International Journal of Mathematical Archive-11(2), 2020, 1-6
Available online through www.ijma.info ISSN 2229 – 5046

PART-III ON NON-ASSOCIATIVE Γ-SEMI SUB NEAR-FIELD SPACES OF A Γ-NEAR-FIELD SPACE OVER NEAR-FIELD (PART-III NA-Γ-SSNFS-Γ-NFS-NF)

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(Received On: 21-12-19; Revised & Accepted On: 20-01-20)

ABSTRACT

In this manuscript we introduce the concept of Thurumella non-associative Γ-semi sub near-field space and also study about the near loop Γ-semi sub near-field space introduced as to be Thurumella-Γ-semi sub near-field space. Several interesting Thurumella concepts are introduced.

Keywords: *loop Γ-semi sub near-field spaces, near loop Γ-semi sub near-field space, Non-associative Γ-semi sub near-field space, Thurumella- non-associative Γ-semi sub near-field space.*

2000 Mathematics Subject Classification: *46H25, 6H99, 46L10, 46M2051 M 10, 51 F 15, 03 B 30, 43A10, 46B28.*

SECTION-1. INTRODUCTION CUM PRELIMINARIES ON SOME SPECIAL THURUMELLA NON-ASSOCIATIVE Γ-SEMI SUB NEAR-FIELD SPACE (T-NA-Γ-SSNFS-Γ-NFS-NF) A Γ-NEAR-FIELD SPACE OVER NEAR-FIELD.

Here in this section, Here also we once again mention loop Γ-semi sub near-field spaces and near loop Γ-semi sub near-field spaces are different for the former has '+' to be non associative where as in the later '+' is associative but '.' Happens to be non associative and the near loop Γ-semi sub near-field spaces are built using a loop and a Γ-semi near-field space over a near-field. Finally we also introduce the identities newly to be the near-field space which are non associative Γ-semi sub near-field space. We introduce the concept of Thurumella right loop – half groupoid near-field space which is the most generalized concept of loop Γ-semi sub near-field space over near-field.

Definition 1.1: Thurumella right loop half groupoid Γ-semi sub near-field space (T-RL-Γ-semi sub near-field space). The system N = (N, '+', '.' ,0) is called a be a Thurumella right loop half groupoid Γ-semi sub near-field space(T-RL-Γ-semi sub near-field space) provided.

a. (N, '+', 0) is a Thurumella loop.
b. (N, '.') is a half groupoid.
c. $(n_1, n_2).n_1 = n_1 . (n_2.n_3)$ for all $n_1, n_2, n_3 \in N$ for which $n_1.n_2$, $n_2.n_3$, $n_1 . (n_2.n_3)$ and $n_1 . (n_2.n_3) \in N$.
d. $(n_1 + n_2) .n_3 = (n_1.n_3) + (n_2.n_3)$ for all $n_1, n_2, n_3 \in N$ for which $(n_1 + n_2) .n_3$, $(n_1.n_3)$, $(n_2.n_3)$ is satisfied then we say that N is a Thurumella left half groupoid Γ-semi sub near-field space (T-Left Half groupoid-Γ-semi sub near-field space)

We say that (L,'+') is a T-loop if L has a proper Γ-semi sub near-field space P such that (P, +) is an additive Γ-semi sub near-field space.

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Definition 1.2: A Thurumella right loop Γ-semi sub near-field space (T-RL-Γ-semi sub near-field space) N is a system (N, +, .) of double composition '+' , and '.' Such that

a. (N, +) is a T-loop (Thurumella – loop)
b. (N, .) is a T-Γ-semi sub near-field space
c. The multiplication '.' is right distributive over addition i.e. for all $n_1, n_2, n_3 \in N$ such that $(n_1 + n_2) .n_3 = (n_1.n_3)+ (n_2.n_3)$.

Example 1.3: Every T-Γ-semi sub near-field space is a T-loop Γ-semi sub near-field space.

