

**PART-IV ON NON-ASSOCIATIVE Γ -SEMI SUB NEAR-FIELD SPACES
OF A Γ -NEAR-FIELD SPACE OVER NEAR-FIELD (PART-IV NA- Γ -SSNFS- Γ -NFS-NF)**

SMT. THURUMELLA MADHAVI LATHA*¹

Author cum research Scholar,
Junior Lecturer, Department of Mathematics, APSWREIS
Tadepalli, Guntur District, Amaravathi, Andhra Pradesh. INDIA.

DR T V PRADEEP KUMAR²

Assistant Professor of Mathematics cum Guide,
A N U College of Engineering & Technology,
Department of Mathematics, Acharya Nagarjuna University
Nambur, Nagarjuna Nagar 522 510, Guntur District. Andhra Pradesh. INDIA.

(Received On: 21-12-19; Revised & Accepted On: 20-01-20)

ABSTRACT

In this manuscript we introduce two new classes of non-associative Γ -semi sub near-field spaces which is define as Thurumella loop Γ -semi sub near-field spaces and Thurumella groupoid Γ -semi sub near-field spaces.

Keywords: *Thurumella loop Γ -semi sub near-field space, Thurumella groupoid Γ -semi sub near-field space, loop Γ -semi sub near-field spaces, near loop Γ -semi sub near-field space, Non-associative Γ -semi sub near-field space, Thurumella- non-associative Γ -semi sub near-field space.*

2000 Mathematics Subject Classification: *46H25, 6H99, 46L10, 46M2051 M 10, 51 F 15,03 B 30, 43A10, 46B28.*

SECTION-1: INTRODUCTION ON THURUMELLA LOOP NON-ASSOCIATIVE Γ -SEMI SUB NEAR-FIELD SPACES (T-L-NA- Γ -SSNFS- Γ -NFS-NF) AND THURUMELLA GROUPOID LOOP NON-ASSOCIATIVE Γ -SEMI SUB NEAR-FIELD SPACES (T-G-L-NA- Γ -SSNFS- Γ -NFS-NF) OF A Γ -NEAR-FIELD SPACE OVER NEAR-FIELD.

Here in this section, we know the class of loop Γ -semi sub near-field spaces is a generalization of group Γ -semi sub near-field spaces and loop near-field spaces. In fact groupoid Γ -semi sub near-field spaces which contain all these classes of Γ -semi sub near-field spaces of a near-field space. We are more interested in the study of their Thurumella nature. The concept of classical identities play a vital role when the structure under study happens to be non-associative. With the advent of a new class of loops of even order and new class of groupoids of all order built using integers we are able to give examples which are concrete.

One of the major drawbacks in the study of near-field spaces or any algebraic structure is the lack of concrete examples. Unless we have concrete examples it becomes difficult to make the subject or that algebraic structure profoundly attractive to many researchers. For instance, the research in near-field theory over ring theory or group theory or semi group theory are carried out by more researchers when compared to the theory of loop, theory of groupoid. So in this section we introduce the Thurumella analogue of these concepts and try our level best to illustrate them whenever possible with examples.

Definition 1.1: The system $N = (N, '+', '\cdot')$ be a non-empty set, N endowed with two binary operations '+' and ' \cdot '. Satisfying the following conditions.

- $(N, +)$ is a Γ -semi sub near-field space
 - (N, \cdot) is a groupoid.
 - $(a + b) \cdot c = a \cdot c + b \cdot c$ for $a, b, c \in N$. $(N, +, \cdot)$ is defined as a non-associative Γ -semi sub near-field space (NA- Γ -semi sub near-field space).
-

**Corresponding Author: Smt. Thurumella Madhavi Latha¹, Junior Lecturer,
Department of Mathematics, APSWREIS, Tadepalli, Guntur District,
Amaravathi, Andhra Pradesh. INDIA.**

But one of the major draw backs is that we do not have several examples of these Γ -semi sub near-field spaces ; so it is become difficult to give examples using Z_p alone or Z alone or any of the known sets. Thus to obtain examples of non-associative Γ -semi sub near-field spaces we define loop Γ -semi sub near-field space and groupoid Γ -semi sub near-field spaces which has been defined earlier.

