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PART-III ON NON-ASSOCIATIVE Γ-SEMI SUB NEAR-FIELD SPACES OF A Γ-NEAR-FIELD SPACE OVER NEAR-FIELD (PART-III NA-Γ-SSNFS-Γ-NFS-NF)

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ABSTRACT

In this manuscript we introduce the concept of Thurumella non-associative Γ-semi sub near-field space and also study about the near loop Γ-semi sub near-field space introduced as to be Thurumella-Γ-semi sub near-field space. Several interesting Thurumella concepts are introduced.

Keywords: *loop Γ-semi sub near-field spaces, near loop Γ-semi sub near-field space, Non-associative Γ-semi sub near-field space, Thurumella- non-associative Γ-semi sub near-field space.*

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SECTION-1. INTRODUCTION CUM PRELIMINARIES ON SOME SPECIAL THURUMELLA NON-ASSOCIATIVE Γ-SEMI SUB NEAR-FIELD SPACE (T-NA-Γ-SSNFS-Γ-NFS-NF) A Γ-NEAR-FIELD SPACE OVER NEAR-FIELD.

Here in this section, Here also we once again mention loop Γ-semi sub near-field spaces and near loop Γ-semi sub near-field spaces are different for the former has '+' to be non associative where as in the later '+' is associative but '.' Happens to be non associative and the near loop Γ-semi sub near-field spaces are built using a loop and a Γ-semi near-field space over a near-field. Finally we also introduce the identities newly to be the near-field space which are non associative Γ-semi sub near-field space. We introduce the concept of Thurumella right loop – half groupoid near-field space which is the most generalized concept of loop Γ-semi sub near-field space over near-field.

Definition 1.1: Thurumella right loop half groupoid Γ-semi sub near-field space (T-RL-Γ-semi sub near-field space). The system N = (N, '+', '.' ,0) is called a be a Thurumella right loop half groupoid Γ-semi sub near-field space(T-RL-Γ-semi sub near-field space) provided.

a. (N, '+', 0) is a Thurumella loop.
b. (N, '.') is a half groupoid.
c. $(n_1, n_2).n_1 = n_1 . (n_2.n_3)$ for all $n_1, n_2, n_3 \in N$ for which $n_1.n_2, n_2.n_3, n_1 . (n_2.n_3)$ and $n_1 . (n_2.n_3) \in N$.
d. $(n_1 + n_2) .n_3 = (n_1.n_3) + (n_2.n_3)$ for all $n_1, n_2, n_3 \in N$ for which $(n_1 + n_2) .n_3$, $(n_1.n_3)$, $(n_2.n_3)$ is satisfied then we say that N is a Thurumella left half groupoid Γ-semi sub near-field space (T-Left Half groupoid-Γ-semi sub near-field space)

We say that (L,'+') is a T-loop if L has a proper Γ-semi sub near-field space P such that (P, +) is an additive Γ-semi sub near-field space.

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Definition 1.2: A Thurumella right loop Γ-semi sub near-field space (T-RL-Γ-semi sub near-field space) N is a system (N, +, .) of double composition '+' , and '.' Such that

a. (N, +) is a T-loop (Thurumella – loop)
b. (N, .) is a T-Γ-semi sub near-field space
c. The multiplication '.' is right distributive over addition i.e. for all $n_1, n_2, n_3 \in N$ such that $(n_1 + n_2).n_3 = (n_1.n_3) + (n_2.n_3)$.

Example 1.3: Every T-Γ-semi sub near-field space is a T-loop Γ-semi sub near-field space.

Definition 1.4: Let (M, +, 0) be a T-loop Γ-semi sub near-field space and Δ be a T-groupoid of M. A set of all endomorphism of M is called a Thurumella Δ centralizer (T-Δ centralizer) of M provided.

a. The zero endomorphism $\delta \in T$
b. T / δ (complement of δ in T) is a Γ-semi sub near-field space of automorphisms of M.
c. $\phi(\Delta) \subset \Delta$ for all $\phi \in T$ where Δ being Thurumella centralizer
d. $\phi, \psi \in T$ and $\phi(\omega) = \psi(\omega)$ for some $\delta \neq \omega \in \Delta$ imply $\phi = \psi$.

Definition 1.5: Let (M, +, 0) be a T-loop Γ-semi sub near-field space. Let Δ be a Γ-semi sub near-field space of M which is a T-groupoid of M and T-Δ centralizer of M. A mapping $\Phi : M \to M$ into itself is called a Thurumella Δ - transformation (T-Δ-Transformation) of M over T provided $\Phi(\phi(\omega)) = \phi(\Phi(\omega))$ for all $\omega \in \Delta$ and $\phi \in T$.

