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NANO IDEAL α - REGULAR CLOSED SETS IN NANO IDEAL TOPOLOGICAL SPACES

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ABSTRACT

The purpose of this paper is to define and study a new class of closed sets called $NI_{\alpha r}$ - closed sets in nano ideal topological spaces. Basic properties of $NI_{\alpha r}$ - closed sets are analyzed and we compared it with some existing and few new closed sets in nano ideal topology introduced in this paper.

MSC (2010): *54A05, 54A10.*

Key words: *$NI_{\alpha r}$ - closed set, Closed sets in nano ideal topology , $NI_{\alpha r}$ - open set, Nano topology.*

1. INTRODUCTION

The concept of ideal topological space was introduced by Kuratowski [4] . In 1990, Jankovic and Hamlett investigated further properties of ideal topological spaces [2]. An ideal I on a nonempty collection of subsets of X which satisfies (i) $A \in I$ and $B \subset A$, implies $B \in I$ and (ii) $A \in I$ and $B \in I$, implies $A \cup B \in I$. Given a topological space (X, τ) with an ideal I on X and if P(X) is the set of all subsets of X , a set operator $(.)^* : P(X) \to P(X)$ called a local function of A with respect to τ and I is defined as follows: for $A \subset X, A^*(I, \tau) = \{x \in X : U \cap A \notin I, \text{ for every} U \in \tau(X)\}$ where $\tau(X) = \{U \in \tau : X \in U\}$. A Kuratowski closure operator cl*(.) for a topology $\tau^*(I, \tau)$ called the *-topology finer than τ, is defined by $cl^*(A) = A \cup A^*(I, \tau)$. When there is no chance of confusion, we will simply write A^* for $A^*(I, \tau)$ and τ^* for $\tau^*(I, \tau)$. If I is an ideal on X, the space (X, τ, I) is called the ideal topological space.

The concept of nano topology was introduced by Lellis Thivagar.M [5], which was defined in terms of approximations and boundary region of a subset of a universe using an equivalence relation on it. He has also defined a Nano continuous functions, Nano open mappings, Nano closed mappings and Nano Homeomorphisms and their representations in terms of Nano closure and Nano interior. In this paper, we introduce and investigate a new class of closed sets called $NI_{\alpha r}$ - closed sets and also discuss the relationship with some new and existing closed sets in nano ideal topological spaces.

2. PRELIMINARIES

Definition 2.1: [5,7] A subset A of a nano ideal topological space $(U, \tau_R(X,), I)$ is called,

(i) NI open if $A \subseteq N\operatorname{int}(A^{*^N})$ and its complement is NI closed.
(ii) NI pre open if $A \subseteq N\operatorname{int}(Ncl^*(A))$ and NI pre closed if $Ncl^*(N\operatorname{int}(A)) \subseteq A$.
(iii) NI semi open if $A \subseteq Ncl^*(N\operatorname{int}(A))$ and NI semi closed if $N\operatorname{int}(Ncl^*(A)) \subseteq A$.
(iv) $NI\alpha$ open if $A \subseteq N\operatorname{int}(Ncl^*(N\operatorname{int}(A)))$ and $NI\alpha$ closed if $Ncl^*(N\operatorname{int}(Ncl^*(A))) \subseteq A$.
(v) $NI\beta$ open if $A \subseteq Ncl^*(N\operatorname{int}(Ncl^*(A)))$ and $NI\beta$ closed if $N\operatorname{int}(Ncl^*(N\operatorname{int}(A))) \subseteq A$.
(vi) NI regular open if $A = N\operatorname{int}(Ncl^*(A))$ and NI regular closed if $Ncl^*(N\operatorname{int}(A)) = A$.