Definition 1.2: Let N be a non-empty set. Define two binary operations ‘+’ and ‘.’ Satisfying the following conditions:

- a. $(N, +)$ is a Γ -semi sub near-field space
- b. (N, \cdot) is a groupoid.
- c. $(a + b) \cdot c = a \cdot c + b \cdot c$ for $a, b, c \in N$. then we call N a Thurumella NA- Γ -semi sub near-field space of level I (T-NA-- Γ -semi sub near-field space of level I).

Now we define T-NA-- Γ -semi sub near-field space of level II.

Definition 1.3: Let $(N, +, \cdot)$ be a non-associative Γ -semi sub near-field space of a Γ -near-field space over near-field. N is said to Thurumella NA- Γ -semi sub near-field space of level II (T-NA- Γ -semi sub near-field space of level II) . If N has a proper Γ -semi sub near-field space $P \subset N$ where P itself a Non Associative Γ -semi sub near-field space of a Γ -near-field space over near-field.

Definition 1.4: Let $(N, +, \cdot)$ be a non-associative Γ -semi sub near-field space of a Γ -near-field space over near-field. N is said to Thurumella pseudo NA- Γ -semi sub near-field space (T-pseudo NA- Γ -semi sub near-field space). If N has a proper Γ -semi sub near-field space $P \subset N$ where P itself a NA- Γ -semi sub near-field space of a Γ -near-field space over operations of a near-field.

Now still we have a level III and level IV definition of Thurumella NA-- Γ -semi sub near-field spaces.

Definition 1.5: Let $(N, +, \cdot)$ be a non-associative Γ -semi sub near-field space of a Γ -near-field space over near-field. N is said to Thurumella NA- Γ -semi sub near-field space of level III (T-NA- Γ -semi sub near-field space of level III). If N has a proper Γ -semi sub near-field space $P \subset N$ where P itself an associative Γ -semi sub near-field space of a Γ -near-field space over near-field.

Finally, we define a level IV definition of Thurumella NA-- Γ -semi sub near-field spaces of level IV and obtain the possible and probable relation between the four levels of T-NA- Γ -semi sub near-field spaces.

Definition 1.6: Let $(N, +, \cdot)$ be a NA- Γ -semi sub near-field space we say N is a Thurumella NA- Γ -semi sub near-field space of level IV (T-NA- Γ -semi sub near-field space of level IV) if N has a proper Γ -semi sub near-field space P such that $(P, +, \cdot)$ is an associative - Γ -semi sub near-field space.

SECTION-2: MAIN RESULTS ON SOME SPECIAL THURUMELLA LOOP NON-ASSOCIATIVE Γ -SEMI SUB NEAR-FIELD SPACES (T-L-NA- Γ -SSNFS- Γ -NFS-NF) AND THURUMELLA GROUPOID LOOP NON-ASSOCIATIVE Γ -SEMI SUB NEAR-FIELD SPACES (T-G-L-NA- Γ -SSNFS- Γ -NFS-NF) OF A Γ -NEAR-FIELD SPACE OVER NEAR-FIELD.

In this section 3, we deduce main results on some special Thurumella loop non-associative Γ -semi sub near-field spaces (T-L-NA- Γ -SSNFS- Γ -NFS-NF) and Thurumella groupoid loop non-associative Γ -semi sub near-field spaces (T-G-L-NA- Γ -SSNFS- Γ -NFS-NF) of a Γ -near-field space over near-field.

Theorem 1.7: Let $(N, +, \cdot)$ be a Thurumella NA- Γ -semi sub near-field space level of IV then $(N, +, \cdot)$ is a T-NA- Γ -semi sub near-field space of level II.

Proof: Here we recall the definitions of level III and level IV of Thurumella NA- Γ -semi sub near-field space.

