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# ON SEMI GENERALIZED TOPOLOGICAL SPACES

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***ABSTRACT***

*The aim of this paper is to introduce and study two new classes of spaces,namely Semi generalised normal and Semi generalized regular spaces and obtained their properties by utilizing Semi generalised closed sets.*

***Keywords:*** *Semi generalised closed set,Semi generalised open sets, Semi generalised regular space and Semi generalised normal space.*

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## 1 INTRODUCTION

Maheshwari and Prasad[8] introduced the new class of spaces called s-normal spacesusing semi-open sets. It was further studied by Noiri and Popa[10],Dorsett[6] andArya [1]. Munshi[9], introduced g-regular and g- normal spaces using g-closed sets of Levine[7]. Later, Benchalli *et al.* [3] and Shik John[12] studied the concept of g*- pre-regular, g*-pre normal and w- normal, w-regular spaces in topological spaces.Recently, Benchalli *et al*. [2,11] introduced and studied the properties of regular weakly closedsets and regular weakly continuous functions.

## 2. PRELIMINARIES

Throughout this paper space (X, τ) and (Y, σ) (or simply X and Y) always denote topological space on which no separation axioms are assumed unless explicitly stated. For a subset A of a space X, Cl(A), Int(A), $A^c$ , and $\alpha$-Cl(A), denote the Closure of A, Interior of A and Compliment of A and $\alpha$-closure of A in X respectively.

**Definition 2.1:** A subset A of a topological space (X, τ) is called
(i)Generalized closed set(briefly g-closed) [7] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in X.
(ii)W-closed set[ 12] if cl(A) $\subseteq$ U whenever A$\subseteq$ U and U is semi-open in X.

**Definition 2.2:** A topological space X is said to be a

1. $\alpha$ - regular [4], if for each $\alpha$ - closed set F of X and each point x $\notin$ F, there existsdisjoint $\alpha$ - open sets U and V such that F $\subseteq$V and x $\epsilon$ U.
2. w-regular[12], if for each closed set F of X and each point x $\notin$ F, there existsdisjoint w-open sets U and V such that F$\subseteq$U and x$\epsilon$V.
3. g-regular[10], if for each g-closed set F of X and each point x $\notin$F,there exists disjoint open sets U and V such that F$\subseteq$U and x $\epsilon$ V.
4. Semi generalized closed set[15] Scl(A) $\subseteq$U When A$\subseteq$U and U is Semi-open in X

**Definition 2.3:** A topological space X is said to be a

1. $\alpha$-normal [4], if for any pair of disjoint α − closed sets A and B, there exists dis-joint $\alpha$-open sets U and V such that A$\subseteq$U and B$\subseteq$V .
2. w-normal [12], if for any pair of disjoint w -closed sets A and B, there existsdisjoint open sets U and V such that A$\subseteq$ U and B$\subseteq$V.
3. g- normal [10], if for any pair of disjoint g-closed sets A and B, there exists disjoint open sets U and V such that A$\subseteq$U and B$\subseteq$V.

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**Definition 2.4:** [2] A topological space X is called $T_{\text{regular weakly}}$ -space if every Semi generalised closed setis closed set.

**Definition 2.5:** A map f: (X, τ) ⟶ (Y, τ) is said to be

(i) Semi generalised continuous map[16]if $f^{-1}$(V)is a Semi generalised closed set of (X, τ) for every closed set V of (Y, τ).
(ii) Semi generalised irresolute map[16] if $f^{-1}$(V)is a Semi generalised closed set of (X, τ) for everySemi generalised closed set V of (Y, τ).

## 3. SEMI GENERALIZED REGULAR SPACE

In this section, we introduce a new class of space called Semi generalised regular space using Semi generalised closed set and obtain some of their characterizations.

**Definition 3.1.** A topological space Xis said to be Semi generalised regular space if for each Semi generalised closedset F and a point x∉F, there exist disjoint open sets G and H such that F⊆G andx∊H.

We have the following interrelationship between Semi generalised regularity and regularity.