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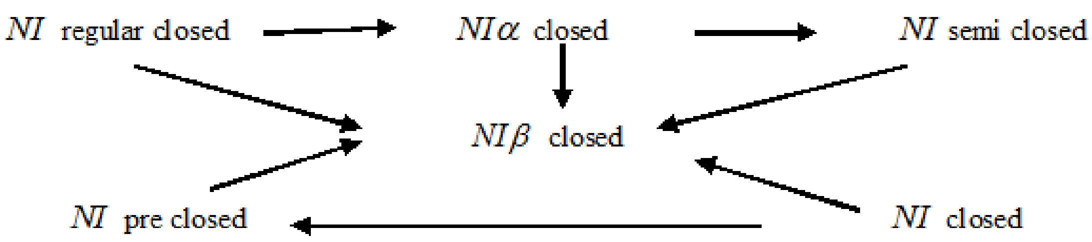
We have the following implications



Nano closed set is independent of NI closed set.

3. SOME CLOSED SETS IN NANO IDEAL TOPOLOGICAL SPACES

Definition 3.1: A subset A of a nano ideal topological space $(U, \tau_R(X,), I)$ is called,

(i) a nano ideal regular generalized closed set (NI_{rg} $-$closed) if $NIcl(A) \subseteq Z$ whenever $A \subseteq Z$ and Z is nano regular open.

(ii) a nano ideal generalized pre closed set (NI_{gp} $-$closed) if $NIpcl(A) \subseteq Z$ whenever $A \subseteq Z$ and Z is nano open.

(iii) a nano ideal $\alpha -$ generalized closed set ($NI_{\alpha g}$ $-$closed) if $NI\alpha cl(A) \subseteq Z$ whenever $A \subseteq Z$ and Z is nano open.

(iv) a nano ideal generalized $\alpha -$closed set ($NI_{g\alpha}$ $-$closed) if $NI\alpha cl(A) \subseteq Z$ whenever $A \subseteq Z$ and Z is nano $\alpha -$open.

(v) a nano ideal generalized semi closed set (NI_{gs} $-$closed) if $NIscl(A) \subseteq Z$ whenever $A \subseteq Z$ and Z is nano open.

(vi) a nano ideal semi generalized closed set (NI_{sg} $-$closed) if $NIscl(A) \subseteq Z$ whenever $A \subseteq Z$ and Z is nano semi open.

(vii) a nano ideal generalized closed set (NI_g $-$closed) if $NIcl(A) \subseteq Z$ whenever $A \subseteq Z$ and Z is nano open.

(viii) a nano ideal generalized pre regular closed set (NI_{gpr} $-$closed) if $NIpcl(A) \subseteq Z$ whenever $A \subseteq Z$ and Z is nano regular open.

(ix) a nano ideal generalized $\beta -$ closed set ($NI_{g\beta}$ $-$closed) if $NI\beta cl(A) \subseteq Z$ whenever $A \subseteq Z$ and Z is nano open.

(x) a nano ideal generalized regular closed set (NI_{gr} $-$closed) if $NIrcl(A) \subseteq Z$ whenever $A \subseteq Z$ and Z is nano open.

4. NANO IDEAL α REGULAR CLOSED SET

Definition 4.1: A subset A of a nano ideal topological space $(U, \tau_R(X,), I)$ is called nano ideal α regular closed set (briefly $NI_{\alpha r}$ - closed) if $NI\alpha cl(A) \subseteq Z$ whenever $A \subseteq Z$ and Z is nano regular open.

Theorem 4.2: In a nano ideal topological space $(U, \tau_R(X), I)$,

(i) every $NI_{\alpha g}$ - closed , $NI_{g\alpha}$ - closed and NI_{gr} - closed set is $NI_{\alpha r}$ - closed

(ii) every $NI_{\alpha r}$ - closed set is NI_{gpr} - closed

Converse of the above theorem need not be true as shown in the following example.

Example 4.3: Let $U=\{a,b,c,d\}$, $U/R=\{\{a\},\{b\},\{c,d\}\}$, $X=\{b,d\}$ and $I=\{\phi,\{a\}\}$. Then $\tau_R(X)=\{\phi,U,\{b\},\{c,d\},\{b,c,d\}\}$.