Definition 1.4: Let (M, +, 0) be a T-loop Γ-semi sub near-field space and Δ be a T-groupoid of M. A set of all endomorphism of M is called a Thurumella Δ centralizer (T-Δ centralizer) of M provided.

a. The zero endomorphism $\delta \in T$
b. T / δ (complement of δ in T) is a Γ-semi sub near-field space of automorphisms of M.
c. $\phi (\Delta) \subset \Delta$ for all $\phi \in T$ where Δ being Thurumella centralizer
d. $\phi, \psi \in T$ and $\phi (\omega) = \psi (\omega)$ for some $\delta \neq \omega \in \Delta$ imply $\phi = \psi$.

Definition 1.5: Let (M, +, 0) be a T-loop Γ-semi sub near-field space. Let Δ be a Γ-semi sub near-field space of M which is a T-groupoid of M and T-Δ centralizer of M. A mapping $\Phi : M \to M$ into itself is called a Thurumella Δ - transformation (T-Δ-Transformation) of M over T provided $\Phi (\phi (\omega)) = \phi (\Phi (\omega))$ for all $\omega \in \Delta$ and $\phi \in T$.

Note 1.6: If $\mathbf{0}$ (zero bar) $\in \Delta$ and Φ is a T-Δ transformation of M over T then Φ fixes 0 i.e. $\Phi (\mathbf{0}) = (\mathbf{0})$. we shall denote the set of all T - Δ transformations of M over T by T(N(T, Δ)). Further we see for any endomorphism ϕ of a T-loop Γ-semi sub near-field space M, $[\phi(m)]_r = \phi (m_r)$ for all $m \in M$.

Definition 1.7: A non empty Γ-semi sub near-field space K of T – loop Γ-semi sub near-field space (N, '+', '.', 0) is said to be a Thurumella sub loop Γ-semi sub near-field space (T-SL-Γ-semi sub near-field space) of N if and only if (K, '+', '.',0) is a T-loop Γ-semi sub near-field space.

Definition 1.8: A T-loop Γ-semi sub near-field space N is said to be Thurumella zero symmetric (T-zero symmetric) if and only if n0 = 0 for every $n \in P \subset N$ where (P, +) is a Γ-semi sub near-field space, here '0' is the additive identity. T-zero symmetric loop Γ-semi sub near-field space will be denoted by $T(N_0)$.

Definition 1.9: A T-loop Γ-semi sub near-field space N is said to be a T-Γ-semi sub near-field space if N contains a proper Γ-semi sub near-field space P such that (P, +, .) is a Γ-semi near-field space over near-field.

Definition 1.10: An element *a* of a T-loop Γ-semi sub near-field space is said to be Thurumella left(or right) zero divisor (T-left(or right) zero divisor) in N if there exists an element $b \neq 0$ in N with a. b = 0 (b. a = 0) and there exists $x, y \in T \sim \{a, b, 0\}$, $x \neq y$ with the following three axioms:

1. a. x = 0 (xa = 0) 2. by = 0 (yb = 0) 3. $xy \neq 0$ ($yx \neq 0$).

Definition 1.11: An element which is both a T-right and T-left zero divisor in N is called a Thurumella two sided zero divisor or simply a Thurumella zero divisor (T-zero divisor) in N.

Definition 1.12: A map θ form a T-loop Γ-semi sub near-field space N into a T-loop Γ-semi sub near-field space N_1 is called a Thurumella homomorphism (T-homomorphism) if $\theta (n_1 + n_2) = \theta (n_1) + \theta (n_2)$ and $\theta (n_1 . n_2) = \theta (n_1) . \theta (n_2)$ for every $n_1, n_2 \in P \subset N$ where (P, +, .) is a Γ-semi sub near-field space and $\phi (n_1), \phi (n_2) \in P_1$ where $(P_1, +, .)$ is a Γ-semi sub near-field space of N_1.

Definition 1.13: Let N be a T-loop Γ-semi sub near-field space. An additive T-sub loop Γ-semi sub near-field space A of N is called a N-sub loop Γ-semi sub near-field space (right N-sub loop Γ-semi sub near-field space) if $NA \subset A$ ($AN \subset A$) where $NA = \{ na / n \in N \text{ and } a \in A\}$.

