A NEW APPROACH FOR SOLVING HEXADECAGONAL FUZZY TRANSPORTATION PROBLEM

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ABSTRACT

T̲ransportation problem is one of the sub classes of linear programming problem. The objective of the transportation problem is to minimize the transportation cost or maximize the profit. Fuzzy set theory has been applied in many fields of science, Engineering and Management. In this paper a new ranking method is proposed for solving hexadecagonal fuzzy transportation problem. Fuzzy transportation problem transformed into crisp transportation problem and solved by MODI method. A numerical example is presented and the optimal solution obtained by using proposed method.

Keywords: Hexadecagonal Fuzzy number, Ranking Method, Numerical Example.

1. INTRODUCTION

Transportation problem is the special case of linear programming problem. The objective of the transportation problem is to minimize the transportation cost or maximizing the profit while satisfying the rim requirements. The methods for solving the transportation problem are available only for when the demand and supply quantities are exactly known. But in some situation the values of the problem are not exactly known. This uncertainty leads fuzziness. A fuzzy transportation problem is a transportation problem in which the transportation costs, supply and demand quantities are fuzzy quantities. A.L Zadeh [14] was first introduced the concept of fuzzy sets to deal with imprecision, vagueness in real life situations. Pandian and G.Natarajan [11] proposed a new method called a fuzzy zero point method to find the fuzzy optimal solution of fuzzy transportation problems. Solaiappan and Jeyaraman [12] investigated the fuzzy transportation problem by using zero termination method. Nagoor Gani and K.Abdul Razak [10] solved two stage fuzzy transportation problem Priyanka A. Pathade, Kirtiwant P. Ghadle [1] were finding optimal solution of balanced and unbalanced fuzzy transportation problem of octagonal fuzzy numbers.. A.Sahaya Sudha, S.Karunambigai [2] solved a fuzzy transportation problem using a heptagonal fuzzy number. Jatinder Pal Singh and Neha Ishesh Thakur [3] proposed a new method for solving fuzzy transportation problem using dodecagonal fuzzy number.L.Sujatha, P. Vinothini and R. Jothilakshmi [6] solved a fuzzy transportation problem using zero point maximum allocation method. S. U. Malini, Felbin and C. Kennedy [4] gave the comparative study of trapezoidal, octagonal and dodecagonal fuzzy numbers in solving fuzzy transportation problem. V.J. Sudhakar and V. Navaneetha Kumar [5] solved the multiobjective two stage fuzzy transportation problem by zero suffix method. S.Krishna prabha, V.Seerengasamy [7] proposed a method for solving unbalanced fuzzy transportation problem for maximizing the profit. S. Narayanamoorthy, S.Saranya & S.Maheswari [8] solved a fuzzy transportation problem using fuzzy russell's method. M. S. Annie Christi [9] finding solution of fuzzy transportation problem using best candidate method. Shugani Poonam, S.H Abbas and V.K Gupta [13] solved fuzzy vector transportation problem in interval integer form

In this paper a new ranking method is proposed solving a hexadecogonal fuzzy transportation problem. Fuzzy transportation problem can be converted in to crisp transportation problem using ranking method and an optimal solution is obtained by using MODI method.

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2. PRELIMINARIES

Definition 2.1: A fuzzy set is characterized by a membership function mapping element of a domain space or the universe of discourse X to the unit interval {0, 1}

(i.e) $A = \{x, \mu_A(x) \; ; x \in X\}$ .Here $\mu_A(x) = 1$

Definition 2.2: A fuzzy set A of the universe of discourse X is called normal fuzzy set implying that there exist atleast one $x \in X$ Such that $\mu_A(x) = 1$

Definition 2.3: The support of fuzzy set in the Universal set X is the set that contains all the elements of X that have anon- zero membership grade in $\tilde{A}$.(i.e) $Supp(\tilde{A}) = \left\{x \in X \,/\, \mu_{\tilde{A}}(x) > 0\right\}$

Definition 2.4: Given a fuzzy set A defined on X and any number $\alpha \in [0,1]$ the α - cut, α_A is the crisp set $\alpha_A = \{x \in X \,/\, A(x) \geq \alpha, \alpha \in [0,1]\}$