**Theorem 3.2:** Every Semi generalised regular space is regular.

**Proof:** Let X be a Semi generalised regular space. Let F be any closed set in X and a point x∉Xsuch that x∉F. By [2], F is Semi generalised topological space-closed and x ∉F. Since X is a Semi generalised regular space, thereexists a pair of disjoint open sets G and H such that F ⊆ G and x ∊H. Hence X is aregular space.

**Remark 3.3:** If X is a regular space and $T_{\text{Semi generalised topological space}}$, then X is Semi generalised regular space then we have the following characterization.

**Theorem 3.4:** The following statements are equivalent for a topological space X.
(i) X is a Semi generalised regular space
(ii) For each x ∊X and each Semi generalised topological spaceopen neighbourhood U of x,there exists an open neighbourhood N of x such that cl(N)⊆U.

**Proof:** (i)implies(ii): Suppose X is a Semi generalised regular space. Let U be any Semi generalised neighbourhood of x. Then there exists Semi generalised open set G such that x ∊ G ⊆U. Now X – G is Semi generalised closed set and x ∉ X - G. Since X is Semi generalised regular space, then there exist open sets Mand N such that X -G⊆M, x ∊ N and M ∩ N = 𝜑and so N ⊆X-M. Nowcl(N) ⊆ cl(X -M) = X-M and X -M ⊆M. This implies X -M⊆ U. Thereforecl(N)⊆U.
(ii)implies (i): Let F be any Semi generalised topological space closed set in X and x ∊X -F and X - F is aSemi generalised topological space open and so X - F is a Semi generalised topological space neighbourhood of x. By hypothesis, there existsan open neighbourhood N of x such that x ∊N and cl(N) ⊆X - F. This impliesF ⊆X - cl(N) is an open set containing F and N ∩ f(X - cl(N)= 𝜑 . Hence X isSemi generalised regular space.

We have another characterization of Semi generalised regularity in the following.

**Theorem 3.5:** A topological space X is Semi generalised regular if and only if for each Semi generalised topological space closedset F of X and each x ∊X - F there exist open sets G and H of X such that x ∊ G,F⊆H and cl(G) ∩ cl(H) = ∅.

**Proof:** Suppose X is Semi generalised regular space. Let F be a Semi generalised topological space closed set in X with x ∉F.Then there exists open sets M and H of X such that x ∊ M, F ⊆H and M∩H =∅.This implies M∩cl(H) = ∅.As X is Semi generalised regular, there exist open sets U and V suchthat x ∊ U, cl(H)⊆V and U∩V = ∅. so cl(U)∩V = ∅.
Let G = M ∩U, then G andH are open sets of X such that x∊G, F ⊆ H and cl(H) ∩cl(H) =∅.
Conversely, if for each Semi generalised closed set F of X and each x ∊ X -F there exists opensets G and H such that x ∊ G, F⊆ H and cl(H)∩cl(H) =∅.This implies x ∊G,F⊆H and G ∩ H = ∅. Hence X is Semi generalised regular.
Now we prove that Semi generalised topological spaces- regularity is a heriditary property.

**Theorem 3.6:** Every subspace of a Semi generalised regular space is Semi generalised regular.

**Proof:** Let X be a Semi generalised regular space. Let Y be a subspace of X. Let x ∊ Y and F bea Semi generalised closed set in Y such that x∉F. Then there is a closed set and so Semi generalised closedset A of X with F = Y ∩ A and x ∉A. Therefore we have x ∊ X, A is Semi generalised closedin X such that x∉A. Since X is Semi generalised regular, then there exist open sets G and H suchthat x ∊G, A⊆H and G∩H = 𝜑. Note that Y ∩G and Y∩H are open sets in Y.Also x ∊ G and x ∊Y, which implies x ∊ Y ∩G and A ⊆ H implies Y∩ G⊆Y ∩H,F⊆Y∩H. Also (Y∩G)∩(Y∩H) = 𝜑. Hence Y is Semi generalised regular space.

