International Journal of Mathematical Archive-11(1), 2020, 24-31
Available online through www.ijma.info ISSN 2229 – 5046

ADDING HOME GUARDS TO THE MATHEMATICAL MODEL OF CRIMINALS AND POLICE FORCES IN A CONTEMPORARY INDIAN COMMUNITY

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(Received On: 20-12-19; Revised & Accepted On: 06-01-20)

ABSTRACT

A mathematical model of criminals and police forces is developed from the population dynamics in a contemporary Indian community, using fifteen major crime data covering the period 1995 to 2011 from Delhi crime statistics. We propose a modification adding Home guards to the mathematical model of criminals and Police forces. We derived a system of non linear first order differential equations. After determination of the fixed points of the system, a local stability analysis was investigated. The result suggests that criminals' population falls rapidly if Home guards engaged in the society to help Police forces population.

Keywords*- population dynamics, fixed point, Eigen value, Jacobian.*

I. INTRODUCTION

Different types of criminal activities occur in societies. Crime has become one of the most critical challenges facing by the NCT of Delhi, India. Rate of cognizable crime per lakh population in 2014 is 767.4 in Delhi (The Hindu, Aug.-19, 2015). There were 147,230 crime cases registered in the year 2014, but total 73,702 crime cases registered in the year 2013.Hence the NCT of Delhi witnessed an alarming increase of 99.22 percent in crime in 2014 over the previous year.

According to the crime statistics of Delhi (spl. commissioner of police report till 15th Dec/2015), total index of street crime increased from 33997(in year 2013) to 96922 (in 2014), and an increase rate of 185.09%. However in recent time, according to the National Crime Records Bureau(NCRB) the level of criminal activities in the past 3 years has increased to 73.96% with the following breakdown: Burglary and Theft 59.51%, Robbed or Mugged 58.45%, Auto theft 59.17%, things from Car stolen 60.74%, Attacked 57.09%,Insulted 52.90%, subject to Physical attack because of Skin Colour, Ethnic Origin or Religion 40.67%, dealing in Drugs 44.33%, Vandalism and theft 57.32%, Violent Crimes(such as assault and armed robbery) 58.16%, Bribery 83.46%.

Many programs are launched by the government to reduce crime but based on empirical evidence its prevalence rate is still high. Hence to combat the propagation of criminals there is the need to deploy of adequate Police forces into the community to combat these criminals. The Police forces includes central and state Police service, the Armed forces, BSF, CID, CBI *etc*. Delhi police service has 3 police ranges, 11 police districts, 54 police subdivision and 84 police stations. The service has man power strength of a over 73,558 civil and district Armed police and 6,909 state Armed police.

Hence total strength of police force is 80,467 which include 11DGP or special DGP, 20 IGP, 19DIG, 46 AIGP or SSP or SP, 32 additional SP, 290 ASP or Dy. SP, 1257 inspector, 4958 SI, 6521 ASI, 18,921 H. constable, 41,483 constable and 6909 state armed police.

In an effort to combat propagation of criminal activities, there is the need for a continuous quantitative monitoring of the criminals by deploying police forces into contemporary community of the NCT of Delhi, and this can be effectively done within the field of mathematical ecology. Hence a mathematical model of Criminals and Police forces could help to solve this problem. The model will be applied to suggest the police forces in bringing crime under control, as to which groups of the society should be targeted most.

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II. RELATED LITERATURES

The problem of crime is a major issue faced by all mankind. To obtain perfect solutions to the uncertainties of crime in a diverse society claim the application of many disciplines.

The modern economics literature on crime essentially follows from G.S. Beckers' (1968) model of rational criminal activity [1]. In this model, individuals in the market for crime are assumed to act according to the rules of optimizing behavior. Another economics literature on crime has developed by Isaac Ehrlich [5, 6, 7]. His model is based on the hypothesis that criminal activity is related to the complexities of production and consumption, demand and supply and loss and gain.