Note 1.6: If **0** (zero bar) $\in \Delta$ and Φ is a T-Δ transformation of M over T then Φ fixes 0 i.e. $\Phi(\mathbf{0}) = (\mathbf{0})$. we shall denote the set of all T - Δ transformations of M over T by T(N(T, Δ)). Further we see for any endomorphism ϕ of a T-loop Γ-semi sub near-field space M, $[\phi(m)]_r = \phi(m_r)$ for all $m \in M$.

Definition 1.7: A non empty Γ-semi sub near-field space K of T – loop Γ-semi sub near-field space (N, '+', '.', 0) is said to be a Thurumella sub loop Γ-semi sub near-field space (T-SL-Γ-semi sub near-field space) of N if and only if (K, '+', '.',0) is a T-loop Γ-semi sub near-field space.

Definition 1.8: A T-loop Γ-semi sub near-field space N is said to be Thurumella zero symmetric (T-zero symmetric) if and only if n0 = 0 for every $n \in P \subset N$ where (P, +) is a Γ-semi sub near-field space, here '0' is the additive identity. T-zero symmetric loop Γ-semi sub near-field space will be denoted by $T(N_0)$.

Definition 1.9: A T-loop Γ-semi sub near-field space N is said to be a T-Γ-semi sub near-field space if N contains a proper Γ-semi sub near-field space P such that (P, +, .) is a Γ-semi near-field space over near-field.

Definition 1.10: An element *a* of a T-loop Γ-semi sub near-field space is said to be Thurumella left(or right) zero divisor (T-left(or right) zero divisor) in N if there exists an element $b \neq 0$ in N with a. b = 0 (b. a = 0) and there exists $x, y \in T \sim \{a, b, 0\}$, $x \neq y$ with the following three axioms:

1. a. x = 0 (xa = 0) 2. by = 0 (yb = 0) 3. $xy \neq 0$ ($yx \neq 0$).

Definition 1.11: An element which is both a T-right and T-left zero divisor in N is called a Thurumella two sided zero divisor or simply a Thurumella zero divisor (T-zero divisor) in N.

Definition 1.12: A map θ form a T-loop Γ-semi sub near-field space N into a T-loop Γ-semi sub near-field space N_1 is called a Thurumella homomorphism (T-homomorphism) if $\theta(n_1 + n_2) = \theta(n_1) + \theta(n_2)$ and $\theta(n_1 . n_2) = \theta(n_1) . \theta(n_2)$ for every $n_1, n_2 \in P \subset N$ where (P, +, .) is a Γ-semi sub near-field space and $\phi(n_1), \phi(n_2) \in P_1$ where $(P_1, +, .)$ is a Γ-semi sub near-field space of N_1.

Definition 1.13: Let N be a T-loop Γ-semi sub near-field space. An additive T-sub loop Γ-semi sub near-field space A of N is called a N-sub loop Γ-semi sub near-field space (right N-sub loop Γ-semi sub near-field space) if $NA \subset A$ ($AN \subset A$) where $NA = \{ na / n \in N \text{ and } a \in A\}$.

Definition 1.14: A non empty Γ-semi sub near-field space J of a T-Γ-semi sub near-field space N is called a Thurumella left ideal (T-left ideal) in N if

(i) (J, +) is a T-normal sub loop Γ-semi sub near-field space of (N, +)
(ii) $n(n_1 + j) + n_r n_1 \in J$ for each $j \in J$ and $n, n_1 \in N$ where n_r denotes the unique right inverse of n.

Definition 1.15: A non empty Γ-semi sub near-field space J of N is called a Thurumella ideal (T-ideal) of N if 1. J is a T-left ideal and 2. $JN \subset J$.

Definition 1.16: A T-loop Γ-semi sub near-field space N is said to be Thurumella left bipotent (T-left bipotent) if $Na = N a^2$ for every a in N.

Example 1.17: Let N = { e, a, b, c, d } be given by the following composition table for addition '+' .

+	e	a	b	c	d
e	e	a	b	c	d
a	a	b	e	c	d
b	b	c	d	a	e
c	c	d	a	e	b
d	d	e	c	b	a

Let '.' be defined on N by x.y = x for every x, y ∈ N. (N, +, .) is a T-loop Γ-semi sub near-field space. For P = {e, c} is a T-loop Γ-semi sub near-field space. Clearly, $Na^2 = Na$ for every a ∈ N. Thus N is a T-left bipotent loop Γ-semi sub near-field space.

Definition 1.18: A T-loop Γ-semi sub near-field space N is said to be a Thurumella T-loop Γ-semi sub near-field space (T-t-loop Γ-semi sub near-field space) if a ∈ Na for every a ∈ N.

Example 1.19: Let the T-loop Γ-semi sub near-field space (N, +,.) be given by the following composition table for '+', '.'.