(i) $A=\{b,d\}$ is $NI_{\alpha r}$ - closed, but not $NI_{\alpha g}$ - closed

(ii) $B=\{b,c\}$ is $NI_{\alpha r}$ - closed, but not $NI_{g\alpha}$ - closed.

(iii) $C=\{c\}$ is NI_{gpr} - closed, but not $NI_{\alpha r}$ - closed.

(iv) $D=\{b,c,d\}$ is $NI_{\alpha r}$ - closed, but not NI_{gr} - closed.

The following example shows that $NI_{\alpha r}$ - closed set is independent of NI_{gp} - closed, NI_{gs} - closed, NI_{sg} - closed, NI_g - closed and $NI_{g\beta}$ - closed sets.

Example 4.4: Let $U=\{a,b,c,d\}$, $U/R=\{\{a\},\{b\},\{c,d\}\}$, $X=\{b,d\}$ and $I=\{\phi,\{a\}\}$. Then $\tau_R(X)=\{\phi,U,\{b\},\{c,d\},\{b,c,d\}\}$.

(i) $NI_{\alpha r}$ - closed set= $\{\phi,\{a\},\{a,b\},\{a,c\},\{a,d\},\{b,c\},\{b,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\},\{b,c,d\},U\}$

(ii) NI_{gp} - closed set= $\{\phi,\{a\},\{c\},\{d\},\{a,b\},\{a,c\},\{a,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\},U\}$

(iii) NI_{gs} - closed set= $\{\phi,\{a\},\{b\},\{c\},\{d\},\{a,b\},\{a,c\},\{a,d\},\{c,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\},U\}$

(iv) NI_{sg} - closed set= $\{\phi,\{a\},\{b\},\{c\},\{d\},\{a,b\},\{a,c\},\{a,d\},\{c,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\},U\}$

(v) NI_g - closed set= $\{\phi,\{a\},\{c\},\{d\},\{a,b\},\{a,c\},\{a,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\},U\}$

(vi) $NI_{g\beta}$ -closed set= $\{\phi,\{a\},\{b\},\{c\},\{d\},\{a,b\},\{a,c\},\{a,d\},\{c,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\},U\}$

Theorem 4.5: Let $(U,\tau_R(X),I)$ be a nano ideal topological space and A be a subset of U. Then

(i) every nano closed set is $NI_{\alpha r}$ - closed

(ii) every $NI\ \ \alpha$ - closed set is $NI_{\alpha r}$ - closed

(iii) every NI regular closed set is $NI_{\alpha r}$ - closed.

Proof: (i) Let $A\subseteq Z$, where Z is nano regular open. By hypothesis and since every nano closed set is $NI\ \ \alpha$ - closed, $NI\alpha cl(A)\subseteq Ncl(A)=A\subseteq Z$. Hence A is $NI_{\alpha r}$ - closed.
Proofs of (ii) & (iii) are similar to (i).

The following example shows that $NI_{\alpha r}$ - closed set is independent of NI - closed, NI pre closed, NI semi closed and $NI\ \ \beta$ closed sets.

Example 4.6: Let $U=\{a,b,c,d\}$, $U/R=\{\{a\},\{b\},\{c,d\}\}$, $X=\{b,d\}$ and $I=\{\phi,\{a\}\}$. Then $\tau_R(X)=\{\phi,U,\{b\},\{c,d\},\{b,c,d\}\}$.

(i) $NI_{\alpha r}$ -closed set= $\{\phi,\{a\},\{a,b\},\{a,c\},\{a,d\},\{b,c\},\{b,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\},\{b,c,d\},U\}$

