*\text{cl}(A) \setminus A$ is b-closed.

Proof: Necessity: Since A is semi*-closed set in (X, τ) , $s^*\text{cl}(A) = A$. Then $s^*\text{cl}(A) \setminus A = \emptyset$, which is a b-closed set in (X, τ) .

Sufficiency: Since A is gs^* -b-closed set in (X, τ) , by Theorem 4.1, $s^*\text{cl}(A) \setminus A$ does not contain any non-empty b-closed set. Therefore, $s^*\text{cl}(A) \setminus A = \emptyset$. Hence $s^*\text{cl}(A) = A$. Thus A is semi*-closed set in (X, τ) .

Theorem 4.6: For every element x in a space X , $X - \{x\}$ is gs^* -b-closed or b-open.

Proof: Case-(i): Suppose $X - \{x\}$ is not b-open. Then X is the only b-open set containing $X - \{x\}$. This implies $s^*\text{cl}(X - \{x\}) \subseteq X$. Hence $X - \{x\}$ is gs^* -b-closed.

Case-(ii): Suppose $X - \{x\}$ is not gs^* -b-closed. Then there exists a b-open set U containing $X - \{x\}$ such that $s^*\text{cl}(X - \{x\})$ does not contain U . Now $s^*\text{cl}(X - \{x\})$ is either $X - \{x\}$ or X . If $s^*\text{cl}(X - \{x\}) = X - \{x\}$, then $X - \{x\}$ is semi*-closed. Since every semi*-closed set is gs^* -b-closed, $X - \{x\}$ is gs^* -b-closed, which is a contradiction. Therefore $s^*\text{cl}(X - \{x\}) = X$. To prove that, $X - \{x\}$ is b-open. Suppose not. Then by case (i), $X - \{x\}$ is gs^* -b-closed. There is a contradiction to our assumption. Hence $X - \{x\}$ is b-open.

Theorem 4.7: If A is both b-open and gs^* -b-closed set in X , then A is semi*-closed set.

Proof: Since A is b-open and gs^* -b-closed in X , $s^*\text{cl}(A) \subseteq A$. But always $A \subseteq s^*\text{cl}(A)$. Therefore, $A = s^*\text{cl}(A)$. Hence A is a semi*-closed set.

Definition 4.8: The intersection of all b-open sets containing A is called the b-kernel of A and it is denoted by $b\text{-ker}(A)$.

Theorem 4.9: A subset A of X is gs^* -b-closed iff $s^*\text{cl}(A) \subseteq b\text{-ker}(A)$.

Proof: Necessity: Let A be a gs^* -b-closed subset of X and $x \in s^*\text{cl}(A)$. Suppose $x \notin b\text{-ker}(A)$. Then there exists a b-open set U containing A such that $x \notin U$. Since A is gs^* -b-closed set, then $s^*\text{cl}(A) \subseteq U$. This implies that, $x \notin s^*\text{cl}(A)$, which is a contradiction to $x \in s^*\text{cl}(A)$. Therefore $s^*\text{cl}(A) \subseteq b\text{-ker}(A)$.

Sufficiency: Suppose $s^*\text{cl}(A) \subseteq b\text{-ker}(A)$. If U is any b-open set containing A , then $b\text{-ker}(A) \subseteq U$. That implies, $s^*\text{cl}(A) \subseteq U$. Hence A is gs^* -b-closed in X .

Remark 4.10: For any subset A of X , $gb\text{-ker}(A) \subseteq b\text{-ker}(A)$.

Theorem 4.11: For any subset A of X , $X_2 \cap s^*\text{cl}(A) \subseteq b\text{-ker}(A)$.

Proof: Since $s^*\text{cl}(A) \subseteq cl(A)$, then $X_2 \cap s^*\text{cl}(A) \subseteq X_2 \cap cl(A)$. Then by Lemma 2.11 and Remark 4.10, $X_2 \cap s^*\text{cl}(A) \subseteq b\text{-ker}(A)$.

Theorem 4.12: A subset A of X is gs^* -b-closed if and only if $X_1 \cap s^*\text{cl}(A) \subseteq A$.

Proof: Necessity: Suppose that A is gs^* -b-closed and $x \in X_1 \cap s^*\text{cl}(A)$. Then $x \in X_1$ and $x \in s^*\text{cl}(A)$. Since $x \in X_1$, then $\text{int}(\text{cl}(\{x\})) = \emptyset$. That implies, $\text{cl}(\text{int}(\text{cl}(\{x\}))) = \emptyset$. Therefore $\{x\}$ is α -closed. Then $\{x\}$ is b-closed. If x does not belong to A , then $U = X - \{x\}$ is a b-open set containing A and so $s^*\text{cl}(A) \subseteq U$. Since $x \in s^*\text{cl}(A), x \in U$. This is a contradiction to x not in U . Hence $X_1 \cap s^*\text{cl}(A) \subseteq A$.

Sufficiency: Let $X_1 \cap s^*\text{cl}(A) \subseteq A$. Then $X_1 \cap s^*\text{cl}(A) \subseteq b\text{-ker}(A)$. Now, $s^*\text{cl}(A) = X \cap s^*\text{cl}(A) = (X_1 \cup X_2) \cap s^*\text{cl}(A) = (X_1 \cap s^*\text{cl}(A)) \cup (X_2 \cap s^*\text{cl}(A)) \subseteq b\text{-ker}(A)$. Then by Theorem 4.9, A is gs^* -b-closed.

Remark 4.13: Union of any two gs^* -b-closed sets in (X, τ) need not be a gs^* -b-closed set which is shown in the following example.