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	0	a

And

.	0	a	b	c
0	0	0	0	0
a	0	a	b	c
b	0	b	0	0
c	0	c	b	c

Every pair (0, a) is a Γ-semi sub near-field space so (N, +) is a T-loop Γ-semi sub near-field space. (N, .) is a T-Γ-semi sub near-field space as { a } is a Γ-semi sub near-field space. Thus (N, +, .) is a T-t-loop Γ-semi sub near-field space.

Definition 1.20: A T-loop Γ-semi sub near-field space N is said to be Thurumella regular (T-regular) if for each a in N there exists x in P (P ⊂ N), P a T-loop Γ-semi sub near-field space such that a = axa.

Definition 1.21: Let N be a Γ-semi sub near-field space. An element e ∈ N is said to be Thurumella idempotent Γ-semi sub near-field space (T-idempotent Γ-semi sub near-field space) in N if

(i) $e^2 = e$
(ii) there exists b ∈ N ~ {e} such that $b^2 = e$ and eb = b (or be = b) or (be = b or eb = b).

Definition 1.22: A T-loop Γ-semi sub near-field space N is said to be Thurumella strictly duo Γ-semi sub near-field space (T-strictly duo) Γ-semi sub near-field space if every T-N-sub loop (T-left ideal) is also a T-right N-sub loop (T-right ideal).

Definition 1.23: For any Γ-semi sub near-field space A of a T-loop Γ-semi sub near-field space N we define,
$\sqrt{A} = \{x \in N / x^n \in A$ for some n }.

Definition 1.24: A T-loop Γ-semi sub near-field space N is called Thurumella irreducible (T-irreducible) (T-simple) if it contains only the trivial T-N-sub loop Γ-semi sub near-field spaces (T-ideals) (0) and N itself.

Definition 1.25: A loop Γ-semi sub near-field space N is called a T-loop Γ-semi near-field space if it satisfies the following:

1. if N is a T-loop Γ-semi sub near-field space and
2. if P ⊆ N contain an identity and each non zero element in P ⊂ N, { (P, +) a Γ-semi sub near-field space } has a multiplicative inverse.

Definition 1.26: A T-ideal P (≠ N) is called Thurumella strictly prime Γ-semi sub near-field space (T-strictly prime Γ-semi sub near-field space) if for any two T-N-sub loop Γ-semi sub near-field spaces (T-ideals) A and B of N such that AB ⊂ P then A ⊂ P or B ⊂ P.

Definition 1.27: As left ideal B of a T-loop Γ-semi sub near-field space N is called Thurumella strictly essential Γ-semi sub near-field space (T-strictly essential Γ-semi sub near-field space) if B ∩ K ≠ {0} for any non zero T-N-sub loop Γ-semi sub near-field space K of N.

Definition 1.28: An element x ∈ N is said to be Thurumella non-singular Γ-semi sub near-field space (T-non singular Γ-semi sub near-field space) if there exists a T-strictly essential left ideal A in N such that Ax = {0}.

Definition 1.29: An element x of a T-loop Γ-semi sub near-field space N is said to be Thurumella central Γ-semi sub near-field space (T-central Γ-semi sub near-field space) if xy = yx for all y ∈ P ⊂ N, (P, +, .) is a Γ-semi sub near-field space.

Definition 1.30: A non-zero T-loop Γ-semi sub near-field space N is said to be Thurumella sub directly irreducible Γ-semi sub near-field space(T-sub directly irreducible Γ-semi sub near-field space) if the intersection of all the non-zero T-ideals of N is non-zero.

SECTION-2. RESULTS ON SOME SPECIAL THURUMELLA Γ-SEMI SUB NEAR-FIELD SPACE (T-Γ-SSNFS-Γ-NFS-NF) A Γ-NEAR-FIELD SPACE OVER NEAR-FIELD.

Here in this section 2, Here we study when loops over Γ-semi sub near-field spaces i.e. near loop Γ-semi sub near-field spaces in terminology Thurumella near loop Γ-semi sub near-field spaces, as loop Γ-semi sub near-field spaces are not non associative Γ-semi sub near-field space analogous to non associative near-field. We proceed to define here the concept of Thurumella non associative Γ-semi sub near-field spaces first in the following:

Theorem 2.1: Let N be a T-right loop Γ-semi sub near-field space then N is a T-right loop half groupoid Γ-semi sub near-field space.

Proof: Obvious by the very definitions.

Theorem 2.2: Let N be a T-right loop half groupoid Γ-semi sub near-field space. Then N is not in general a T-right loop Γ-semi near-field space.

Proof: Obvious. (N, .) is only a half groupoid so it can never be a T-Γ-semi near-field space.