Definition 2.5: A fuzzy set $\tilde{A}$ defined on the set of real numbers R is said to be fuzzy number if its membership function $\mu_A(x): R \to [0,1]$ has the following properties
(i)   A must be a normal and convex fuzzy set
(ii) α_A must be a closed interval for every $\alpha \in (0,1]$
(iii) The support of $\tilde{A}$ must be bounded

Definition 2.6: A fuzzy number $\tilde{A}$ is called triangular function is denoted by $\tilde{A} = (a_1, a_2, a_3)$ whose membership function is defined as follows

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & x < a_1 \\ \dfrac{x - a_1}{a_2 - a_1} & a_1 \leq x \leq a_2 \\ \dfrac{a_3 - x}{a_3 - a_2} & a_2 \leq x \leq a_3 \\ 0 & x > a_3 \end{cases}$$

3. HEXADECAGONAL FUZZY NUMBER

A fuzzy number $\tilde{A}$ is a Hexadecagonal fuzzy number defined by $\tilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16})$ where

$a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}$ are real numbers and its membership function is given by

$$\mu_A(x) = \begin{cases} 0 & x \le a_1 \\ k_1\left(\dfrac{x-a_1}{a_2-a_1}\right) & a_1 \le x \le a_2 \\ k_1 & a_2 \le x \le a_3 \\ k_1 + (k_2-k_1)\left(\dfrac{x-a_3}{a_4-a_3}\right) & a_3 \le x \le a_4 \\ k_2 & a_4 \le x \le a_5 \\ k_2 + (k_3-k_2)\left(\dfrac{x-a_5}{a_6-a_5}\right) & a_5 \le x \le a_6 \\ k_3 & a_6 \le x \le a_7 \\ k_3 + (1-k_3)\left(\dfrac{x-a_7}{a_8-a_7}\right) & a_7 \le x \le a_8 \\ 1 & a_8 \le x \le a_9 \\ k_3 + (1-k_3)\left(\dfrac{a_{10}-x}{a_{10}-a_9}\right) & a_9 \le x \le a_{10} \\ k_3 & a_{10} \le x \le a_{11} \\ k_2 + (k_3-k_2)\left(\dfrac{a_{12}-x}{a_{12}-a_{11}}\right) & a_{11} \le x \le a_{12} \\ k_2 & a_{12} \le x \le a_{13} \\ k_1 + (k_2-k_1)\left(\dfrac{a_{14}-x}{a_{14}-a_{13}}\right) & a_{13} \le x \le a_{14} \\ k_1 & a_{14} \le x \le a_{15} \\ k_1\left(\dfrac{a_{16}-x}{a_{16}-a_{15}}\right) & a_{15} \le x \le a_{16} \\ 0 & a_{16} \le x \end{cases}$$

where $0 < k_1 < k_2 < k_3 < 1$

3.1. Arithmetic operations on Dodecagonal Fuzzy number

Let $\widetilde{A}_{HXDFN} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16})$ &
$\widetilde{B}_{HXDFN} = (b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8, b_9, b_{10}, b_{11}, b_{12}, b_{13}, b_{14}, b_{15}, b_{16})$

be two hexadecogonal fuzzy numbers then the addition and subtraction can be performed as

$$\widetilde{A}_{HXDFN} + \widetilde{B}_{HXFDN} = [a_1+b_1, a_2+b_2, a_3+b_3, a_4+b_4, a_5+b_5, a_6+b_6, a_7+b_7, a_8+b_8, a_9+b_9, a_{10}+b_{10},$$
$$a_{11}+b_{11}, a_{12}+b_{12}, a_{13}+b_{13}, a_{14}+b_{14}, a_{15}+b_{15}, a_{16}+b_{16}]$$

$$\widetilde{A}_{HXFDN} - \widetilde{B}_{Dode} = [a_1-b_1, a_2-b_2, a_3-b_3, a_4-b_4, a_5-b_5, a_6-b_6, a_7-b_7, a_8-b_8, a_9-b_9, a_{10}-b_{10},$$
$$a_{11}-b_{11}, a_{12}-b_{12}, a_{13}-b_{13}, a_{14}-b_{14}, a_{15}-b_{15}, a_{16}-b_{16}]$$

