

NANO IDEAL α - REGULAR CLOSED SETS IN NANO IDEAL TOPOLOGICAL SPACES

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ABSTRACT

The purpose of this paper is to define and study a new class of closed sets called NI_{cr} - closed sets in nano ideal topological spaces. Basic properties of NI_{cr} - closed sets are analyzed and we compared it with some existing and few new closed sets in nano ideal topology introduced in this paper.

MSC (2010): 54A05, 54A10.

Key words: NI_{cr} - closed set, Closed sets in nano ideal topology, NI_{cr} - open set, Nano topology.

1. INTRODUCTION

The concept of ideal topological space was introduced by Kuratowski [4]. In 1990, Jankovic and Hamlett investigated further properties of ideal topological spaces [2]. An ideal I on a nonempty collection of subsets of X which satisfies (i) $A \in I$ and $B \subset A$, implies $B \in I$ and (ii) $A \in I$ and $B \in I$, implies $A \cup B \in I$. Given a topological space (X, τ) with an ideal I on X and if $P(X)$ is the set of all subsets of X , a set operator $(.)^* : P(X) \rightarrow P(X)$ called a local function of A with respect to τ and I is defined as follows: for $A \subset X$, $A^*(I, \tau) = \{x \in X : U \cap A \notin I, \text{ for every } U \in \tau(X)\}$ where $\tau(X) = \{U \in \tau : X \in U\}$. A Kuratowski closure operator $cl^*(.)$ for a topology $\tau^*(I, \tau)$ called the $*$ -topology finer than τ , is defined by $cl^*(A) = A \cup A^*(I, \tau)$. When there is no chance of confusion, we will simply write A^* for $A^*(I, \tau)$ and τ^* for $\tau^*(I, \tau)$. If I is an ideal on X , the space (X, τ, I) is called the ideal topological space.

The concept of nano topology was introduced by Lellis Thivagar.M [5], which was defined in terms of approximations and boundary region of a subset of a universe using an equivalence relation on it. He has also defined a Nano continuous functions, Nano open mappings, Nano closed mappings and Nano Homeomorphisms and their representations in terms of Nano closure and Nano interior. In this paper, we introduce and investigate a new class of closed sets called NI_{cr} - closed sets and also discuss the relationship with some new and existing closed sets in nano ideal topological spaces.

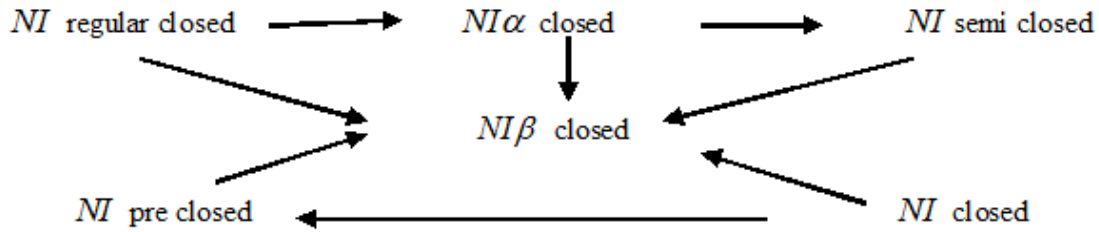
2. PRELIMINARIES

Definition 2.1: [5,7] A subset A of a nano ideal topological space $(U, \tau_R(X, I))$ is called,

- (i) NI open if $A \subseteq N \text{int}(A^{*N})$ and its complement is NI closed.
- (ii) NI pre open if $A \subseteq N \text{int}(Ncl^*(A))$ and NI pre closed if $Ncl^*(N \text{int}(A)) \subseteq A$.
- (iii) NI semi open if $A \subseteq Ncl^*(N \text{int}(A))$ and NI semi closed if $N \text{int}(Ncl^*(A)) \subseteq A$.
- (iv) $NI\alpha$ open if $A \subseteq N \text{int}(Ncl^*(N \text{int}(A)))$ and $NI\alpha$ closed if $Ncl^*(N \text{int}(Ncl^*(A))) \subseteq A$.
- (v) $NI\beta$ open if $A \subseteq Ncl^*(N \text{int}(Ncl^*(A)))$ and $NI\beta$ closed if $N \text{int}(Ncl^*(N \text{int}(A))) \subseteq A$.
- (vi) NI regular open if $A = N \text{int}(Ncl^*(A))$ and NI regular closed if $Ncl^*(N \text{int}(A)) = A$.