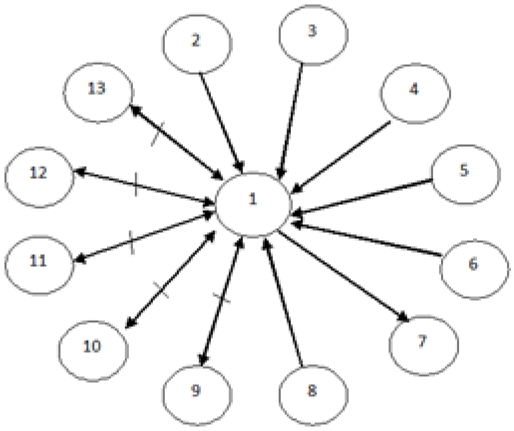
(ii) NI - closed set= $\{\phi,\{a\},\{c\},\{d\},\{a,b\},\{a,c\},\{a,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\},U\}$

(iii) NI - pre closed set= $\{\phi,\{a\},\{c\},\{d\},\{a,b\},\{a,c\},\{a,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\},U\}$

(iv) NI - semi closed set= $\{\phi,\{a\},\{b\},\{a,b\},\{c,d\},\{a,c,d\},U\}$

(v) $NI\beta$ closed set=
$\{\phi,\{a\},\{b\}\{c\},\{d\},\{a,b\},\{a,c\},\{a,d\},\{b,c\},\{b,d\},\{c,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\},U\}$

The above discussions are summarized in the following diagram



1. $NI_{\alpha r}$ - closed 2. nano closed 3. NI α - closed 4. NI r – Closed 5. NI_{gr} - closed 6. $NI_{g\alpha}$ - closed 7. NI_{gpr} - closed 8. $NI_{\alpha g}$ - closed 9. NI_{g} - closed 10. NI_{sg} - closed 11. NI_{gs} - closed 12. NI_{gp} - closed 13. $NI_{g\beta}$ - closed

Theorem 4.7: Finite union of two $NI_{\alpha r}$ - closed sets is $NI_{\alpha r}$ - closed.

Proof: Let A, B be two $NI_{\alpha r}$ - closed sets. Then $NI\alpha cl(A) \subseteq Z_1$ and $NI\alpha cl(B) \subseteq Z_2$, whenever $A \subseteq Z_1$ and $B \subseteq Z_2$ and Z_1, Z_2 are nano regular open. $NI\alpha cl(A) \cup NI\alpha cl(B) \subseteq Z_1 \cup Z_2$.That is, $NI\alpha cl(A \cup B) \subseteq Z_1 \cup Z_2 \subseteq Z$ (say). $\therefore A \cup B$ is $NI_{\alpha r}$ - closed.

The following example shows that the intersection of two $NI_{\alpha r}$ - closed sets need not be $NI_{\alpha r}$ - closed.

Example 4.8: Let $U = \{a,b,c,d\}$, $U/R = \{\{a\},\{c\},\{b,d\}\}$, $X = \{a,b\}$ and $I = \{\phi,\{a\}\}$. Then $\tau_R(X) = \{\phi, U, \{a\}, \{b,d\}, \{a,b,d\}\}$.

$NI_{\alpha r}$ - closed set = $\{\phi,\{c\},\{a,b\},\{a,c\},\{a,d\},\{b,c\},\{c,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\},\{b,c,d\},U\}$.

(i) $\{a,d\} \cup \{c\} = \{a,c,d\} \in NI_{\alpha r}$ - closed set (ii) $\{a,b\} \cap \{a,c\} = \{a\} \notin NI_{\alpha r}$ -closed set.

Theorem 4.9: Let A be $NI_{\alpha r}$ - closed in a nano ideal topological space $(U, \tau_R(X), I)$. Then for all $x \in NI\alpha cl(A)$, $Nrcl(\{x\}) \cap A \neq \phi$.

Proof: Let A be $NI_{\alpha r}$ - closed. Suppose $x \in NI\alpha cl(A)$, $Nrcl(\{x\}) \cap A = \phi$. Then $A \subseteq U - Nrcl(\{x\})$. This implies $NI\alpha cl(A) \subseteq U - Nrcl(\{x\})$, which is a contradiction, since $x \in NI\alpha cl(A)$. $\therefore Nrcl(\{x\}) \cap A \neq \phi$.