Glaeser *et al*. [8, 9] have developed models on crime and social interactions. Their models suggest that social interactions create enough covariance across individuals to explain the high cross-city variance of crime rates. Glaeser and Scheinkman [10] presented a model almost similar to the Glaeser *et al.* [9] model with a slight extension. They presented the model on measuring social interactions with local interactions but using a continuous action space and starting with optimizing behavior. Further, Glaeser *et.al*. [11] has developed an interesting model on the social multiplier. They presented the size of the social multiplier in the impact of education on wages, the impact of demographics on crime.

Calvo-Armengo *et al.* (2003), in their social networks and crime decisions model, show that younger agents may benefit each other in their crime business [12]. They show that tentative identical agents connected through a network can end up with very different equilibrium outcomes; either employed or isolated criminal or criminals in networks. Michael Cambell & Ormerod, P. (1998) presented a model on social interaction and the dynamics of crime. They proposed to give a general description of how crime rate changes over time. They formulated a system of differential equations to model the flows between these groups over time.

Juan C. Nuño *et al*. (2008) developed a model describing the three sociological species, termed as owners, criminals and security Guards [13]. They proposed a system of three ordinary differential equations to account for the dynamics of owners, criminals and security Guards.

Alfred Lotka (1930s) and Vitto Volterra (1920s) independently reduced Darwin's prey-predator interactions to mathematical models [2]. The simplest model of prey and predator association includes only natural growth or decay and the pre-predator interaction itself. All other relationships are assumed to be negligible.

In our model we shall consider, the number of Criminals as the prey population and the number of Police forces population as the predator population in a society. The existence of Police forces depends on the society. Because, the society bear the cost to keep the actual number of Police forces to encounter with the criminals.

III. FUNDAMENTAL ASSUMPTIONS FOR THE MODEL

We consider a society where criminal activities are present. The society allows Police forces to interact with criminals. We consider the following assumptions [3]. ***(i)*** If there are no Police forces population, there are no encounter with criminals, so the criminals will grow exponentially at a rate proportional to the number of the criminals**.** **(*ii*)** If there are no criminal population in the society, then the Police forces population will decline at a rate proportional to the Police forces population. Because in a criminal free society, the society will not bear the cost of Police forces population**.** **(*iii*)** In presence of both Police forces and criminals, the Police forces are beneficial to growth of Police forces population and the criminals are harmful to growth of criminal population.

IV. THE BASIC MODEL

Let**,** $X(t)$ be the number of criminals at time t and $Y(t)$ be the number of Police forces at time t in the society.

From assumption (*i*), if, $Y(t)=0$ then $\frac{d}{dt}X(t)=AX(t),\ A>0$ (1)

As per the assumption (*iii*) that is an increase in Police forces in a criminal prone society will reduce the criminals at rates proportional to the both Police forces $Y(t)$ and criminals $X(t)$ population. It can be written as: $\frac{d}{dt}X(t)=-BX(t)Y(t), B>0$ (2)

From assumption (ii) if, $X(t)=0$ then $Y(t)$ will decline exponentially at a rate to their populations. (because society will not bear the cost to keep Police forces in the criminal free society)

This can be expressed as: $\frac{d}{dt}Y(t)=-CY(t), C>0$ (3)

Again from the assumption (*iii*) , the criminals-Police forces interaction is modeled by mass action terms- as the Police forces population grows in number to prevent the rapid growth of criminal population, the interaction (encounter) rate between the Police forces and criminals also increases at the rate proportional to the product of both populations.

It can be expressed as: $\frac{d}{dt}Y(t) = DX(t)Y(t),\ D>0$ (4)

Now combining equations (1) and (2) we have, the criminals equation

$$\frac{d}{dt}X(t) = X(t)\{A - BY(t)\} \tag{5}$$

Similarly, combining equations (3) and (4), we have the Police forces equation

$$\frac{d}{dt}Y(t) = -Y(t)\{\mathrm{C} - DX(t)\} \tag{6}$$

The equation (5) and (6) together is the system of non-linear first order ordinary differential equations, where A and D are the growth rate constants of criminals and Police force respectively, B is the measures of effect of their interactions and C is the decline rate of Police forces.