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We have the following implications



Nano closed set is independent of NI closed set.

3. SOME CLOSED SETS IN NANO IDEAL TOPOLOGICAL SPACES

Definition 3.1: A subset A of a nano ideal topological space $(U, \tau_R(X, I))$ is called,

- (i) a nano ideal regular generalized closed set (NI_{rg} - closed) if $NIcl(A) \subseteq Z$ whenever $A \subseteq Z$ and Z is nano regular open.
- (ii) a nano ideal generalized pre closed set (NI_{gp} - closed) if $NIpcl(A) \subseteq Z$ whenever $A \subseteq Z$ and Z is nano open.
- (iii) a nano ideal α - generalized closed set ($NI_{\alpha g}$ - closed) if $NI\alpha cl(A) \subseteq Z$ whenever $A \subseteq Z$ and Z is nano open.
- (iv) a nano ideal generalized α - closed set ($NI_{g\alpha}$ - closed) if $NI\alpha cl(A) \subseteq Z$ whenever $A \subseteq Z$ and Z is nano α - open.
- (v) a nano ideal generalized semi closed set (NI_{gs} - closed) if $NI scl(A) \subseteq Z$ whenever $A \subseteq Z$ and Z is nano open.
- (vi) a nano ideal semi generalized closed set (NI_{sg} - closed) if $NI scl(A) \subseteq Z$ whenever $A \subseteq Z$ and Z is nano semi open.
- (vii) a nano ideal generalized closed set (NI_g - closed) if $NIcl(A) \subseteq Z$ whenever $A \subseteq Z$ and Z is nano open.
- (viii) a nano ideal generalized pre regular closed set (NI_{gpr} - closed) if $NIpcl(A) \subseteq Z$ whenever $A \subseteq Z$ and Z is nano regular open.
- (ix) a nano ideal generalized β - closed set ($NI_{g\beta}$ - closed) if $NI\beta cl(A) \subseteq Z$ whenever $A \subseteq Z$ and Z is nano open.
- (x) a nano ideal generalized regular closed set (NI_{gr} - closed) if $NIrc cl(A) \subseteq Z$ whenever $A \subseteq Z$ and Z is nano open.

4. NANO IDEAL α REGULAR CLOSED SET

Definition 4.1: A subset A of a nano ideal topological space $(U, \tau_R(X, I))$ is called nano ideal α regular closed set (briefly $NI_{\alpha r}$ - closed) if $NI\alpha cl(A) \subseteq Z$ whenever $A \subseteq Z$ and Z is nano regular open.

Theorem 4.2: In a nano ideal topological space $(U, \tau_R(X, I))$,

- (i) every $NI_{\alpha g}$ - closed, $NI_{g\alpha}$ - closed and NI_{gr} - closed set is $NI_{\alpha r}$ - closed
- (ii) every $NI_{\alpha r}$ - closed set is NI_{gpr} - closed

Converse of the above theorem need not be true as shown in the following example.

Example 4.3: Let $U = \{a, b, c, d\}$, $U/R = \{\{a\}, \{b\}, \{c, d\}\}$, $X = \{b, d\}$ and $I = \{\emptyset, \{a\}\}$. Then $\tau_R(X) = \{\emptyset, U, \{b\}, \{c, d\}, \{b, c, d\}\}$.

- (i) $A = \{b, d\}$ is NI_{cr} - closed, but not NI_{ag} - closed
- (ii) $B = \{b, c\}$ is NI_{cr} - closed, but not $NI_{g\alpha}$ - closed.
- (iii) $C = \{c\}$ is NI_{gpr} - closed, but not NI_{cr} - closed.
- (iv) $D = \{b, c, d\}$ is NI_{cr} - closed, but not NI_{gr} - closed.