Converse of the above theorem does not hold, which is shown in the following example.

Example 4.10: In example 4.8, let $A = \{b,d\}$, $NI\alpha cl(A) = \{b,c,d\}$. Take $\{b\} \in NI\alpha cl(A)$, $Nrcl(\{b\}) \cap A = \{b,c,d\} \cap \{b,d\} = \{b,d\} \neq \phi$. But $\{b,d\} \notin NI_{\alpha r}$ - closed set.

Theorem 4.11: Let A be $NI_{\alpha r}$ - closed in a nano ideal topological space $(U, \tau_R(X), I)$. Then $NI\alpha cl(A) - A$ contains no non empty nano regular closed set.

Proof: Let G be a nano regular closed set such that $G \subseteq NI\alpha cl(A) - A$. Then $G \subseteq U - A$, which implies $A \subseteq U - G$. Then $NI\alpha cl(A) \subseteq U - G$. $G \subseteq U - NI\alpha cl(A)$. Also $G \subseteq NI\alpha cl(A)$. $\therefore G \subseteq (U - NI\alpha cl(A)) \cap (NI\alpha cl(A)) = \phi$. Therefore $NI\alpha cl(A) - A$ contains no non empty nano regular closed set.

Converse of the above theorem does not hold as seen in the following example.

Example 4.12: In example 4.8, let $A = \{d\}$, then $NI\alpha cl(A) - A = \{b, c, d\} - \{d\} = \{b, c\}$ which does not contain any non empty nano regular closed set. Also A is not $NI_{\alpha r}$ - closed.

Theorem 4.13: Let A be $NI_{\alpha r}$ - closed in a nano ideal topological space $(U, \tau_R(X), I)$. Then A is $NI\alpha$ - closed iff $NI\alpha cl(A) - A$ is nano regular closed.

Proof: Let A be $NI\alpha$ - closed and hence $NI\alpha cl(A) = A$, that implies $NI\alpha cl(A) - A = \phi$, which is nano regular closed. Conversely, suppose $NI\alpha cl(A) - A$ is nano regular closed and let it be ϕ. Hence A is $NI\alpha$ - closed, since $NI\alpha cl(A) = A$.

Theorem 4.14: If A is $NI_{\alpha r}$ - closed $A \subset B \subset NI\alpha cl(A)$, then B is $NI_{\alpha r}$ - closed.

Proof: Let $B \subseteq Z$ and Z is nano regular open. Since $A \subset B, A \subset Z$, also $NI\alpha cl(A) \subseteq Z$. Since $A \subset B, NI\alpha cl(B) \subset NI\alpha cl(A) \subseteq Z$. This shows that B is $NI_{\alpha r}$ - closed.

5. NANO IDEAL α REGULA OPEN SET

Definition 5.1: A set A in a nano ideal topological space $(U, \tau_R(X), I)$ is called nano ideal α regular open ($NI_{\alpha r}$ - open) if and only if its complement is nano ideal α regular closed.

Remark 5.2: $NI\alpha cl(U - A) = U - NI\alpha \operatorname{int}(A)$

Remark 5.3: The following example shows that

(i) Finite Intersection of two $NI_{\alpha r}$ -open sets is $NI_{\alpha r}$ -open.

(ii) Union of two $NI_{\alpha r}$ -open sets needs not be $NI_{\alpha r}$ -open.

Example 5.4: In example : 4.8, $NI_{\alpha r}$ -open sets are { ϕ, {a}, {b}, {c}, {d}, {a,b}, {a,d}, {b,c}, {b,d},{c,d}, {a,b,d},U}

(i) $\{a, b\} \cap \{b, c\} = \{b\} \in NI_{\alpha r}$ -open set.

(ii) $\{a, b\} \cup \{c\} = \{a, b, c\} \notin NI_{\alpha r}$ -open set.

Theorem 5.5: In a nano ideal topological space $(U, \tau_R(X), I)$, $A \subseteq U$ is $NI_{\alpha r}$ -open if and only if $F \subseteq NI\alpha \operatorname{int}(A)$, whenever F is nano regular closed and $F \subseteq A$.