Definition 1.14: A non empty Γ-semi sub near-field space J of a T-Γ-semi sub near-field space N is called a Thurumella left ideal (T-left ideal) in N if

(i) (J, +) is a T-normal sub loop Γ-semi sub near-field space of (N, +)
(ii) $n (n_1 + j) + n_r n_1 \in J$ for each $j \in J$ and $n, n_1 \in N$ where n_r denotes the unique right inverse of n.

Definition 1.15: A non empty Γ-semi sub near-field space J of N is called a Thurumella ideal (T-ideal) of N if 1. J is a T-left ideal and 2. $JN \subset J$.

Definition 1.16: A T-loop Γ-semi sub near-field space N is said to be Thurumella left bipotent (T-left bipotent) if $Na = N a^2$ for every a in N.

Example 1.17: Let N = { e, a, b, c, d } be given by the following composition table for addition '+' .

+	e	a	b	c	d
e	e	a	b	c	d
a	a	b	e	c	d
b	b	c	d	a	e
c	c	d	a	e	b
d	d	e	c	b	a

Let '.' be defined on N by x.y = x for every x, y ∈ N. (N, +, .) is a T-loop Γ-semi sub near-field space. For P = {e, c} is a T-loop Γ-semi sub near-field space. Clearly, $Na^2 = Na$ for every a ∈ N. Thus N is a T-left bipotent loop Γ-semi sub near-field space.

Definition 1.18: A T-loop Γ-semi sub near-field space N is said to be a Thurumella T-loop Γ-semi sub near-field space (T-t-loop Γ-semi sub near-field space) if a ∈ Na for every a ∈ N.

Example 1.19: Let the T-loop Γ-semi sub near-field space (N, +,.) be given by the following composition table for '+', .'.

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	0	a

And

.	0	a	b	c
0	0	0	0	0
a	0	a	b	c
b	0	b	0	0
c	0	c	b	c

Every pair (0, a) is a Γ-semi sub near-field space so (N, +) is a T-loop Γ-semi sub near-field space. (N, .) is a T-Γ-semi sub near-field space as { a } is a Γ-semi sub near-field space. Thus (N, +, .) is a T-t-loop Γ-semi sub near-field space.

Definition 1.20: A T-loop Γ-semi sub near-field space N is said to be Thurumella regular (T-regular) if for each a in N there exists x in P (P ⊂ N), P a T-loop Γ-semi sub near-field space such that a = axa.

Definition 1.21: Let N be a Γ-semi sub near-field space. An element e ∈ N is said to be Thurumella idempotent Γ-semi sub near-field space (T-idempotent Γ-semi sub near-field space) in N if

(i) $e^2 = e$

(ii) there exists b ∈ N ~ {e} such that $b^2 = e$ and eb = b (or be = b) or (be = b or eb = b).

Definition 1.22: A T-loop Γ-semi sub near-field space N is said to be Thurumella strictly duo Γ-semi sub near-field space (T-strictly duo) Γ-semi sub near-field space if every T-N-sub loop (T-left ideal) is also a T-right N-sub loop (T-right ideal).

Definition 1.23: For any Γ-semi sub near-field space A of a T-loop Γ-semi sub near-field space N we define,
$\sqrt{A} = \{x \in N / x^n \in A$ for some n }.

Definition 1.24: A T-loop Γ-semi sub near-field space N is called Thurumella irreducible (T-irreducible) (T-simple) if it contains only the trivial T-N-sub loop Γ-semi sub near-field spaces (T-ideals) (0) and N itself.

Definition 1.25: A loop Γ-semi sub near-field space N is called a T-loop Γ-semi near-field space if it satisfies the following:

1. if N is a T-loop Γ-semi sub near-field space and
2. if P ⊆ N contain an identity and each non zero element in P ⊂ N, { (P, +) a Γ-semi sub near-field space } has a multiplicative inverse.