Let $(N, +, \cdot)$ be a non-associative Γ -semi sub near-field space of a Γ -near-field space over near-field. N is said to Thurumella NA- Γ -semi sub near-field space of level III (T-NA- Γ -semi sub near-field space of level III) . If N has a proper Γ -semi sub near-field space $P \subset N$ where P itself an associative Γ -semi sub near-field space of a Γ -near-field space over near-field.

Let $(N, +, \cdot)$ be a NA- Γ -semi sub near-field space we say N is a Thurumella NA- Γ -semi sub near-field space of level IV (T-NA- Γ -semi sub near-field space of level IV) if N has a proper Γ -semi sub near-field space P such that $(P, +, \cdot)$ is an associative - Γ -semi sub near-field space.

It is clear that a T-NA- Γ -semi sub near-field space of level III which is not a Thurumella NA- Γ -semi sub near-field space of level IV. Thus we see the class of T-NA- Γ -semi sub near-field space of level IV is strictly contained in the class of T-NA- Γ -semi sub near-field space of level III. This completes the proof of the theorem.

Theorem 1.8: Let $(N, +, \cdot)$ be a T-NA- Γ -semi sub near-field space of level IV then $(N, +, \cdot)$ is a T-NA- Γ -semi sub near-field space of level II.

Proof: It is obvious that the fact the class of all associative Γ -semi sub near-field spaces is contained in the class of non-associative Γ -semi sub near-field spaces as for example we have every near-field is a loop and not vice versa.

Thus we see the class of T-NA- Γ -semi sub near-field space of level IV is strictly contained in the class of T-NA- Γ -semi sub near-field space of level II. It is easily seen that a T-NA- Γ -semi sub near-field space of level II can in general never be a T-NA- Γ -semi sub near-field space of level IV. This completes the proof of the theorem.

Example 1.9: Let $Z_6 = \{0, 1, 2, 3, 4, 5\}$ Define 'x' and '.' On Z_6 by 'x' is the usual multiplication modulo 6 and '.' is defined by $a \cdot b = a$ for all $a, b \in Z_6$. Let N be a groupoid with 1. The groupoid near-field space Z_6N is a T-NA- Γ -semi sub near-field space of level III as $Z_6 \subset Z_6N$.

ACKNOWLEDGMENT

I, Smt. T Madhavi Latha being a junior lecturer Department of Mathematics, APSWREIS, Tadepalli, Guntur District, Amaravathi, Andhra Pradesh. INDIA as an author under the guidance of my guide Dr T V Pradeep Kumar, Assistant professor, ANU college of Engineering, ANU from this article PART-IV on non-associative Γ -semi sub near-field spaces of a Γ -near-field space over near-field (Part-IV-NA- Γ -SSNFS- Γ -NFS-NF) being is indebted to the referee for his various valuable comments leading to the improvement of the advanced research article in algebra of Mathematics. For the academic and financial year 2020-'21, this work was supported by Director/Secretary, Department of Mathematics, APSWREIS, Tadepalli, Guntur District, Amaravathi, Andhra Pradesh. INDIA.

