

**PART -II ON NON-ASSOCIATIVE  $\Gamma$ -SEMI SUB NEAR-FIELD SPACES  
OF A  $\Gamma$ -NEAR-FIELD SPACE OVER NEAR-FIELD (NA- $\Gamma$ -SSNFS- $\Gamma$ -NFS-NF)**

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**ABSTRACT**

*In this manuscript we introduce Thurumella non-associative  $\Gamma$ -semi sub near-field space of type II and define some interesting results about them. We see that T-NA- $\Gamma$ -semi sub near-field space of type I happen to have a very meagre property for we in the definition of T-NA- $\Gamma$ -semi sub near-field spaces of type I replace the groupoid by T-groupoids this does not make a big change except one of the obvious properties that these non-associative  $\Gamma$ -semi sub near-field spaces contain associative  $\Gamma$ -semi sub near-field spaces as structures. Except for this we were not able to achieve many more properties we proceed on to define Thurumella NA- $\Gamma$ -semi sub near-field spaces.*

**Keywords:** Non-associative  $\Gamma$ -semi sub near-field space, Thurumella- non-associative  $\Gamma$ -semi sub near-field space,  $\Gamma$ -semi sub near-field space,  $\Gamma$ -near-field space;  $\Gamma$ -Semi sub near-field space of  $\Gamma$ -near-field space; Semi near-field space of  $\Gamma$ -near-field space, T-insertion of factor property(T-IFP).

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**SECTION-1: INTRODUCTION CUM PRELIMINARIES ON SOME SPECIAL THURUMELLA NON-ASSOCIATIVE  $\Gamma$ -SEMI SUB NEAR-FIELD SPACE OF TYPE II IN (T-NA- $\Gamma$ -SSNFS- $\Gamma$ -NFS-NF) A  $\Gamma$ -NEAR-FIELD SPACE OVER NEAR-FIELD.**

Here in this section, we introduce Thurumella non-associative  $\Gamma$ -semi sub near-field space of type II in a  $\Gamma$ -near-field space over near-field and define some interesting results of its.

Since these T-NA- $\Gamma$ -semi sub near-field spaces are non-associative structures we would justified in studying the cases when these T-NA- $\Gamma$ -semi sub near-field spaces satisfy the special identities mentioned thereof.

As we do not have a very large class of T-NA- $\Gamma$ -semi sub near-field spaces for that matter NA- $\Gamma$ -semi sub near-field spaces of a  $\Gamma$ -semi near-field space over near-field only by using loops and groupoids and these newly constructed structure will be called as loop  $\Gamma$ -semi sub near-field spaces and groupoid  $\Gamma$ -semi sub near-field spaces.

To the best of the authors' knowledge no researcher has studied such type of identities in these structures. Further in view of the author the very study of non-associative  $\Gamma$ -semi sub near-field spaces and  $\Gamma$ -semi near-field spaces is very meagre(of something provided or available). It is all the more important to state especially these non-associative  $\Gamma$ -semi sub near-field spaces do not find any place in any text books on near-rings. Even the important concept of  $\Gamma$ -semi sub near-field spaces is dealt very sparingly in some of the text books. In case of near-rings there are only less than a dozen text books.

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Thus we have tried to give the definitions as well, we proceed on to define the Thurumella sub structures and Thurumella special elements in these NA- $\Gamma$ -semi near-field spaces. Throughout this paper we only assume T- $\Gamma$ -semi sub near-field space of level II or T- $\Gamma$ -semi sub near-field space of level I type B in a  $\Gamma$ -semi near-field space over a near-field.

**Definition 1.1: Thurumella non-associative  $\Gamma$ -semi sub near-field space II (T-NA- $\Gamma$ -semi sub near-field space II).**

Let  $(N, '+', \cdot)$  be a NA- $\Gamma$ -semi sub near-field space, we say that N is a Thurumella NA- $\Gamma$ -semi sub near-field space II (T-NA- $\Gamma$ -semi sub near-field space II) if N has a proper  $\Gamma$ -semi sub near-field space P which is a associative  $\Gamma$ -semi sub near-field space.

**Theorem 1.2:** Let N be a T-NA- $\Gamma$ -semi sub near-field space II then N is a T-NA- $\Gamma$ -semi sub near-field space I of type A.

