

**PART-IV ON NON-ASSOCIATIVE Γ -SEMI SUB NEAR-FIELD SPACES
OF A Γ -NEAR-FIELD SPACE OVER NEAR-FIELD (PART-IV NA- Γ -SSNFS- Γ -NFS-NF)**

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ABSTRACT

In this manuscript we introduce two new classes of non-associative Γ -semi sub near-field spaces which is define as Thurumella loop Γ -semi sub near-field spaces and Thurumella groupoid Γ -semi sub near-field spaces.

Keywords: *Thurumella loop Γ -semi sub near-field space, Thurumella groupoid Γ -semi sub near-field space, loop Γ -semi sub near-field spaces, near loop Γ -semi sub near-field space, Non-associative Γ -semi sub near-field space, Thurumella- non-associative Γ -semi sub near-field space.*

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SECTION-1: INTRODUCTION ON THURUMELLA LOOP NON-ASSOCIATIVE Γ -SEMI SUB NEAR-FIELD SPACES (T-L-NA- Γ -SSNFS- Γ -NFS-NF) AND THURUMELLA GROUPOID LOOP NON-ASSOCIATIVE Γ -SEMI SUB NEAR-FIELD SPACES (T-G-L-NA- Γ -SSNFS- Γ -NFS-NF) OF A Γ -NEAR-FIELD SPACE OVER NEAR-FIELD.

Here in this section, we know the class of loop Γ -semi sub near-field spaces is a generalization of group Γ -semi sub near-field spaces and loop near-field spaces. In fact groupoid Γ -semi sub near-field spaces which contain all these classes of Γ -semi sub near-field spaces of a near-field space. We are more interested in the study of their Thurumella nature. The concept of classical identities play a vital role when the structure under study happens to be non-associative. With the advent of a new class of loops of even order and new class of groupoids of all order built using integers we are able to give examples which are concrete.

One of the major drawbacks in the study of near-field spaces or any algebraic structure is the lack of concrete examples. Unless we have concrete examples it becomes difficult to make the subject or that algebraic structure profoundly attractive to many researchers. For instance, the research in near-field theory over ring theory or group theory or semi group theory are carried out by more researchers when compared to the theory of loop, theory of groupoid. So in this section we introduce the Thurumella analogue of these concepts and try our level best to illustrate them whenever possible with examples.

Definition 1.1: The system $N = (N, '+', '\cdot')$ be a non-empty set, N endowed with two binary operations '+' and ' \cdot '. Satisfying the following conditions.

- $(N, +)$ is a Γ -semi sub near-field space
 - (N, \cdot) is a groupoid.
 - $(a + b) \cdot c = a \cdot c + b \cdot c$ for $a, b, c \in N$. $(N, +, \cdot)$ is defined as a non-associative Γ -semi sub near-field space (NA- Γ -semi sub near-field space).
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But one of the major draw backs is that we do not have several examples of these Γ -semi sub near-field spaces ; so it is become difficult to give examples using Z_p alone or Z alone or any of the known sets. Thus to obtain examples of non-associative Γ -semi sub near-field spaces we define loop Γ -semi sub near-field space and groupoid Γ -semi sub near-field spaces which has been defined earlier.

Definition 1.2: Let N be a non-empty set. Define two binary operations ‘+’ and ‘.’ Satisfying the following conditions:

- a. $(N, +)$ is a Γ -semi sub near-field space
- b. (N, \cdot) is a groupoid.
- c. $(a + b) \cdot c = a \cdot c + b \cdot c$ for $a, b, c \in N$. then we call N a Thurumella NA- Γ -semi sub near-field space of level I (T-NA-- Γ -semi sub near-field space of level I).

Now we define T-NA-- Γ -semi sub near-field space of level II.

Definition 1.3: Let $(N, +, \cdot)$ be a non-associative Γ -semi sub near-field space of a Γ -near-field space over near-field. N is said to Thurumella NA- Γ -semi sub near-field space of level II (T-NA- Γ -semi sub near-field space of level II) . If N has a proper Γ -semi sub near-field space $P \subset N$ where P itself a Non Associative Γ -semi sub near-field space of a Γ -near-field space over near-field.

Definition 1.4: Let $(N, +, \cdot)$ be a non-associative Γ -semi sub near-field space of a Γ -near-field space over near-field. N is said to Thurumella pseudo NA- Γ -semi sub near-field space (T-pseudo NA- Γ -semi sub near-field space). If N has a proper Γ -semi sub near-field space $P \subset N$ where P itself a NA- Γ -semi sub near-field space of a Γ -near-field space over operations of a near-field.