3.2. Measure of fuzzy number

The measure of $\tilde{A}_\omega$ is a measure is a function $M_o : R_\omega(I) \to R^+$ which assign a non negative real numbers $M_o{}^{HXDFN}\left(\tilde{A}_\omega\right)$ that expresses the measure of

$$M_o{}^{HXDFN}\left(\tilde{A}_\omega\right) = \frac{1}{2}\int_\alpha^{k_1}\left(f_1(r) + \bar{f}_1(r)\right)dr + \frac{1}{2}\int_{k_1}^{k_2}\left(g_1(s) + \bar{g}_1(s)\right)ds + \frac{1}{2}\int_{k_2}^{k_3}\left(h_1(t) + \bar{h}_1(t)\right)dt + \frac{1}{2}\int_{k_3}^{\omega}\left(l_1(w) + \bar{l}_1(w)\right)dw$$

where $0 \le \alpha < 1$

4. RANKING METHOD

Let $\tilde{A}$ be a normal Hexadecagonal fuzzy number. The measure of $\tilde{A}$ is calculated as follows

$$M_o{}^{HXDFN}\left(\tilde{A}_\omega\right) = \frac{1}{2}\int_0^{k_1}\left(f_1(r) + \bar{f}_1(r)\right)dr + \frac{1}{2}\int_{k_1}^{k_2}\left(g_1(s) + \bar{g}_1(s)\right)ds + \frac{1}{2}\int_{k_2}^{k_3}\left(h_1(t) + \bar{h}_1(t)\right)dt + \frac{1}{2}\int_{k_3}^{1}\left(l_1(w) + \bar{l}_1(w)\right)dw$$

$$M_o{}^{HXDFN}(\tilde{A}) = \frac{1}{4}\left\{\begin{array}{l}(a_1 + a_2 + a_{15} + a_{16})k_1 + (a_3 + a_4 + a_{13} + a_{14})(k_2 - k_1) \\ + (a_5 + a_6 + a_{11} + a_{12})(k_3 - k_2) + (a_7 + a_8 + a_9 + a_{10})(1 - k_3)\end{array}\right\}$$

$$where \; 0 < k_1 < k_2 < k_3 < 1$$

5. MATHEMATICAL FORMULATION OF FUZZY TRANSPORTATION PROBLEM

The general form of fuzzy transportation problem is given by

$$Minimize \; Z = \sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij}c_{ij} \; \text{Subject to the constraints}$$

$$\sum_{j=1}^{n} x_{ij} = a_i, \; i = 1,2,...m$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \; j = 1,2,...n$$

and $x_{ij} \ge 0$ for all i and j

where a_i = the fuzzy supply of the product at source i, b_j = the fuzzy demand of the product at source j,

c_{ij} = transportation cost of commodity from i^{th} source to j^{th} destination, x_{ij} = transported quantity from i^{th} source to j^{th} destination

6. NUMERICAL EXAMPLE

Consider the following hexadecagonal fuzzy transportation problem.

	D_1	D_2	D_3	Supply
S_1	(1,2,3,4,5,6,7,8,9,10,11,12,13,14, 15,16)	(0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15)	(1,3,5,7,9,11,13,15,17,19,21,23,25,27, 29,31)	(1,3,4,5,7,9,10,12,14,15, 16,18,20,21, 22,24)
S_2	(2,4,6,8,10,12,14,16,18,20,22,24,26,28,30,32)	(2,3,5,7,9,11,13,17,19,23,29,31,35,37, 41,43)	(1,4,7,10,13,16,19,22,25,28,31,34,37,40,43, 46)	(0,4,6,9,11,12,13,14,15,16,19,21,23,25,28,30)
S_3	(1,2,3,4,7,10,13,15,16,17,22,26,30,34, 35,36)	(2,4,6,8,9,13,15,16,18,20,21,25,27,28, 30,31)	(1,2,3,4,5,7,9,11,13,17,21,25,27,31,32, 34)	(2,4,8,9,11,13,16,19,20,22,24,25,27,29, 30,31)
Demand	(1,2,3,4,8,9,10,12,13,15,17,18,20,22,24, 26)	(0,2,3,5,6,7,9,10,13,15,16,18,21,24,29, 34)	(3,4,7,10,11,13,15,16,18,21,24,28,30,34,36,44)	