**Proof:** It is obvious by the very definition. Now using the T-mixed direct product definition of  $\Gamma$ -semi sub near-field spaces we can extend it to the case of NA- $\Gamma$ -semi sub near-field spaces of a  $\Gamma$ -semi near-field space over near-field. This method will help to build a class of T-NA- $\Gamma$ -semi sub near-field spaces of type II. This completes the proof of the theorem.

**Definition 1.3: T-mixed direct product of  $\Gamma$ -semi sub near-field spaces.**

Let  $N_1, N_2, \dots, N_k$  be k  $\Gamma$ -semi sub near-field spaces of a  $\Gamma$ -semi near-field space over near-field where at least one of the  $\Gamma$ -semi sub near-field spaces is non-associative and at least one is associative. The direct product of these  $\Gamma$ -semi sub near-field spaces  $N = N_1 \times N_2 \times \dots \times N_k$  is called the T-mixed direct product of  $\Gamma$ -semi sub near-field spaces (T-mixed direct product of  $\Gamma$ -semi sub near-field spaces). The operation in N is carried out component wise. Clearly N is a T- $\Gamma$ -semi sub near-field space II and T- $\Gamma$ -semi sub near-field space I of type A.

**Definition 1.4: T-mixed direct product of  $\Gamma$ -semi sub near-field spaces.**

Let  $N_1, N_2, \dots, N_k$  be a collection of  $\Gamma$ -semi sub near-field spaces, non-associative  $\Gamma$ -semi sub near-field spaces and  $\Gamma$ -semi near-field spaces over near-field. The direct product of these  $\Gamma$ -semi sub near-field spaces  $N = N_1 \times N_2 \times \dots \times N_k$  is called the Thurumella strong mixed direct product (T-strong mixed direct product) of NA- $\Gamma$ -semi sub near-field spaces. Under the component wise operation on N we get N to be a NA- $\Gamma$ -semi sub near-field space. Clearly N is a T-NA- $\Gamma$ -semi sub near-field space I of type A and B and also T-NA- $\Gamma$ -semi sub near-field space II.

**Definition 1.5: T-left N  $\Gamma$ -semi sub near-field space.**

Let N be a T- $\Gamma$ -semi sub near-field space II. An additive T- $\Gamma$ -semi sub near-field space A of N is said to be a T-left N  $\Gamma$ -semi sub near-field space (right-N  $\Gamma$ -semi sub near-field space) if  $PA \subset A$  and  $(AP \subset A)$  where P is a proper  $\Gamma$ -semi sub near-field space of N and P is a  $\Gamma$ -semi sub near-field space which is associative,  $PA = \{pa / p \in P \text{ and } a \in A\}$ .

**Definition 1.6:** Let N and  $N_1$  be any two T-NA- $\Gamma$ -semi sub near-field spaces A mapping  $\phi : N \rightarrow N_1$  is called a

Thurumella – NA- $\Gamma$ -semi sub near-field space homomorphism (T-NA- $\Gamma$ -semi sub near-field space homo) if  $\phi$  maps every  $p \in P \subset N$  (p a associative  $\Gamma$ -semi sub near-field space associative) into a unique element  $\phi(p) \in P_1 \subset N_1$  where  $P_1$  is an associative  $\Gamma$ -semi sub near-field space such that  $\phi(p + p') = \phi(p) + \phi(p')$  and  $\phi(p \cdot p') = \phi(p) \cdot \phi(p')$  and  $\phi(p_1 p_2) = \phi(p_1) \phi(p_2)$  for every  $p_1, p_2 \in P \subset N$ .

**Note 1.7:** It is important that  $\phi$  need not be defined on the whole of N it is sufficient if  $\phi$  is defined on a  $\Gamma$ -semi sub near-field space say P of a  $\Gamma$ -near-field space is an associative  $\Gamma$ -semi sub near-field space. Thus if  $P \subset N$  and  $P_1 \subset N_1$  and  $\phi : N \rightarrow N_1$  is such that  $\phi$  is one to one and on to from P to  $P_1$  the two T-NA- $\Gamma$ -semi sub near-field spaces would become isomorphic even if they are not having same number of elements in them.

**Definition 1.8:** Let N be a T-NA- $\Gamma$ -semi sub near-field space. An additive T- $\Gamma$ -semi sub near-field space A of N is said

to be a Thurumella left ideal (T-left ideal) of N if it is an ideal of the  $\Gamma$ -semi sub near-field space  $(N, +)$  with conditions.  $q_1(q_2 + a) - q_1q_2 \in A$  for each  $a \in A$ ;  $q_1, q_2 \in P \subset N$ ; P is a  $\Gamma$ -semi sub near-field space.