Now still we have a level III and level IV definition of Thurumella NA-- Γ -semi sub near-field spaces.

Definition 1.5: Let $(N, +, \cdot)$ be a non-associative Γ -semi sub near-field space of a Γ -near-field space over near-field. N is said to Thurumella NA- Γ -semi sub near-field space of level III (T-NA- Γ -semi sub near-field space of level III). If N has a proper Γ -semi sub near-field space $P \subset N$ where P itself an associative Γ -semi sub near-field space of a Γ -near-field space over near-field.

Finally, we define a level IV definition of Thurumella NA-- Γ -semi sub near-field spaces of level IV and obtain the possible and probable relation between the four levels of T-NA- Γ -semi sub near-field spaces.

Definition 1.6: Let $(N, +, \cdot)$ be a NA- Γ -semi sub near-field space we say N is a Thurumella NA- Γ -semi sub near-field space of level IV (T-NA- Γ -semi sub near-field space of level IV) if N has a proper Γ -semi sub near-field space P such that $(P, +, \cdot)$ is an associative - Γ -semi sub near-field space.

SECTION-2: MAIN RESULTS ON SOME SPECIAL THURUMELLA LOOP NON-ASSOCIATIVE Γ -SEMI SUB NEAR-FIELD SPACES (T-L-NA- Γ -SSNFS- Γ -NFS-NF) AND THURUMELLA GROUPOID LOOP NON-ASSOCIATIVE Γ -SEMI SUB NEAR-FIELD SPACES (T-G-L-NA- Γ -SSNFS- Γ -NFS-NF) OF A Γ -NEAR-FIELD SPACE OVER NEAR-FIELD.

In this section 3, we deduce main results on some special Thurumella loop non-associative Γ -semi sub near-field spaces (T-L-NA- Γ -SSNFS- Γ -NFS-NF) and Thurumella groupoid loop non-associative Γ -semi sub near-field spaces (T-G-L-NA- Γ -SSNFS- Γ -NFS-NF) of a Γ -near-field space over near-field.

Theorem 1.7: Let $(N, +, \cdot)$ be a Thurumella NA- Γ -semi sub near-field space level of IV then $(N, +, \cdot)$ is a T-NA- Γ -semi sub near-field space of level II.

Proof: Here we recall the definitions of level III and level IV of Thurumella NA- Γ -semi sub near-field space.

Let $(N, +, \cdot)$ be a non-associative Γ -semi sub near-field space of a Γ -near-field space over near-field. N is said to Thurumella NA- Γ -semi sub near-field space of level III (T-NA- Γ -semi sub near-field space of level III) . If N has a proper Γ -semi sub near-field space $P \subset N$ where P itself an associative Γ -semi sub near-field space of a Γ -near-field space over near-field.

Let $(N, +, \cdot)$ be a NA- Γ -semi sub near-field space we say N is a Thurumella NA- Γ -semi sub near-field space of level IV (T-NA- Γ -semi sub near-field space of level IV) if N has a proper Γ -semi sub near-field space P such that $(P, +, \cdot)$ is an associative - Γ -semi sub near-field space.

It is clear that a T-NA- Γ -semi sub near-field space of level III which is not a Thurumella NA- Γ -semi sub near-field space of level IV. Thus we see the class of T-NA- Γ -semi sub near-field space of level IV is strictly contained in the class of T-NA- Γ -semi sub near-field space of level III. This completes the proof of the theorem.

Theorem 1.8: Let $(N, +, \cdot)$ be a T-NA- Γ -semi sub near-field space of level IV then $(N, +, \cdot)$ is a T-NA- Γ -semi sub near-field space of level II.

Proof: It is obvious that the fact the class of all associative Γ -semi sub near-field spaces is contained in the class of non-associative Γ -semi sub near-field spaces as for example we have every near-field is a loop and not vice versa.

Thus we see the class of T-NA- Γ -semi sub near-field space of level IV is strictly contained in the class of T-NA- Γ -semi sub near-field space of level II. It is easily seen that a T-NA- Γ -semi sub near-field space of level II can in general never be a T-NA- Γ -semi sub near-field space of level IV. This completes the proof of the theorem.

Example 1.9: Let $Z_6 = \{0, 1, 2, 3, 4, 5\}$ Define 'x' and '.' On Z_6 by 'x' is the usual multiplication modulo 6 and '.' is defined by $a.b = a$ for all $a, b \in Z_6$. Let N be a groupoid with 1. The groupoid near-field space Z_6N is a T-NA- Γ -semi sub near-field space of level III as $Z_6 \subset Z_6N$.

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