Definition 1.26: A T-ideal P (≠ N) is called Thurumella strictly prime Γ-semi sub near-field space (T-strictly prime Γ-semi sub near-field space) if for any two T-N-sub loop Γ-semi sub near-field spaces (T-ideals) A and B of N such that AB ⊂ P then A ⊂ P or B ⊂ P.

Definition 1.27: As left ideal B of a T-loop Γ-semi sub near-field space N is called Thurumella strictly essential Γ-semi sub near-field space (T-strictly essential Γ-semi sub near-field space) if B ∩ K ≠ {0} for any non zero T-N-sub loop Γ-semi sub near-field space K of N.

Definition 1.28: An element x ∈ N is said to be Thurumella non-singular Γ-semi sub near-field space (T-non singular Γ-semi sub near-field space) if there exists a T-strictly essential left ideal A in N such that Ax = {0}.

Definition 1.29: An element x of a T-loop Γ-semi sub near-field space N is said to be Thurumella central Γ-semi sub near-field space (T-central Γ-semi sub near-field space) if xy = yx for all y ∈ P ⊂ N, (P, +, .) is a Γ-semi sub near-field space.

Definition 1.30: A non-zero T-loop Γ-semi sub near-field space N is said to be Thurumella sub directly irreducible Γ-semi sub near-field space(T-sub directly irreducible Γ-semi sub near-field space) if the intersection of all the non-zero T-ideals of N is non-zero.

SECTION-2. RESULTS ON SOME SPECIAL THURUMELLA Γ-SEMI SUB NEAR-FIELD SPACE (T-Γ-SSNFS-Γ-NFS-NF) A Γ-NEAR-FIELD SPACE OVER NEAR-FIELD.

Here in this section 2, Here we study when loops over Γ-semi sub near-field spaces i.e. near loop Γ-semi sub near-field spaces in terminology Thurumella near loop Γ-semi sub near-field spaces, as loop Γ-semi sub near-field spaces are not non associative Γ-semi sub near-field space analogous to non associative near-field. We proceed to define here the concept of Thurumella non associative Γ-semi sub near-field spaces first in the following:

Theorem 2.1: Let N be a T-right loop Γ-semi sub near-field space then N is a T-right loop half groupoid Γ-semi sub near-field space.

Proof: Obvious by the very definitions.

Theorem 2.2: Let N be a T-right loop half groupoid Γ-semi sub near-field space. Then N is not in general a T-right loop Γ-semi near-field space.

Proof: Obvious. (N, .) is only a half groupoid so it can never be a T-Γ-semi near-field space.

Theorem 2.3: Let N be a T-loop Γ-semi sub near-field space. Then N contains a proper Γ-semi sub near-field space P such that (P, +) is a Γ-semi near-field space and (P, .) is a Γ-semi sub near-field space so that P is a Γ-semi sub near-field space.

Proof: By the very definition of T-loop Γ-semi sub near-field space, we have (N, +) is a T-loop Γ-semi sub near-field space so N has a proper Γ-semi sub near-field space say, P which is a Γ-semi near-field space under '+' now (P, +, .) is Γ-semi sub near-field space. This completes the proof of the theorem.

Note 2.4: All T-idempotents in T-loop Γ-semi sub near-field spaces are also idempotents and non-conversely.

Note 2.5: Let N be a T-t-loop Γ-semi sub near-field space, then N is T-regular if and only if for each a (≠ 0) in N there exists a T-idempotent e such that Na = Ne.

Remark 2.6: Let N be a T-Γ-semi sub near-field space and L be a T-Γ-semi sub near-field space loop. Then the near loop Γ-semi near-field space NL is a T-Γ-semi sub near-field space loop near-field.