The following example shows that NI_{cr} - closed set is independent of NI_{gp} - closed, NI_{gs} - closed, NI_{sg} - closed, NI_g - closed and $NI_{g\beta}$ - closed sets.

Example 4.4: Let $U = \{a, b, c, d\}$, $U/R = \{\{a\}, \{b\}, \{c, d\}\}$, $X = \{b, d\}$ and $I = \{\emptyset, \{a\}\}$. Then $\tau_R(X) = \{\emptyset, U, \{b\}, \{c, d\}, \{b, c, d\}\}$.

- (i) NI_{cr} - closed set = $\{\emptyset, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, U\}$
- (ii) NI_{gp} - closed set = $\{\emptyset, \{a\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, U\}$
- (iii) NI_{gs} - closed set = $\{\emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, U\}$
- (iv) NI_{sg} - closed set = $\{\emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, U\}$
- (v) NI_g - closed set = $\{\emptyset, \{a\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, U\}$
- (vi) $NI_{g\beta}$ - closed set = $\{\emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, U\}$

Theorem 4.5: Let $(U, \tau_R(X), I)$ be a nano ideal topological space and A be a subset of U. Then

- (i) every nano closed set is NI_{cr} - closed
- (ii) every NI_{α} - closed set is NI_{cr} - closed
- (iii) every NI regular closed set is NI_{cr} - closed.

Proof: (i) Let $A \subseteq Z$, where Z is nano regular open. By hypothesis and since every nano closed set is NI_{α} - closed, $NI_{\alpha}cl(A) \subseteq Ncl(A) = A \subseteq Z$. Hence A is NI_{cr} - closed.

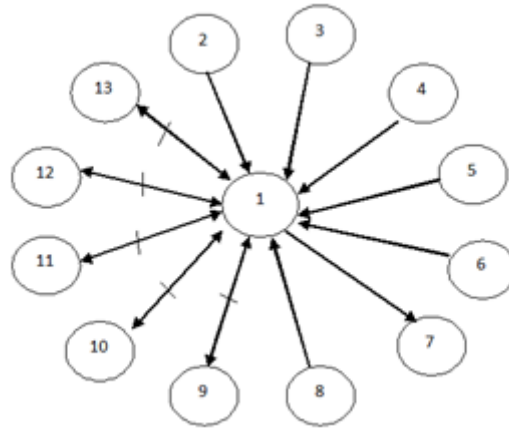
Proofs of (ii) & (iii) are similar to (i).

The following example shows that NI_{cr} - closed set is independent of NI - closed, NI pre closed, NI semi closed and NI_{β} closed sets.

Example 4.6: Let $U = \{a, b, c, d\}$, $U/R = \{\{a\}, \{b\}, \{c, d\}\}$, $X = \{b, d\}$ and $I = \{\emptyset, \{a\}\}$. Then $\tau_R(X) = \{\emptyset, U, \{b\}, \{c, d\}, \{b, c, d\}\}$.

- (i) NI_{cr} - closed set = $\{\emptyset, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, U\}$
- (ii) NI - closed set = $\{\emptyset, \{a\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, U\}$
- (iii) NI - pre closed set = $\{\emptyset, \{a\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, U\}$
- (iv) NI - semi closed set = $\{\emptyset, \{a\}, \{b\}, \{a, b\}, \{c, d\}, \{a, c, d\}, U\}$
- (v) NI_{β} closed set = $\{\emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, U\}$

The above discussions are summarized in the following diagram



1. NI_{ar} - closed 2. nano closed 3. NI_{α} - closed 4. NI_r - Closed 5. NI_{gr} - closed
 6. $NI_{g\alpha}$ - closed 7. NI_{gpr} - closed 8. $NI_{\alpha g}$ - closed 9. NI_g - closed 10. NI_{sg} - closed
 11. NI_{gs} - closed 12. NI_{gp} - closed 13. $NI_{g\beta}$ - closed

Theorem 4.7: Finite union of two NI_{ar} - closed sets is NI_{ar} - closed.