If we put $B = \frac{A}{L}$ where constant $L > 0$ and $D = \frac{C}{V}$ where $V > 0$ Then the equations (5) and (6) can be re-written as follows:-

$$\frac{dX}{dt} = AX\left(1 - \frac{Y}{L}\right) \tag{7a}$$

$$\frac{dY}{dt} = -CY\left(1 - \frac{X}{V}\right) \tag{7b}$$

The fixed points are obtained from system of equations (7) by equating $\frac{dX}{dt}$ and $\frac{dY}{dt}$ to zero. If (X^*, Y^*) be the fixed point, then $(X^*, Y^*) = (0,0)$, $(X^*, Y^*) = (V, L)$

V. ANALYSIS OF STABILITY OF THE FIXED POINTS (X^*, Y^*)

The stability is determined of system (7) with the help of Jacobian corresponding to the fixed points. $(X^*, Y^*) = (0,0),(V, L)$

The system (7) can be rewritten as $\frac{dX}{dt} = AX\left(1 - \frac{Y}{L}\right) = f(X,Y)$, say

$$\frac{dY}{dt} = -CY\left(1 - \frac{X}{V}\right) = g(X,Y),\text{say}$$

The Jacobian matrix of the above system at the fixed point (X^*, Y^*) is given by

$$\mathrm{J}(X^*, Y^*) = \begin{bmatrix} \frac{\partial f}{\partial X} & \frac{\partial f}{\partial Y} \\ \frac{\partial g}{\partial X} & \frac{\partial g}{\partial Y} \end{bmatrix}_{(X^*, Y^*)}$$

The Jacobian corresponding to the fixed point $(0,0)$ is $J_* = \mathrm{J}(0,0) = \begin{bmatrix} A & 0 \\ 0 & -C \end{bmatrix}$

The characteristic equation is given by $\Rightarrow \det\begin{bmatrix} A-\lambda & 0 \\ 0 & -C-\lambda \end{bmatrix} = 0$

Hence the eigenvalues are $\lambda_1 = A$ and $\lambda_2 = -C$, so the fixed point $(0,0)$ is unstable.

The Jacobian matrix corresponding to the fixed point (V, L) is

$$J_* = J(V,L) = \begin{bmatrix} \frac{\partial f}{\partial X} & \frac{\partial f}{\partial Y} \\ \frac{\partial g}{\partial X} & \frac{\partial g}{\partial Y} \end{bmatrix}_{(V,L)} = \begin{bmatrix} 0 & -\frac{AV}{L} \\ \frac{CL}{V} & 0 \end{bmatrix}$$

The characteristic equation for the Eigen value λ is $\det(J_* - \lambda I) = 0$

$$\Rightarrow \det\left(\begin{bmatrix} 0-\lambda & -\frac{AV}{L} \\ \frac{CL}{V} & 0-\lambda \end{bmatrix}\right) = 0 \Rightarrow \lambda = \pm i\sqrt{AC} \tag{8}$$

The relation (8) shows that the real parts of Eigen values of the Jacobian matrix are zero for the fixed point (V, L).So the fixed point (V, L) is a neutrally stable centre or vortex (as the Eigen values are purely imaginary). The off-diagonal terms $\frac{CL}{V}$ and $-\frac{AV}{L}$ are opposite of sign and diagonal terms are zero. This analysis predicts oscillations about the equilibrium state.

VI. MODIFIED MODEL

In this model we introduce two more parameters namely recruitment rate of Home guards and the rate at which the Home guards come into contact with the criminals.