Proof: Let A be $NI_{\alpha r}$ -open and F is nano regular closed , $F \subseteq A$. Then $U - A \subseteq U - F$, U-F is nano regular open. $NI\alpha cl(U - A) \subseteq U - F \Rightarrow U - NI\alpha \operatorname{int}(A) \subseteq U - F \Rightarrow F \subseteq NI\alpha \operatorname{int}(A)$.

Conversely, suppose F is nano regular closed and $F \subseteq A$ implies $F \subseteq NI\alpha \operatorname{int}(A)$. Let $U - A \subseteq Z$, where Z is nano regular open. Then $U - Z \subseteq A$ where U- Z is nano regular closed. By hypothesis, $U - Z \subseteq NI\alpha \operatorname{int}(A) \Rightarrow U - NI\alpha \operatorname{int}(A) \subseteq Z$. By remark : 5.2, $NI\alpha cl(U - A) \subseteq Z$. Then U-A is $NI_{\alpha r}$ - closed and hence A is $NI_{\alpha r}$ -open.

Theorem 5.6: If $NI\alpha \operatorname{int}(A) \subset B \subset A$ and A is $NI_{\alpha r}$ -open, then B is $NI_{\alpha r}$ -open.

Proof: $NI\alpha \operatorname{int}(A) \subset B \subset A$ implies $U - A \subset U - B \subset U - NI\alpha \operatorname{int}(A) = NI\alpha cl(U - A)$. Since U-A is $NI_{\alpha r}$ -closed, by theorem-4.14, U-B is $NI_{\alpha r}$ -closed and hence B is $NI_{\alpha r}$ -open.

Remark 5.7: If $A \subseteq U, NI\alpha\operatorname{int}(NI\alpha cl(A) - A) = \phi$.

Theorem 5.8: If $A \subseteq U$ is $NI_{\alpha r}$ -closed, then $NI\alpha cl(A) - A$ is $NI_{\alpha r}$ -open.

Proof: Let A be $NI_{\alpha r}$ -closed and let G be a nano regular closed set such that $G \subseteq NI\alpha cl(A) - A$. Then by theorem: 4.11, $G = \phi$ and hence by remark: 5.7 $G \subset NI\alpha\operatorname{int}(NI\alpha cl(A) - A)$. This shows that $NI\alpha cl(A) - A$ is $NI_{\alpha r}$ -open.

The converse of the above theorem is not true as shown in the following example.

Example 5.9: From example: 4.12 let $A = \{d\}, NI\alpha cl(A) - A = \{b, c\}$ which is $NI_{\alpha r}$ -open. But A is not $NI_{\alpha r}$ -closed.

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REFERENCES

1. Y.Gnanambal, “On generalized preregular closed sets in topological spaces”, Indian J.Pure appl.Math., 28(3): 351-360 (1997).
2. Jankovic D. and Hamlett T. R. “New topologies from old via ideals”, Amer. Math. Monthly,(1990) 97(4), 295-310.
3. S.Krishnaprakash, R.Ramesh, R.Suresh, “Nano compactness and Nano connectedness in nano topological space” Indian J.Pure appl.Math. 119(13): 107-115 (2018).
4. K.Kuratowski, Topology, Vol I, Academic Press (New York, 1966).
5. M.Lellis Thivagar and V.Sutha Devi, “New sort of operators in nano ideal topology”, Ultra Scientist Vol.28 (1) A, 51-64 (2016).
6. A.Pandi, G.Thanavalli, K.Banumathi,V.BA.Vijeyrani and B.Theanmalar, “New types of continuous maps and Hausdorff spaces in nano ideal spaces” International Journal of Research in Advent Technology, Vol. 6, 2018.
7. Rishwanth M Parimala and Saeid Jafari “On some new notions in nano ideal topological spaces” IBJM (2018), VOL.1, NO. 3, 85-92.

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