We have yet another characterization of Semi generalised topological spaces-regularity in the following.

**Theorem 3.7:** The following statements about a topological space X are equivalent:
(i) X is Semi generalised regular
(ii) For each x $\epsilon$ X and each Semi generalised topological space open set U in X such that x $\epsilon$ U there exists anopen set V in X such that x $\epsilon$ V⊆cl(V)⊆U.
(iii) For each point x $\epsilon$X and for each Semi generalised topological space closed set A with x ∉ A, then there exists anopen set V containing x such that cl(V)∩A = $\varphi$.

**Proof:**
(i) implies(ii): Follows from Theorem 3.5.
(ii) implies(iii): Suppose (ii) holds. Let x $\epsilon$ X and A be an Semi generalised topological spaceclosed set of X suchthat x ∉ A.Then X - A is a Semi generalised topological spaceopen set with x$\epsilon$X -A. By hypothesis, thereexists an open set V such that x $\epsilon$V ⊆ cl(V ) ⊆ X - A. That is x$\epsilon$ V , V⊆ cl(A) andcl(A) ⊆ X - A. So x $\epsilon$V and cl(V)∩A = $\varphi$.
(iii) implies(i): Let x $\epsilon$ X and U be an Semi generalised topological space open set in X such that x $\epsilon$ U. ThenX - U is an Semi generalised topological spaceclosed set and x∉ X - U. Then by hypothesis, there exists an openset V containing x such that cl(A) ∩(X -U) = Á. Therefore x $\epsilon$V , cl(V )⊆U sox $\epsilon$ V⊆ cl(V)⊆U.

The invariance of Semi generalised topological space regularity is given in the following.

**Theorem 3.8:** Let f : X ⟶Y be a bijective, Semi generalised topological space irresolute and open map from a Semi generalised topological space regular space X into a topological space Y, then Y is Semi generalised topological spaces-regular.

**Proof:** Let y $\epsilon$ Y and F be a Semi generalised topological space closed set in Y with y ∉ F. Since F is Semi generalised topological spaceirresolute, $f^{-1}$(F) is Semi generalised topological space closed set in X. Let f(x) = y so that x = $f^{-1}$ (y) andx ∉ $f^{-1}$(F). Again X is Semi generalised-regular space, then there exist open sets U and V such thatx $\epsilon$ U and $f^{-1}$(F) ⊆ G, U ∩ V = $\varphi$. Since f is open and bijective, we have y $\epsilon$ f(U),F ⊆ f(V ) and f(U) ∩f(V) = f(U∩V ) = f($\varphi$) = $\varphi$ . Hence Y is Semi generalised regular space.

**Theorem 3.9:** Let f : X ⟶Y be a bijective, Semi generalised closed and open map from atopological space X into a Semi generalised regular space Y . If X is $T_{\text{Semi generalised topological spaces}}$, then X is Semi generalised regular.

**Proof:** Let x $\epsilon$X and F be an Semi generalised closed set in X with x ∉F. Since X is $T_{\text{Semi generalised topological spaces}}$,F is closed in X. Then f(F) is Semi generalised closed set with f(x)∉ f(F) in Y , since f is Semi generalised closed. As Y is Semi generalised regular,then there exist open sets U and V such that x $\epsilon$ Uandf(x) $\epsilon$ U and f(F) ⊆V . Therefore x $\epsilon$ $f^{-1}$(U) and F⊆ $f^{-1}$(V ). Hence X is Semi generalised regular space.

**Theorem 3.10:** If f : X ⟶Y is w-irresolute, continuous injection and Y is Semi generalised topological spaces-regular space,then X is Semi generalised topological spaces- regular.

**Proof:** Let F be any closed set in X with x∉F. Since f is w-irresolute, f is Semi generalised topological space closed set in Yand f(x) $\epsilon$ f(F). Since Y is Semi generalised regular,then there exists open sets U and V such that f(x) $\epsilon$U and f(F) ⊆ V . Thus x $\epsilon$$f^{-1}$(U),F ⊆ $f^{-1}$(V ) and $f^{-1}$ (U) ∩ $f^{-1}$(V ) = $\varphi$. Hence X is Semi generalised regular space.