Example 2.7: Let Z_2 = {0, 1} be a Γ-semi sub near-field space and L be a T-loop Γ-semi near-field space.
The near loop Γ-semi sub near-field space Z_2L is a T-Γ-semi sub near-field space where the T-loop Γ-semi sub near-field space is given by the following composition table:

.	e	a1	a2	a3	a4	a5
e	e	a1	a2	a3	a4	a5
a1	a1	e	a4	a2	a5	a3
a2	a2	a4	e	a5	a3	a1
a3	a3	a2	a5	e	a1	a4
a4	a4	a5	a3	a1	e	a2
a5	a5	a3	a1	a4	a2	e

Now $Z_2L = \{1, a_1, a_2, a_3, a_4, a_5$, sums taken 2 at a time ...} where 1.e = e.1 = 1 is the assumption made so that 1 acts as the identity. When we say Z_2L is a near loop Γ-semi near-field space. We assume $L \subset Z_2L$. Clearly, Z_2L is a non associative Γ-semi sub near-field space and is a T--Γ-semi near-field space.

SECTION-3. MAIN RESULTS ON SOME SPECIAL THURUMELLA NON ASSOCIATIVE Γ-SEMI SUB NEAR-FIELD SPACE (T-NA-Γ-SSNFS-Γ-NFS-NF) A Γ-NEAR-FIELD SPACE OVER NEAR-FIELD.

In this section 3, we deduce main results on some special Thurumella NA-Γ-semi sub near-field spaces.

Theorem 3.1: Let L be a loop and N be a T-Γ-semi sub near-field space. NL is a T-Γ-semi sub near-field space which is non-associative Γ-semi sub near-field space over near-field.

Proof: Let $N \subset NL$ and $L \subset NL$ and as L is non associative Γ-semi sub near-field space and N is a T-Γ-semi sub near-field space NL is a T-Γ-semi sub near-field space. This completes the proof of the theorem.

Theorem 3.2: Let L be any loop and N any Γ-semi sub near-field space. The near loop Γ-semi near-field space NL is a T-non associative Γ-semi sub near-field space.

Proof: It is fact that (NL, +) is a Γ-semi near-field space, hence trivially a T-Γ-semi sub near-field space and (NL, .) is a T-groupoid as $N \subset NL$ is a Γ-semi sub near-field space. Hence this completes the proof of the theorem.

Definition 3.3: Let N be a non associative loop Γ-semi sub near-field space. We say N is a additively power associative loop Γ-semi sub near-field space if every element under '+' of N generates a Γ-semi near-field space. The Γ-semi sub near-field space N is said to be a multiplicatively power associative loop Γ-semi sub near-field space if every element different from 0 generates a Γ-semi near-field space under product. A loop Γ-semi sub near-field space N is said to be power associative Γ-semi sub near-field space if

1. every element $n \in N$ under '+' generates a Γ-semi near-field space
2. every element $n \in N \sim \{ 0 \}$ under 'x' or '.' Generates a Γ-semi near-field space.

Definition 3.4: Let N be a non associative Γ-semi near-field space. We say N is power associative if every element of $N \sim \{0\}$ generates a Γ-semi near-field space with respect to.

Definition 3.5: Let N be a non associative Γ-semi sub near-field space. We say N is power associative Γ-semi sub near-field space if every element in N generates a Γ-semi sub near-field space under multiplication.

Definition 3.6: Let N be a non associative Γ-semi sub near-field space we say N is a Thurumella power associative (T-power associative) if (N, .) has a proper Γ-semi sub near-field space P where P is a T-sub groupoid of N and P is power associative i.e. every element $p \in P$ generates a Γ-semi sub near-field space.

ACKNOWLEDGMENT

I, Smt. T Madhavi Latha being a junior lecturer Department of Mathematics, APSWREIS, Tadepalli, Guntur District, Amaravathi, Andhra Pradesh. INDIA as an author under the guidance of my guide Dr T V Pradeep Kumar, Assistant professor, ANU college of Engineering, ANU from this article PART-III on non-associative Γ-semi sub near-field spaces of a Γ-near-field space over near-field (Part-III-NA-Γ-SSNFS-Γ-NFS-NF) being is indebted to the referee for his various valuable comments leading to the improvement of the advanced research article in algebra of Mathematics. For the academic and financial year 2020-'21, this work was supported by Director, Department of Mathematics, APSWREIS, Tadepalli, Guntur District, Amaravathi, Andhra Pradesh. INDIA.