**Definition 1.9:** A  $\Gamma$ -semi sub near-field space I of N is called T-ideal if it is a T-left ideal and  $IP \subset I$  where  $P \subset N$  is the

associative  $\Gamma$ -semi sub near-field space of a  $\Gamma$ -semi near-field space over near-field.

**Definition 1.10:** A T-NA- $\Gamma$ -semi sub near-field space N has Thurumella-IFP (T-IFP) (insertion of factor property) if

for  $a, b \in P$ ,  $ab = 0$  implies  $a.p.B = 0$  for all  $p \in P \subset N$  since N is non-associative we have to restrict our selves only to the associative structure to define IFP property as  $(nb) \neq (an) b$  in general for all  $a, n, b \in N$ .

**Remark 1.11:** Now for T-NA- $\Gamma$ -semi sub near-field space of level I type B we define all the above concepts with the only modification we replace the  $\Gamma$ -semi sub near-field space P by the  $\Gamma$ -semi near-field space in all the definitions.

**Remark 1.12:** Certainly we can not say T-ideals in T-NA- $\Gamma$ -semi sub near-field space of level II will be the same as T-ideals of T-NA- $\Gamma$ -semi sub near-field space I of type B even if N is both a T-NA- $\Gamma$ -semi sub near-field space II and T-NA- $\Gamma$ -semi sub near-field space I type B. So all results will be different. The reader can verify them. We give only example.

**Example 1.13:** Let  $N = N_1 \times N_2 \times N_3$  where  $N_1$  any non-associative  $\Gamma$ -semi sub near-field space,  $N_2 = Z_{24}$  where ' $\times$ ' is the usual multiplication modulo 24 and ' $\cdot$ ' is  $a \cdot b = a$  for all  $a, b \in Z_{24}$  is an associative  $\Gamma$ -semi sub near-field space and  $N_3 = Z_{12}$  where '+' is the operation i.e. addition modulo 12 and  $a \cdot b = a$  for all  $a, b \in N_3$ ; clearly  $N_3$  is an associative  $\Gamma$ -semi sub near-field space. N by the very definition is a NA- $\Gamma$ -semi sub near-field space. Now N is a T-NA- $\Gamma$ -semi sub near-field space II for it has  $N_2$  as a proper  $\Gamma$ -semi sub near-field space which is an associative  $\Gamma$ -semi sub near-field space. N is also a T-NA- $\Gamma$ -semi sub near-field space I type B as  $N_3$  is a proper  $\Gamma$ -semi sub near-field space which is a  $\Gamma$ -semi near-field space of N. Thus we see simultaneously N is T-NA- $\Gamma$ -semi sub near-field space II and T-NA- $\Gamma$ -semi sub near-field space I type B. It is left for the reader to define T-ideals in them and see how they are different sub near field spaces behaving in a distinct way, in case of T-mixed direct product of  $\Gamma$ -semi sub near-field space II or T-NA- $\Gamma$ -semi sub near-field space I of type A. this is clearly evident from the example.

**Example 1.14:** Let  $N = Z \times Z_{15} \times N_1$  where Z is a  $\Gamma$ -semi near-field space in which  $(Z, +)$  is a near-field space and for all  $x, y \in Z$ ;  $x \cdot y = x$ ,  $(Z_{15}, \times, \cdot)$  is a  $\Gamma$ -semi sub near-field space under usual multiplication modulo 15 and ' $\cdot$ ' As  $a \cdot b = a$  for all  $a, b \in Z_{15}$  and  $N_1$  some non-associative  $\Gamma$ -semi sub near-field space. N is T-NA- $\Gamma$ -semi sub near-field space II and T-NA- $\Gamma$ -semi sub near-field space I of type B.

**Note 1.15:** these examples are not still in a position to find natural examples of NA- $\Gamma$ -semi sub near-field space, we have a class of associative  $\Gamma$ -semi sub near-field space given by  $Z_n$  where  $(Z_n, \times)$  is a  $\Gamma$ -semi near-field space under multiplication modulo n and  $a \cdot b = a$  for all  $a, b \in Z_n$ . Thus we discuss further completely devoted to finding a class of NA- $\Gamma$ -semi sub near-field spaces of a  $\Gamma$ -semi near-field space over near-field.

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