REFERENCES

1. G. L. Booth A note on Γ -near-rings Stud. Sci. Math. Hung. 23 (1988) 471-475.
2. G. L. Booth Jacobson radicals of Γ -near-rings Proceedings of the Hobart Conference, Longman Sci. & Technical (1987) 1-12.
3. G Pilz Near-rings, Amsterdam, North Holland.
4. P. S. Das Fuzzy groups and level subgroups J. Math. Anal. and Appl. 84 (1981) 264-269.
5. V. N. Dixit, R. Kumar and N. Ajal On fuzzy rings Fuzzy Sets and Systems 49 (1992) 205-213.
6. S. M. Hong and Y. B. Jun A note on fuzzy ideals in Γ -rings Bull. Honam Math. Soc. 12 (1995) 39-48.
7. Y. B. Jun and S. Lajos Fuzzy (1; 2)-ideals in semigroups PU. M. A. 8(1) (1997) 67-74.
8. Y. B. Jun and C. Y. Lee Fuzzy \square -rings Pusan Kyongnam Math. J. 8(2) (1992) 163-170.
9. Y. B. Jun, J. Neggers and H. S. Kim Normal L-fuzzy ideals in semirings Fuzzy Sets and Systems 82 (1996) 383-386.
10. Smt. T Madhavi Latha, Dr T V Pradeep Kumar and Dr N V Nagendram, "Commutative Prime Γ -near-field spaces with permuting Tri-derivations over near-field " ,IJMA Dec, 2017, Vol.8, No,12, ISSN NO.2229 – 5046, Pg No. 1 – 9.
11. Smt. T Madhavi Latha, Dr T V Pradeep Kumar and Dr N V Nagendram, "Fuzzy sub near-field spaces in Γ - near-field space over a near-field", IJMA Nov, 2017, Vol.8, No, 12, ISSN NO.2229 – 5046, Pg No.188– 196.
12. Smt. T Madhavi Latha, Dr T V Pradeep Kumar and Dr N V Nagendram, "Gamma Semi Sub near-field spaces in gamma near-field space over a near-field PART I", IJMA Jan, 2018, Vol. 9, No, 2, ISSN NO.2229 – 5046, Pg No.135 – 145.
13. Smt. T Madhavi Latha, Dr T V Pradeep Kumar and Dr N V Nagendram, "Gamma Semi Sub near-field spaces in gamma near-field space over a near-field PART II", IJMA 14 Feb, 2018, Vol. 9, No, 3, ISSN NO.2229 – 5046, Pg No.6 – 12.
14. Smt. T Madhavi Latha, Dr T V Pradeep Kumar and Dr N V Nagendram, "Gamma Semi Sub near-field spaces in gamma near-field space over a near-field PART III", IJMA 26 Feb, 2018, Vol. 9, No, 3, ISSN NO.2229 – 5046, Pg No.86 – 95.
15. Smt. T Madhavi Latha, Dr T V Pradeep Kumar and Dr N V Nagendram, "Gamma Semi Sub near-field spaces in gamma near-field space over a near-field PART IV", IJMA 09 Mar, 2018, Vol. 9, No, 4, ISSN NO.2229 – 5046, Pg No.1 – 14.

16. T Madhavi Latha, Dr T V Pradeep Kumar and Dr N V Nagendram “Part III Characters of Nagendram Gamma semi sub near-field spaces of a Gamma-near-field space over near-field” Nov, 2019, IJMA, Vol. xx, No, xx, ISSN NO.2229 – 5046, Pg No .xx – xx.
17. T Madhavi Latha, Dr T V Pradeep Kumar “PART- I on non-associative Γ -semi sub near-field spaces of a Γ -near-field space over near-field (Part I NA- Γ -SSNFS- Γ -NFS-NF)” Dec, 2019, IJMA, Vol. 10, No, 12, ISSN NO.2229 – 5046, Pg No .47 – 51.
18. T Madhavi Latha, Dr T V Pradeep Kumar “PART- II on non-associative Γ -semi sub near-field spaces of a Γ -near-field space over near-field (Part II NA- Γ -SSNFS- Γ -NFS-NF)” Jan. 2020, IJMA, Vol. 11, No, 1, ISSN NO.2229 – 5046, Pg No .20 – 23.
19. T Madhavi Latha, Dr T V Pradeep Kumar “PART- III on non-associative Γ -semi sub near-field spaces of a Γ -near-field space over near-field (Part III NA- Γ -SSNFS- Γ -NFS-NF)” Feb., 2020, IJMA, Vol. 11, No, 2, ISSN NO.2229 – 5046, Pg No .1 – 6.

Source of support: Nil, Conflict of interest: None Declared.

[Copy right © 2020. This is an Open Access article distributed under the terms of the International Journal of Mathematical Archive (IJMA), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.]