REFERENCES

1. G. L. Booth A note on Γ-near-rings Stud. Sci. Math. Hung. 23 (1988) 471-475.
2. G. L. Booth Jacobson radicals of Γ-near-rings Proceedings of the Hobart Conference, Longman Sci. & Technical (1987) 1-12.
3. G Pilz Near-rings, Amsterdam, North Holland.
4. P. S. Das Fuzzy groups and level subgroups J. Math. Anal. and Appl. 84 (1981) 264-269.
5. V. N. Dixit, R. Kumar and N. Ajal On fuzzy rings Fuzzy Sets and Systems 49 (1992) 205-213.
6. S. M. Hong and Y. B. Jun A note on fuzzy ideals in Γ-rings Bull. Honam Math. Soc. 12 (1995) 39-48.
7. Y. B. Jun and S. Lajos Fuzzy (1; 2)-ideals in semigroups PU. M. A. 8(1) (1997) 67-74.
8. Y. B. Jun and C. Y. Lee Fuzzy □-rings Pusan Kyongnam Math. J. 8(2) (1992) 163-170.
9. Y. B. Jun, J. Neggers and H. S. Kim Normal L-fuzzy ideals in semirings Fuzzy Sets and Systems 82 (1996) 383-386.
10. Smt. T Madhavi Latha, Dr T V Pradeep Kumar and Dr N V Nagendram, "Commutative Prime Γ-near-field spaces with permuting Tri-derivations over near-field", IJMA Dec, 2017, Vol.8, No,12, ISSN NO.2229 – 5046, Pg No. 1 – 9.
11. Smt. T Madhavi Latha, Dr T V Pradeep Kumar and Dr N V Nagendram, "Fuzzy sub near-field spaces in Γ-near-field space over a near-field", IJMA Nov, 2017, Vol.8, No, 12, ISSN NO.2229 – 5046, Pg No.188 – 196.
12. Smt. T Madhavi Latha, Dr T V Pradeep Kumar and Dr N V Nagendram, "Gamma Semi Sub near-field spaces in gamma near-field space over a near-field PART I", IJMA Jan, 2018, Vol. 9, No, 2, ISSN NO.2229 – 5046, Pg No.135 – 145.
13. Smt. T Madhavi Latha, Dr T V Pradeep Kumar and Dr N V Nagendram, "Gamma Semi Sub near-field spaces in gamma near-field space over a near-field PART II", IJMA 14 Feb, 2018, Vol. 9, No, 3, ISSN NO.2229 – 5046, Pg No.6 – 12.
14. Smt. T Madhavi Latha, Dr T V Pradeep Kumar and Dr N V Nagendram, "Gamma Semi Sub near-field spaces in gamma near-field space over a near-field PART III", IJMA 26 Feb, 2018, Vol. 9, No, 3, ISSN NO.2229 – 5046, Pg No.86 – 95.
15. Smt. T Madhavi Latha, Dr T V Pradeep Kumar and Dr N V Nagendram, "Gamma Semi Sub near-field spaces in gamma near-field space over a near-field PART IV", IJMA 09 Mar, 2018, Vol. 9, No, 4, ISSN NO.2229 – 5046, Pg No.1 – 14.
16. T Madhavi Latha, Dr T V Pradeep Kumar and Dr N V Nagendram3 "Part III Characters of Nagendram Gamma semi sub near-field spaces of a Gamma-near-field space over near-field" Nov, 2019, IJMA, Vol. xx, No, xx, ISSN NO.2229 – 5046, Pg No .xx – xx.
17. T Madhavi Latha, Dr T V Pradeep Kumar "PART- I on non-associative Γ-semi sub near-field spaces of a Γ-near-field space over near-field (Part I NA-Γ-SSNFS-Γ-NFS-NF)" Dec, 2019, IJMA, Vol. 10, No, 12, ISSN NO.2229 – 5046, Pg No .47 – 51.

Source of support: Nil, Conflict of interest: None Declared.