Let G be the Home Guards engaged in the society and α be the rate of the criminals come into contact with the Home Guards. Then the model can be written as

$$\frac{d}{dt}X(t) = X(t)\{A - BY(t)\} - \alpha GX \tag{9a}$$

$$\frac{d}{dt}Y(t) = -Y(t)\{C - DX(t)\} + \alpha GX \tag{9b}$$

The fixed points are obtained from system of equations (9) by equating $\frac{dX}{dt}$ and $\frac{dY}{dt}$ to zero. If (X^*, Y^*) be the fixed points, then $(X^*, Y^*) = (0,0)$ and $(X^*, Y^*) = \left(\frac{CA - \alpha GC}{DA - \alpha GD + \alpha GB}, \frac{A - \alpha G}{B}\right)$

VII. ANALYSIS OF STABILITY OF THE FIXED POINTS

The stability is determined of system (9) with the help of Jacobian corresponding to the fixed points.
The systems can be re-written as

$$\frac{d}{dt}X(t) = X(t)\{A - BY(t)\} - \alpha GX = f(X,Y), \text{ say}$$

$$\frac{d}{dt}Y(t) = -Y(t)\{C - DX(t)\} + \alpha GX = g(X,Y), \text{ say}$$

The Jacobian matrix of the above systems at the fixed point $(0,0)$ is given by

$$J_* = \mathrm{J}(0,0) = \begin{bmatrix} A - BY - \alpha G & -BX \\ DY + \alpha G & -C + DX \end{bmatrix}_{(0,0)} = \begin{bmatrix} A - \alpha G & 0 \\ \alpha G & -C \end{bmatrix}$$

The characteristic equation is given by $|J_* - \lambda I| = 0$, where is the eigen values of J_*

$$\Rightarrow \begin{vmatrix} A - \alpha G - \lambda & 0 \\ \alpha G & -C - \lambda \end{vmatrix} = 0$$ and hence the eigen value are $\lambda_1 = A - \alpha G$, $\lambda_2 = -C$

$$\lambda_1 = 0.12 - 0.00001 \times 0.0002 = 0.12, \lambda_2 = -0.05$$

Hence the first fixed point is unstable. Again for the second fixed point

$$(X^*, Y^*) = \left(\frac{CA - \alpha GC}{DA - \alpha GD + \alpha GB}, \frac{A - \alpha G}{B} \right) = (u, v), say$$

the Jacobian matrix of the above systems is

$$J_* = J(u,v) = \begin{bmatrix} A - BY - \alpha G & -BX \\ DY + \alpha G & -C + DX \end{bmatrix}_{(u,v)} = \begin{bmatrix} A - Bv - \alpha G & -Bu \\ Dv + \alpha G & -C + Du \end{bmatrix} = \begin{bmatrix} P & Q \\ R & S \end{bmatrix}$$

The characteristic equation is

$$\left| J_* - \lambda I \right| = 0 \Rightarrow \begin{vmatrix} P - \lambda & Q \\ R & S - \lambda \end{vmatrix} = 0 \Rightarrow \lambda^2 - (P+S)\lambda + (PS - RQ) = 0$$

$$trace = T_1 = P + S = A - Bv - \alpha G + (-C + Du) = \frac{-\alpha GBC}{DA - \alpha GD + \alpha GB} = -2.38 \times 10^{-9}$$

$$\det = D_1 = PS - RQ = (A - Bv - \alpha G)(-C + Du) - (Dv + \alpha G)(-Bu) = 5.7144 \times 10^{-3}$$

Since $T_1 < 0$ and $D_1 > 0$ hence the system is asymptotically stable.

Table-1: Parameters and their values

Parameter	Parameter Definition	Values
A	Growth rate coefficient of criminals	0.12
B	Constant of proportionality that measures the probability that a Police forces-criminals encounter removes one of the criminals.	0.002
C	The decline rate of Police forces populations	0.05
D	Growth rate coefficient that measure the efficiency of Police forces populations to increase the encounter for reducing criminal activities.	0.0007
X_0	Initial number of criminals	60
Y_0	Initial number of Police forces	40
N_0	Initial population	100
α	rate of the criminals come into contact with the Home Guards	0.00001
G	Home Guards engaged	0.0002