## 4. SEMI GENERALISED NORMAL SPACES

In this section, we introduce the concept of Semi generalised normal spaces and study some of theircharacterizations.

**Definition 4.1:** A topological space X is said to be Semi generalised normal if for each pair ofdisjoint Semi generalised topological spacesclosed sets A and B in X,then there exists a pair of disjoint open sets U andV in X such that A ⊆ U and B ⊆V

We have the following interrelationship.

**Theorem 4.2:** Every Semi generalised normal space is normal.

**Proof:** Let X be a Semi generalised normal space. Let A and B be a pair of disjoint closed sets inX. From [2], A and B are Semi generalised topological spacesclosed sets in X. Since X is Semi generalised normal, then there existsa pair of disjoint open sets G and H in X such that A⊆ G and B ⊆ H. Hence X is normal.

**Remark 4.3:** The converse need not be true in general as seen from the following example.

 

**Example 4.4:** Let X = Y ={a,b,c,d},τ ={X, ∅,{a},{c},{a,c},{b,c,d}}.Then
the space X is normal but not Semi generalised normal, since the pair of disjoint Semi generalised topological spacesclosed setsnamely, A = {a,d}and B = {b,c}for which there do not exists disjoint open sets Gand H such that A ⊆ G and B⊆ H.

**Remark 4.5:** If X is normal and $T_{\text{Semi generalised topological spaces}}$, then X is Semi generalised -normal.

Hereditary property of Semi generalised normality is given in the following.

**Theorem 4.6:** A Semi generalised closed subspace of a Semi generalised normal space is Semi generalised normal.We have the following characterization.

**Theorem 4.7:** The following statements for a topological space X are equivalent:

(i) X is Semi generalised topological spaces is normal

(ii) For each Semi generalised closed set A and each pre generalized pre regular weakly topological space open set U such that A⊆U, there existsan open set V such that A⊆V⊆cl(V)⊆U

(iii) For any Semi generalised closed sets A, B, there exists an open set V such that A⊆V andcl(V)∩B = $\varphi$.

(iv) For each pair A, B of disjoint Semi generalised closed sets then there exist open sets U and V suchthat A ⊆ U,B ⊆V and cl(U) ∩cl(V ) = $\varphi$.

**Proof:**

(i) implies(ii): Let A be a Semi generalised closed set and U be a Semi generalised open set such that A⊆ U.Then A and X - U are disjoint Semi generalised closed sets in X. Since X is Semi generalised normal , then thereexists a pair of disjoint open sets V and W in X such that A ⊆ V and X -U⊆W.Now X -W ⊆ X -(X -U), so X -W⊆ U also V∩W = $\varphi$.implies V ⊆ X -W, so cl(V)⊆cl(X -W) which implies cl(V ) ⊆ X -W. Therefore cl(V )⊆X -W⊆U. So cl(V ) ⊆ U. Hence A⊆V⊆cl(V)⊆U.

(ii) implies(iii): Let A and B be a pair of disjoint Semi generalised closed sets in X. Now A∩B = $\varphi$,so A⊆X -B, where A is Semi generalised closed and X - B is Semi generalised open . Then by (ii) thereexists an open set V such that A⊆V⊆cl(V)⊆X-B. Now cl(V)⊆X - B impliescl(V)∩B = $\varphi$. Thus A⊆V and cl(V)∩B = $\varphi$.

(iii) implies(iv): Let A and B be a pair of disjoint Semi generalised closed sets in X.Then from (iii)there exists an open set U such that A⊆U and cl(U)∩B = $\varphi$. Since cl(V ) is closed, soSemi generalised closed set.Therefore cl(V ) and B are disjoint Semi generalised closed sets in X. By hypothesis,then their exists an open set V , such that B⊆V and cl(U) ∩cl(V ) = $\varphi$.