Proof: Let A, B be two NI_{ar} - closed sets. Then $NI_{acl}(A) \subseteq Z_1$ and $NI_{acl}(B) \subseteq Z_2$, whenever $A \subseteq Z_1$ and $B \subseteq Z_2$ and Z_1, Z_2 are nano regular open. $NI_{acl}(A) \cup NI_{acl}(B) \subseteq Z_1 \cup Z_2$. That is, $NI_{acl}(A \cup B) \subseteq Z_1 \cup Z_2 \subseteq Z$ (say). $\therefore A \cup B$ is NI_{ar} - closed.

The following example shows that the intersection of two NI_{ar} - closed sets need not be NI_{ar} - closed.

Example 4.8: Let $U = \{a, b, c, d\}$, $U/R = \{\{a\}, \{c\}, \{b, d\}\}$, $X = \{a, b\}$ and $I = \{\phi, \{a\}\}$. Then $\tau_R(X) = \{\phi, U, \{a\}, \{b, d\}, \{a, b, d\}\}$.

NI_{ar} - closed set = $\{\phi, \{c\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, U\}$.

- (i) $\{a, d\} \cup \{c\} = \{a, c, d\} \in NI_{ar}$ - closed set (ii) $\{a, b\} \cap \{a, c\} = \{a\} \notin NI_{ar}$ - closed set.

Theorem 4.9: Let A be NI_{ar} - closed in a nano ideal topological space $(U, \tau_R(X), I)$. Then for all $x \in NI_{acl}(A)$, $Nrcl(\{x\}) \cap A \neq \phi$.

Proof: Let A be NI_{ar} - closed. Suppose $x \in NI_{acl}(A)$, $Nrcl(\{x\}) \cap A = \phi$. Then $A \subseteq U - Nrcl(\{x\})$. This implies $NI_{acl}(A) \subseteq U - Nrcl(\{x\})$, which is a contradiction, since $x \in NI_{acl}(A)$. $\therefore Nrcl(\{x\}) \cap A \neq \phi$.

Converse of the above theorem does not hold, which is shown in the following example.

Example 4.10: In example 4.8, let $A = \{b, d\}$, $NI_{acl}(A) = \{b, c, d\}$. Take $\{b\} \in NI_{acl}(A)$, $Nrcl(\{b\}) \cap A = \{b, c, d\} \cap \{b, d\} = \{b, d\} \neq \phi$. But $\{b, d\} \notin NI_{ar}$ - closed set.

Theorem 4.11: Let A be NI_{ar} - closed in a nano ideal topological space $(U, \tau_R(X), I)$. Then $NI_{acl}(A) - A$ contains no non empty nano regular closed set.

Proof: Let G be a nano regular closed set such that $G \subseteq NI_{acl}(A) - A$. Then $G \subseteq U - A$, which implies $A \subseteq U - G$. Then $NI_{acl}(A) \subseteq U - G$. $G \subseteq U - NI_{acl}(A)$. Also $G \subseteq NI_{acl}(A)$. $\therefore G \subseteq (U - NI_{acl}(A)) \cap (NI_{acl}(A)) = \phi$. Therefore $NI_{acl}(A) - A$ contains no non empty nano regular closed set.

Converse of the above theorem does not hold as seen in the following example.

Example 4.12: In example 4.8, let $A = \{d\}$, then $NI\alpha cl(A) - A = \{b, c, d\} - \{d\} = \{b, c\}$ which does not contain any non empty nano regular closed set. Also A is not NI_{cr} - closed.

Theorem 4.13: Let A be NI_{cr} - closed in a nano ideal topological space $(U, \tau_R(X), I)$. Then A is $NI\alpha$ - closed iff $NI\alpha cl(A) - A$ is nano regular closed.

Proof: Let A be $NI\alpha$ - closed and hence $NI\alpha cl(A) = A$, that implies $NI\alpha cl(A) - A = \phi$, which is nano regular closed. Conversely, suppose $NI\alpha cl(A) - A$ is nano regular closed and let it be ϕ . Hence A is $NI\alpha$ - closed, since $NI\alpha cl(A) = A$.