VIII. DERIVATION OF PARAMETER TO THE MODEL

For the better estimation of parameter for growth and decay rate constants, the initial conditions can be choice from the crime data during the year 1995 to 2011. Considering the initial conditions as $X(0) = X_0 = 60$, $Y(0) = Y_0 = 40$ to find the value of parameters we average the data between two maximum or minimum, so that we obtain a reasonable estimation of equilibrium for these data. Averaging the data between 1996 and 2006 for the criminals and the data between 1995 and 2006 for the Police forces, the following equilibrium is estimated $X_* = V = (C/D) = 74.1$ and $Y_* = L = (A/B) = 56.2$. The data shows that the Police forces population is low around 2000 and at this time criminal population grow rapidly. So we can take $X_0 = 68.1$ (in 2000), $X(t) = 76.7$ (in 2001) and time period, $t = 1$ year, for criminals, we have $X(t) = X_0 e^{At} \Rightarrow A = 0.12$

The Steepest decay of Police forces occurs between 2004and 2005. So, we can take $Y_0 = 57.2$, $Y(t) = 54.2$ and duration, $t = 1$ year, for Police forces $Y(t) = Y_0 e^{-Ct} \Rightarrow C = 0.05$

Using these values of growth and decay constant and assuming the rate of Home Guards engaged in the Society we get the initial estimation for the parameters D , B , α and G .

IX. NUMERICAL SOLUTIONS OF THE MODEL

The numbers of criminals $X(t)$ and Police forces $Y(t)$ to be determined from the systems (7) and (9) by Runge-Kutta's numerical solution methods. The functions $X(t)$ and $Y(t)$ for all t in an interval are called solutions of systems (7) and (9) respectively. The value X_0 of $X(t)$ and Y_0 of $Y(t)$ at some t_0 can be estimated, and must surely be a critical factor in predicting later values of $X(t)$ and $Y(t)$. The condition $X(t_0) = X_0$ and $Y(t_0) = Y_0$ are respectively called an initial condition of systems (7) and (9).

Measuring time forward from the time t_0, we have created a problem whose solution $X(t)$ and $Y(t)$ are predicted number of criminals and Police forces at future times-

In (9), for the given constants A, C , L, and V and the value t_0 and X_0, Y_0 we find functions $X(t), Y(t)$ for which

$$X' = AX\left(1 - \frac{Y}{L}\right), X(t_0) = X_0 \tag{10a}$$

$$Y' = -CY(1 - \frac{X}{V}),\ Y(t_0) = Y_0 \tag{10b}$$

on some t -interval containing t_0. The ODEs and the initial conditions in (10a) and (10b) form an initial value problem (IVP) for $X(t)$ and $Y(t)$
We can write the IVP (10a) and (10b) as below-

$$X' = f\left(t, X, Y\right), \quad X(t_0) = X_0 \tag{11a}$$

$$Y' = g\left(t, X, Y\right), \quad Y(t_0) = Y_0 \tag{11b}$$

Similarly the IVP for the ODEs (9a) and (9b) can be written as

$$X' = X(A - BY - \alpha G), \quad X(t_0) = X_0 \tag{12a}$$

$$Y' = Y(-C + DX) + \alpha GX,\ Y(t_0) = Y_0 \tag{12b}$$

Where $X' = \Phi\left(t, X, Y\right), \quad X(t_0) = X_0$ and $Y' = \Psi\left(t, X, Y\right), \quad Y(t_0) = Y_0$

The IVP of the system (10) and (12) have unique solutions on an interval containing t_0 if the rate functions $f(t, X, Y)$, $g(t, X, Y)$, $\Phi\left(t, X, Y\right)$ and $\Psi\left(t, X, Y\right)$ are well enough behaved.

We discuss basic numerical procedures called Runge-Kutta method for finding approximate values for the solutions $X(t)$ and $Y(t)$ of the IVP (10) and (12) at a discrete set of times near t_0.