(iv) implies(i): Let A and B be a pair of disjoint Semi generalised closed sets in X.Then from (iv)then there exist an open sets U and V in X such that A⊆U, B⊆V and cl(U)∩cl(V ) = $\varphi$.So A ⊆ U , B⊆V and U∩V = $\varphi$.Hence X is Semi generalised normal.

**Theorem 4.8:** Let X be a topological space. Then X is Semi generalised normal if and only if forany pair A, B of disjoint Semi generalised closed setthen there exist open sets U and V of X such thatA⊆U,B⊆V and cl(U) ∩cl(V ) = $\varphi$.

**Theorem 4.9:** Let X be a topological space. Then the following are equivalent:

(i) X is normal

(ii) For any disjoint closed sets A and B,then there exist disjoint Semi generalised topological spaces- open sets U and Vsuch that A⊆U,B⊆V .

(iii) For any closed set A and any open set V such that A⊆ V, there exists an Semi generalised open set U of X such that A⊆U⊆$\alpha$cl(U)⊆ V .

**Proof:**

(i) implies(ii): Suppose X is normal. Since every open set is Semi generalised open [2], (ii)follows.

(ii) implies(iii): Suppose (ii) holds. Let A be a closed set and V be an open set containing A. Then A and X - V are disjoint closed sets. By (ii), then there exist disjoint Semi generalised open sets U and W such that A⊆U and X - V ⊆ W, since X -V is closed, soSemi generalised is closed. From [2], we have X -V⊆$\alpha$-int(W) and U ∩ $\alpha$-int(W) =$\varphi$.and so wehave $\alpha$-cl(U) ∩ $\alpha$-int(W) = $\varphi$. Hence A ⊆ U ⊆$\alpha$-cl(U) ⊆ X – $\alpha$-int(W)⊆V . ThusA ⊆ U ⊆$\alpha$-cl(U) ⊆ V .

(iii) implies(i): Let A and B be a pair of disjoint closed sets of X.Then A ⊆X - B andX -B is open. There exists a Semi generalised open set G of X such that A ⊆ G ⊆$\alpha$-cl(G) ⊆X-B.Since A is closed, it is w- closed, we have A ⊆$\alpha$-int(G). Take U = int(cl(int($\alpha$-int(G))))and V = int(cl(int(X –$\alpha$-cl(G)))). Then U and V are disjoint open sets of X such thatA ⊆U and B ⊆ V Hence X is normal.

We have the following characterization of Semi generalised topological spaces- normality and Semi generalised topological spaces- normality.

**Theorem 4.10:** Let X be a topological space. Then the following are equivalent:

(i) X is $\alpha$-normal.

(ii) For any disjoint closed sets A and B, there exist disjoint Semi generalised topological space- open sets U and Vsuch that A⊆U,B⊆V and U∩ V = $\varphi$.

**Proof:**

(i) implies(ii): Suppose X is $\alpha$- normal. Let A and B be a pair of disjoint closedsets of X. Since X is $\alpha$-normal,there exist disjoint $\alpha -$ open sets U and V such thatA⊆U and B⊆V and U ∩ V = $\varphi$.

(ii) implies(i):Let A and B be a pair of disjoint closed sets of X.The by hypothesis thereexist disjoint Semi generalised open sets U and V such that A⊆U and B ⊆ V and U ∩V = $\varphi$ .Sincefrom [2], A⊆$\alpha$-intU and B ⊆$\alpha -$ int(V)and $\alpha$ –intU∩ $\alpha$-intV = $\varphi$. Hence X is $\alpha$-normal.

**Theorem 4.11:** Let X bea $\alpha$- normal, then the following hold good:

(i) For each closed set A and every Semi generalised open set B such that A⊆B ther exists a αopen set U such that A⊆U⊆α-cl(U)⊆ B.

(ii) For every Semi generalised closed set A and every open set B containing A, there exist a α-open set U such that A⊆U⊆ α-cl(U)⊆B.

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