Theorem 4.14: If A is NI_{cr} - closed $A \subset B \subset NI\alpha cl(A)$, then B is NI_{cr} - closed.

Proof: Let $B \subseteq Z$ and Z is nano regular open. Since $A \subset B, A \subset Z$, also $NI\alpha cl(A) \subseteq Z$. Since $A \subset B, NI\alpha cl(B) \subset NI\alpha cl(A) \subseteq Z$. This shows that B is NI_{cr} - closed.

5. NANO IDEAL α REGULA OPEN SET

Definition 5.1: A set A in a nano ideal topological space $(U, \tau_R(X), I)$ is called nano ideal α regular open (NI_{cr} - open) if and only if its complement is nano ideal α regular closed.

Remark 5.2: $NI\alpha cl(U - A) = U - NI\alpha int(A)$

Remark 5.3: The following example shows that

- (i) Finite Intersection of two NI_{cr} -open sets is NI_{cr} -open.
- (ii) Union of two NI_{cr} -open sets needs not be NI_{cr} -open.

Example 5.4: In example : 4.8, NI_{cr} -open sets are $\{\phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a,b\}, \{a,d\}, \{b,c\}, \{b,d\}, \{c,d\}, \{a,b,d\}, U\}$

- (i) $\{a,b\} \cap \{b,c\} = \{b\} \in NI_{cr}$ -open set.
- (ii) $\{a,b\} \cup \{c\} = \{a,b,c\} \notin NI_{cr}$ -open set.

Theorem 5.5: In a nano ideal topological space $(U, \tau_R(X), I)$, $A \subseteq U$ is NI_{cr} -open if and only if $F \subseteq NI\alpha int(A)$, whenever F is nano regular closed and $F \subseteq A$.

Proof: Let A be NI_{cr} -open and F is nano regular closed, $F \subseteq A$. Then $U - A \subseteq U - F$, U-F is nano regular open. $NI\alpha cl(U - A) \subseteq U - F \Rightarrow U - NI\alpha int(A) \subseteq U - F \Rightarrow F \subseteq NI\alpha int(A)$.

Conversely, suppose F is nano regular closed and $F \subseteq A$ implies $F \subseteq NI\alpha int(A)$. Let $U - A \subseteq Z$, where Z is nano regular open. Then $U - Z \subseteq A$ where U- Z is nano regular closed. By hypothesis, $U - Z \subseteq NI\alpha int(A) \Rightarrow U - NI\alpha int(A) \subseteq Z$. By remark : 5.2, $NI\alpha cl(U - A) \subseteq Z$. Then U-A is NI_{cr} - closed and hence A is NI_{cr} -open.

Theorem 5.6: If $NI\alpha int(A) \subset B \subset A$ and A is NI_{cr} -open, then B is NI_{cr} -open.

Proof: $NI\alpha int(A) \subset B \subset A$ implies $U - A \subset U - B \subset U - NI\alpha int(A) = NI\alpha cl(U - A)$. Since U-A is NI_{cr} -closed, by theorem-4.14, U-B is NI_{cr} -closed and hence B is NI_{cr} -open.

Remark 5.7: If $A \subseteq U$, $NI\alpha \text{int}(NI\alpha cl(A) - A) = \phi$.

Theorem 5.8: If $A \subseteq U$ is NI_{cr} -closed, then $NI\alpha cl(A) - A$ is NI_{cr} -open.

Proof: Let A be NI_{cr} -closed and let G be a nano regular closed set such that $G \subseteq NI\alpha cl(A) - A$. Then by theorem: 4.11, $G = \phi$ and hence by remark: 5.7 $G \subset NI\alpha \text{int}(NI\alpha cl(A) - A)$. This shows that $NI\alpha cl(A) - A$ is NI_{cr} -open.

The converse of the above theorem is not true as shown in the following example.

Example 5.9: From example: 4.12 let $A = \{d\}$, $NI\alpha cl(A) - A = \{b, c\}$ which is NI_{cr} -open. But A is not NI_{cr} -closed.

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