Runge-Kutta Fourth Order (RK4) Method of numerical solution of the model has been discussed in [4]. The general RK4 method for the Criminals and Police forces for the systems (10) and (12) are

$$t_n = t_{n-1} + h$$

$$X_n = X_{n-1} + \frac{h}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right), n = 1, 2, ..., N \ Y_n = Y_{n-1} + \frac{h}{6}\left(p_1 + 2p_2 + 2p_3 + p_4\right), n = 1, 2, ..., N$$

Where h is the fixed step size and for the system (10)

$$k_1 = f\left(t_{n-1}, X_{n-1}, Y_{n-1}\right), p_1 = g\left(t_{n-1}, X_{n-1}, Y_{n-1}\right)$$

$$k_1 = AX_{n-1}\left(t_{n-1}\right)\left(1 - \frac{Y_{n-1}\left(t_{n-1}\right)}{L}\right), p_1 = -CY_{n-1}\left(t_{n-1}\right)\left(1 - \frac{X_{n-1}\left(t_{n-1}\right)}{V}\right)$$

$$k_2 = f\left(t_{n-1} + \frac{h}{2}, X_{n-1} + \frac{h}{2}k_1, Y_{n-1} + \frac{h}{2}p_1\right), p_2 = g\left(t_{n-1} + \frac{h}{2}, X_{n-1} + \frac{h}{2}k_1, Y_{n-1} + \frac{h}{2}p_1\right)$$

$$k_2 = A\left(X_{n-1}\left(t_{n-1} + \frac{h}{2}\right) + \frac{h}{2}k_1\right)\left(1 - \frac{1}{L}\left(Y_{n-1}\left(t_{n-1} + \frac{h}{2}\right) + \frac{h}{2}p_1\right)\right)$$

$$p_2 = -C\left(Y_{n-1}\left(t_{n-1}+\frac{h}{2}\right)+\frac{h}{2}p_1\right)\left(1-\frac{1}{V}\left(X_{n-1}\left(t_{n-1}+\frac{h}{2}\right)+\frac{h}{2}k_1\right)\right)$$

$$k_3 = f\left(t_{n-1}+\frac{h}{2}, X_{n-1}+\frac{h}{2}k_2, Y_{n-1}+\frac{h}{2}p_2\right), p_3 = g\left(t_{n-1}+\frac{h}{2}, X_{n-1}+\frac{h}{2}k_2, Y_{n-1}+\frac{h}{2}p_2\right)$$

$$k_3 = A\left\{X_{n-1}\left(t_{n-1}+\frac{h}{2}\right)+\frac{h}{2}k_2\right\}\left\{1-\frac{1}{L}\left(Y_{n-1}\left(t_{n-1}+\frac{h}{2}\right)+\frac{h}{2}p_2\right)\right\}$$

$$p_3 = -C\left\{Y_{n-1}\left(t_{n-1}+\frac{h}{2}\right)+\frac{h}{2}p_2\right\}\left\{1-\frac{1}{V}\left(X_{n-1}\left(t_{n-1}+\frac{h}{2}\right)+\frac{h}{2}k_2\right)\right\}$$

$$k_4 = f\left(t_{n-1}+h, X_{n-1}+hk_3, Y_{n-1}+hp_3\right), p_4 = g\left(t_{n-1}+h, X_{n-1}+hk_3, Y_{n-1}+hp_3\right)$$

$$k_4 = A\left\{X_{n-1}\left(t_{n-1}+h\right)+hk_3\right\}\left\{1-\frac{1}{L}\left(Y_{n-1}\left(t_{n-1}+h\right)+hp_3\right)\right\}$$

$$p_4 = -C\left\{Y_{n-1}\left(t_{n-1}+h\right)+hp_3\right\}\left\{1-\frac{1}{V}\left(X_{n-1}\left(t_{n-1}+h\right)+hk_3\right)\right\}$$