Example 4.14: Let $X = \{a, b, c\}$ with $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$. The gs^* -b-closed sets in (X, τ) are $\emptyset, X, \{a\}, \{b\}, \{c\}, \{b, c\}, \{a, c\}$. The sets $\{a\}$ and $\{b\}$ are gs^* -b-closed sets but their union $\{a, b\}$ is not a gs^* -b-closed set.

Theorem 4.15: Arbitrary intersection of gs^* -b-closed sets is gs^* -b-closed.

Proof: Let $\{A_i\}$ be the collection of gs^* -b-closed sets of X . Let $A = \bigcap A_i$. Since $A \subseteq A_i$, for each i , then $s^*\text{cl}(A) \subseteq s^*\text{cl}(A_i)$. That implies, $X_1 \cap s^*\text{cl}(A) \subseteq X_1 \cap s^*\text{cl}(A_i)$. Since each A_i is gs^* -b-closed, then by Theorem 4.12, $X_1 \cap s^*\text{cl}(A_i) \subseteq A_i$, for each i . Now, $X_1 \cap s^*\text{cl}(A) = X_1 \cap s^*\text{cl}(\bigcap A_i) \subseteq \bigcap (X_1 \cap s^*\text{cl}(A_i)) \subseteq \bigcap A_i = A$. Again by Theorem 4.12, A is gs^* -b-closed.

Remark 4.16: The set of all gs^* -b-closed sets in a topological space X , form a topology on X .

Theorem 4.17: Let A be a gs*b-closed in X. Then

- (i) $sint(A)$ is gs*b-closed.
- (ii) If A is regular open, then $pint(A)$ and $scl(A)$ are also gs*b-closed.
- (iii) If A is regular closed, then $pcl(A)$ is also gs*b-closed.

Proof: Let A be a gs*b-closed set of X.

- (i) Since $cl(int(A))$ is closed, then by Theorem 3.2(i), $cl(int(A))$ is gs*b-closed, $sint(A)$ is gs*b-closed.
- (ii) Suppose A is regular open, then $int(cl(A))=A$. By Lemma 2.8, $scl(A)=A$. Since A is gs*b-closed, then $scl(A)$ is gs*b-closed. Similarly $pint(A)$ is gs*b-closed.
- (iii) Suppose A is regular closed, then $cl(int(A))=A$. Then by Lemma 2.8, $pcl(A)=A$, and hence gs*b-closed.

5. GENERALIZED s*b-OPEN

Definition 5.1: A subset A of (X, τ) is said to be generalized s*b-open (briefly gs*b-open) set if its complement $X \setminus A$ is gs*b-closed in X. The family of all gs*b-open sets in X is denoted by $gs^*b-O(X)$.

Theorem 5.2: Let (X, τ) be a topological space and $A \subseteq X$. Then A is a gs*b-open if and only if $F \subseteq s^*int(A)$, whenever $F \subseteq A$ and F is b-closed.

Proof: Necessity: Let A be a gs*b-open set in (X, τ) . Let $F \subseteq A$ and F is b-closed. Then $X \setminus A$ is gs*b-closed and it is contained in the b-open set $X \setminus F$. Therefore $s^*cl(X \setminus A) \subseteq X \setminus F$. This implies that $X \setminus s^*int(A) \subseteq X \setminus F$. Hence $F \subseteq s^*int(A)$.

Sufficiency: If F is b-closed set such that $F \subseteq s^*int(A)$ whenever $F \subseteq A$. It follows that $X \setminus A \subseteq X \setminus F$ and $X \setminus s^*int(A) \subseteq X \setminus F$. Therefore $s^*cl(X \setminus A) \subseteq X \setminus F$. Hence $X \setminus A$ is gs*b-closed and hence A is gs*b-open.

Theorem 5.3: If a set A is gs*b-open and $B \subseteq X$ such that $s^*int(A) \subseteq B \subseteq A$, then B is gs*b-open.

Proof: If $s^*int(A) \subseteq B \subseteq A$, then $X \setminus A \subseteq X \setminus B \subseteq X \setminus s^*int(A)$. That is, $X \setminus A \subseteq X \setminus B \subseteq s^*cl(X \setminus A)$. Since $X \setminus A$ is gs*b-closed, then by Theorem 4.2, $X \setminus B$ is gs*b-closed and hence B is gs*b-open.

Theorem 5.4: If a subset A is gs*b-open in X and G is b-open in X with $s^*int(A) \cup (X \setminus G) \subseteq G$ then $X = G$.

Proof: Suppose that G is b-open and $s^*int(A) \cup (X \setminus G) \subseteq G$. This implies, $X \setminus G \subseteq (X \setminus s^*int(A)) \cap A = s^*cl(X \setminus A) \setminus (X \setminus A)$. Since $X \setminus A$ is gs*b-closed and $X \setminus G$ is b-closed, then by Theorem 4.1, $X \setminus G = \emptyset$. Hence $X = G$.

Remark 5.5: Union of gs*b-open sets is gs*b-open in a topological space X.

Remark 5.6: Intersection of any two gs*b-open sets in (X, τ) need not be a gs*b-open set.

Theorem 5.7: If B is gs*b-open and $s^*int(B) \subseteq A$, then $A \cap B$ is gs*b-open.

Proof: Suppose B is gs*b-open and $s^*int(B) \subseteq A$. Then $s^*int(A \cap B) \subseteq A \cap B \subseteq B$. By Theorem 5.3, $A \cap B$ is gs*b-open.

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Source of support: Nil, Conflict of interest: None Declared.

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