Theorem 2.3: Let N be a T-loop Γ-semi sub near-field space. Then N contains a proper Γ-semi sub near-field space P such that (P, +) is a Γ-semi near-field space and (P, .) is a Γ-semi sub near-field space so that P is a Γ-semi sub near-field space.

Proof: By the very definition of T-loop Γ-semi sub near-field space, we have (N, +) is a T-loop Γ-semi sub near-field space so N has a proper Γ-semi sub near-field space say, P which is a Γ-semi near-field space under '+' now (P, +, .) is Γ-semi sub near-field space. This completes the proof of the theorem.

Note 2.4: All T-idempotents in T-loop Γ-semi sub near-field spaces are also idempotents and non-conversely.

Note 2.5: Let N be a T-t-loop Γ-semi sub near-field space, then N is T-regular if and only if for each a (≠ 0) in N there exists a T-idempotent e such that Na = Ne.

Remark 2.6: Let N be a T-Γ-semi sub near-field space and L be a T-Γ-semi sub near-field space loop. Then the near loop Γ-semi near-field space NL is a T-Γ-semi sub near-field space loop near-field.

Example 2.7: Let Z_2 = {0, 1} be a Γ-semi sub near-field space and L be a T-loop Γ-semi near-field space.
The near loop Γ-semi sub near-field space Z_2L is a T-Γ-semi sub near-field space where the T-loop Γ-semi sub near-field space is given by the following composition table:

.	e	a1	a2	a3	a4	a5
e	e	a1	a2	a3	a4	a5
a1	a1	e	a4	a2	a5	a3
a2	a2	a4	e	a5	a3	a1
a3	a3	a2	a5	e	a1	a4
a4	a4	a5	a3	a1	e	a2
a5	a5	a3	a1	a4	a2	e

Now $Z_2L = \{1, a_1, a_2, a_3, a_4, a_5$, sums taken 2 at a time ...} where 1.e = e.1 = 1 is the assumption made so that 1 acts as the identity. When we say Z_2L is a near loop Γ-semi near-field space. We assume $L \subset Z_2L$. Clearly, Z_2L is a non associative Γ-semi sub near-field space and is a T--Γ-semi near-field space.

SECTION-3. MAIN RESULTS ON SOME SPECIAL THURUMELLA NON ASSOCIATIVE Γ-SEMI SUB NEAR-FIELD SPACE (T-NA-Γ-SSNFS-Γ-NFS-NF) A Γ-NEAR-FIELD SPACE OVER NEAR-FIELD.

In this section 3, we deduce main results on some special Thurumella NA-Γ-semi sub near-field spaces.

Theorem 3.1: Let L be a loop and N be a T-Γ-semi sub near-field space. NL is a T-Γ-semi sub near-field space which is non-associative Γ-semi sub near-field space over near-field.

Proof: Let N ⊂ NL and L ⊂ NL and as L is non associative Γ-semi sub near-field space and N is a T-Γ-semi sub near-field space NL is a T-Γ-semi sub near-field space. This completes the proof of the theorem.

Theorem 3.2: Let L be any loop and N any Γ-semi sub near-field space. The near loop Γ-semi near-field space NL is a T-non associative Γ-semi sub near-field space.

Proof: It is fact that (NL, +) is a Γ-semi near-field space, hence trivially a T-Γ-semi sub near-field space and (NL, .) is a T-groupoid as N ⊂ NL is a Γ-semi sub near-field space. Hence this completes the proof of the theorem.

Definition 3.3: Let N be a non associative loop Γ-semi sub near-field space. We say N is a additively power associative loop Γ-semi sub near-field space if every element under '+' of N generates a Γ-semi near-field space. The Γ-semi sub near-field space N is said to be a multiplicatively power associative loop Γ-semi sub near-field space if every element different from 0 generates a Γ-semi near-field space under product. A loop Γ-semi sub near-field space N is said to be power associative Γ-semi sub near-field space if

1. every element n ∈ N under '+' generates a Γ-semi near-field space
2. every element n ∈ N ~ { 0 } under 'x' or '.' Generates a Γ-semi near-field space.

Definition 3.4: Let N be a non associative Γ-semi near-field space. We say N is power associative if every element of N ~ {0} generates a Γ-semi near-field space with respect to.

Definition 3.5: Let N be a non associative Γ-semi sub near-field space. We say N is power associative Γ-semi sub near-field space if every element in N generates a Γ-semi sub near-field space under multiplication.

Definition 3.6: Let N be a non associative Γ-semi sub near-field space we say N is a Thurumella power associative (T-power associative) if (N, .) has a proper Γ-semi sub near-field space P where P is a T-sub groupoid of N and P is power associative i.e. every element p ∈ P generates a Γ-semi sub near-field space.

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