7. RANKING OF HEXADECAGONAL FUZZY NUMBER

Taking the value of $k_1 = 0.3$, $k_2 = 0.5$, $k_3 = 0.8$. The ranking of fuzzy numbers is done by using

Hexadecagonal Number	Ranking values
$C_{11} = (1,2,3,4,5,6,7,8,9,10, 11,12,13,14,15,16)$	$M_o^{HDF} = 8.5$
$C_{12} = (0,1,2,3,4,5,6,7,8,9,10, 11,12,13,14,15)$	$M_o^{HDF} = 7.5$
$C_{13} = (1,3,5,7,9,11,13,15,17,19,21,23,25,27,29,31)$	$M_o^{HDF} = 16$
$C_{21} = (2,4,6,8,10,12,14,16, 18,20,22,24,26,28,30,32)$	$M_o^{HDF} = 17$
$C_{22} = (2,3,5,7,9,11,13,17,19, 23,29,31,35,37,41,43)$	$M_o^{HDF} = 20.5$
$C_{23} = (1,4,7,10,13,16,19,22, 25,28,31,34,37,40,43, 46)$	$M_o^{HDF} = 23.5$
$C_{31} = (1,2,3,4,7,10,13,15,16,17,22,26,30,34,35,36)$	$M_o^{HDF} = 17$
$C_{32} = (2,4,6,8,9,13,15,16,18, 20,21,25,27,28,30,31)$	$M_o^{HDF} = 15.3$
$C_{33} = (1,2,3,4,5,7,9,11,13,17,21,25,27,31,32,34)$	$M_o^{HDF} = 20.6$

The ranking of fuzzy supply is

$S_1 =(1,3,4,5,7,9, 10,12,14,15, 16,18,20,21, 22,24)$	$M_o^{HDF} = 12.6$
$S_2 =(0,4,6,9,11,12,13,14, 15,16,19,21,23,25,28,30)$	$M_o^{HDF} = 15.4$
$S_3 =(2,4,8,9,11,13,16,19, 20, 22,24,25,27,29,30,31)$	$M_o^{HDF} = 18$

The ranking of fuzzy demand is

$D_1 =(1,2,3,4,8,9,10,12,13,15,17,18,20,22,24,26)$	$M_o^{HDF} = 12.8$
$D_2 = (0,2,3,5,6,7,9,10,13,15,16,18,21,24,29,34)$	$M_o^{HDF} = 13.4$
$D_3 = (3,4,7,10,11,13,15,16,18,21,24,28,30,34,36,44)$	$M_o^{HDF} = 19.8$

The crisp transportation problem is

	D_1	D_2	D_3	Supply
S_1	8.5	7.5	16	12.6
S_2	17	20.5	17	15.4
S_3	17	15.5	15.3	18
Demand	12.8	13.4	19.8	

Here fuzzy total demand = 46, fuzzy total supply =46

Therefore this is a balanced transportation problem

By applying the Vogel's approximation method, we find the initial basic feasible solution.

	D_1		D_2		D_3		Supply
S_1	12.6	8.5		7.5		16	12.6
S_2	0.2	17		20.5		17	15.4
S_3		17	13.4	15.5	4.6	15.3	18
Demand	12.8		13.4		19.8		

The initial basic feasible solution is 388.6. Using the MODI method to find the optimal solution

The optimal solution of the transportation problem is 388.6

CONCLUSION

In this a new method is proposed for solving hexadecagonal fuzzy transportation problem. Fuzzy transportation problem transformed into crisp transportation problem and solved by VAM and MODI method. Numerical example is presented and the optimal solution is obtained.

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