For the systems (12)

$$k_1 = \Phi\left(t_{n-1}, X_{n-1}, Y_{n-1}\right), p_1 = \Psi\left(t_{n-1}, X_{n-1}, Y_{n-1}\right)$$

$$k_1 = X_{n-1}\left(t_{n-1}\right)[A - BY_{n-1}\left(t_{n-1}\right) - \alpha G]$$

$$p_1 = -CY_{n-1}\left(t_{n-1}\right) + DX_{n-1}\left(t_{n-1}\right)Y_{n-1}\left(t_{n-1}\right) + \alpha GX_{n-1}\left(t_{n-1}\right)$$

$$k_2 = \Phi\left(t_{n-1}+\frac{h}{2}, X_{n-1}+\frac{h}{2}k_1, Y_{n-1}+\frac{h}{2}p_1\right), p_2 = \Psi\left(t_{n-1}+\frac{h}{2}, X_{n-1}+\frac{h}{2}k_1, Y_{n-1}+\frac{h}{2}p_1\right)$$

$$k_2 = \left(X_{n-1}\left(t_{n-1}+\frac{h}{2}\right)+\frac{h}{2}k_1\right)\left(A - B\left(Y_{n-1}\left(t_{n-1}+\frac{h}{2}\right)+\frac{h}{2}p_1\right)-\alpha G\right)$$

$$p_2 = -C\left(Y_{n-1}\left(t_{n-1}+\frac{h}{2}\right)+\frac{h}{2}p_1\right)+\left(X_{n-1}\left(t+\frac{h}{2}\right)+\frac{h}{2}k_1\right)\left[D\left(Y_{n-1}\left(t_{n-1}+\frac{h}{2}\right)+\frac{h}{2}p_1\right)+\alpha G\right]$$

$$k_3 = \Phi\left(t_{n-1}+\frac{h}{2}, X_{n-1}+\frac{h}{2}k_2, Y_{n-1}+\frac{h}{2}p_2\right), p_3 = \Psi\left(t_{n-1}+\frac{h}{2}, X_{n-1}+\frac{h}{2}k_2, Y_{n-1}+\frac{h}{2}p_2\right)$$

$$k_3 = \left\{X_{n-1}\left(t_{n-1}+\frac{h}{2}\right)+\frac{h}{2}k_2\right\}\left\{A - B\left(Y_{n-1}\left(t_{n-1}+\frac{h}{2}\right)+\frac{h}{2}p_2\right)-\alpha G\right\}$$

$$p_3 = -C\left\{Y_{n-1}\left(t_{n-1}+\frac{h}{2}\right)+\frac{h}{2}p_2\right\}+\left(X_{n-1}\left(t_{n-1}+\frac{h}{2}\right)+\frac{h}{2}k_2\right)[D\left\{Y_{n-1}\left(t_{n-1}+\frac{h}{2}\right)+\frac{h}{2}p_2\right\}+\alpha G]$$

$$k_4 = \Phi\left(t_{n-1}+h, X_{n-1}+hk_3, Y_{n-1}+hp_3\right), p_4 = \Psi\left(t_{n-1}+h, X_{n-1}+hk_3, Y_{n-1}+hp_3\right)$$

$$k_4 = \left\{X_{n-1}\left(t_{n-1}+h\right)+hk_3\right\}\left\{A - B\left(Y_{n-1}\left(t_{n-1}+h\right)+hp_3\right)-\alpha G\right\}$$

$$p_4 = -C\left\{Y_{n-1}\left(t_{n-1}+h\right)+hp_3\right\}+\left(X_{n-1}\left(t_{n-1}+h\right)+hk_3\right)[D\left\{Y_{n-1}\left(t_{n-1}+h\right)+hp_3\right\}+\alpha G]$$

CONCLUSION

From the analysis of the models it can be observed that the models are pre-predator models.

The numerical result shows that the fixed point (0, 0) of the system is unstable which shows that the Police forces or criminal population decreasing to zero or increasing without limit. The fixed point (V, L) and (u, v) of the systems are stable which predicts oscillations about the equilibrium state of the two populations.

The result suggests that criminals' population falls rapidly if Home guards engaged in the society to help Police forces population.

The general RK4 method of Criminals and Police forces equations are derived for the two models, which yields the numerical solutions of criminals and Police forces.

ACKNOWLEDGEMENTS

The author thanks Dr. Atowar Rahman, Associate Professor of the Department of mathematics, B.P. Chaliha College, Nagarbera for many suggestions.

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Source of support: Nil, Conflict of interest: None Declared.