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PART - I ON NON-ASSOCIATIVE Γ-SEMI SUB NEAR-FIELD SPACES OF A Γ-NEAR-FIELD SPACE OVER NEAR-FIELD (NA-Γ-SSNFS-Γ-NFS-NF)

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ABSTRACT

In this manuscript we solely devoted to the study and introduction of non-associative Γ-semi sub near-field space of a Γ-near-field space over near-field. The study of non-associative Γ-semi sub near-field space of a Γ-near-field space over near-field which also called the loop near-field space which is a non-associative structure with respect to the operation '+' which is very unusual in rings. Here in section one we just define non-associative Γ-semi sub near-field space of a Γ-near-field space over near-field and define several of its properties. In section two we construct several new types of non-associative Γ-semi sub near-field space of a Γ-near-field space over near-field and semi Γ-semi sub near-field space of a Γ-near-field space over near-field using mixed direct product.

Keywords: *Γ-semi sub near-field space, Γ-near-field space; Γ-Semi sub near-field space of Γ-near-field space; Semi near-field space of Γ-near-field space, non-associative Γ-semi sub near-field space, Thurumella- non-associative Γ-semi sub near-field space.*

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SECTION-1: PRELIMINARIES ON NON-ASSOCIATIVE Γ-SEMI SUB NEAR-FIELD SPACE OF A Γ-NEAR-FIELD SPACE OVER NEAR-FIELD.

Here in this section, we just define non-associative Γ-semi sub near-field space of a Γ-near-field space over near-field and define several of its properties.

Near-field spaces are one of the generalized structures of Near-rings over rings. The study and research on Near-field spaces are one of the generalized structures of Near-rings over rings is very systematic and continuous.

Conferences devoted solely to near-rings are held once in a year or every two years. There are about half a dozen seminars, work-shops on near-rings apart from the conference proceedings. Above all there is a online searchable database and bibliography on Near-field spaces over near-rings.

As a result the author feels it is very essential to have a in depth study on Near-field spaces over near-rings where the analogues of the Near-field spaces over near-ring concepts are developed. The reader is expected to have a good background both in algebra and in Near-field spaces over near-rings; for, several results are to be proved by the reader as an exercise.

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We just define the notion of non-associative Γ-semi sub near-field space of a Γ-near-field space over near-field and introduce several properties enjoyed by these structures to Γ-semi sub near-field space. This study is interesting and innovative as this gives a new class of non-associative Γ-semi sub near-field space of a Γ-near-field space over near-field. To the best of our knowledge no one has introduced non-associative Γ-semi sub near-field space of a Γ-near-field space over near-field.

Definition 1.1: non-associative right Γ-semi sub near-field space. Let (N, '+', '.') be a non-empty set endowed with two binary operations '+' and '.' Satisfying the following axioms.

(i) (N, +) is a Γ-semi sub near-field space,
(ii) (N, .) is a groupoids and
(iii) (a + b) c = a. c + b. c for all a, b, c ∈ N; (N. '+', '.') is called the right Γ-semi sub near-field space which is non-associative.

Definition 1.2: non-associative left Γ-semi sub near-field space. Let (N, '+', '.') be a non-empty set endowed with two binary operations '+' and '.' Satisfying the following axioms.

(i) (N, +) is a Γ-semi sub near-field space,
(ii) (N, .) is a groupoids and
(iii) a. (b + c) = a. b + a. c for all a, b, c ∈ N; (N. '+', '.') is called the left Γ-semi sub near-field space which is non-associative.

Note 1.3: In this text denote by (N, +, .) a non-associative right Γ-semi sub near-field space and by default of notation call N just a non-associative Γ-semi sub near-field space of a Γ-near-field space over near-field.

Note 1.4: The theory of right or left non-associative Γ-semi sub near-field space of a Γ-near-field space over near-field runs completely parallel in both cases.

Note 1.5: (N,.) will be only groupoids and never semi near-field space for we assume non-associativity of semi sub near-field space of a Γ-near-field space over near-field.

Example 1.6: Let Z_2 = {0, 1} be a Γ-semi sub near-field space and N a Γ-semi near-field space. The Γ-semi near-field space Z_2N is a non-associative Γ-semi sub near-field space of a Γ-near-field space over near-field.

Example 1.7: Let Z_{12} = {0, 1, 2,..., 11} be a Γ-semi sub near-field space of a Γ-near-field space with operation '+' and '.' Therefore (N, .) be a Γ-near-field space given by the following table:

.	0	1	2	3	4	5
0	0	3	0	3	0	3
1	1	4	1	4	1	4
2	2	5	2	5	2	5
3	3	0	3	0	3	0
4	4	1	4	1	4	1
5	5	2	5	2	5	2

$Z_{12}N$ is a non-associativity of semi sub near-field space of a Γ-near-field space over near-field. For = $Z_{12}N$ under multiplication is only a Γ-near-field space.

Definition 1.8: Let (N, +, .) be a Γ-semi sub near-field space of a Γ-near-field space which is not associative. A Γ-semi near-field space P of N is said to be a Γ-semi sub near-field space if (P, +, .) is a Γ-semi sub near-field space of a Γ-near-field space over near-field.

Definition 1.9: Let N be a non-associative Γ-semi sub near-field space of a Γ-near-field space over near-field. Ann Additive Γ-semi sub near-field space A of N is called the N-Γ-semi sub near-field space (right N - Γ-semi sub near-field space) if NA ⊆ A (AN ⊂ A) where NA = {na / n ∈ N, a ∈ A}.

Definition 1.10: A non-empty subset J of N is called left ideal in N if
(i) (I, +) is a normal Γ-semi sub near-field space of a Γ-near-field space over near-field of (N, +).
(ii) n (n_1 + i) + nn_1 ∈ I for each i ∈ I, n, n_1 ∈ N.

Definition 1.11: Let N be a non-associative Γ-semi sub near-field space of a Γ-near-field space over near-field. A non-empty subset I of N is called an ideal in N if (i) I is left ideal (ii) IN ⊂ I.

Definition 1.12: A non-associative Γ-semi sub near-field space of a Γ-near-field space over near-field N is called left bi-potent if $Na = Na^2$ for a ∈ N.

Definition 1.13: A non-associative Γ-semi sub near-field space of a Γ-near-field space over near-field N is said to be a N-Γ-semi sub near-field space of a Γ-near-field space over near-field if a ∈ Na for each a ∈ N.

The following definitions about strictly prime ideals would be of interest when we further develop non-associative Γ-semi sub near-field space of a Γ-near-field space over near-field.

Definition 1.14: An ideal P (≠ N) is called strictly prime if any two N-Γ-semi sub near-field spaces of a Γ-near-field space over near-field such that AB ⊂ P then A ⊂ P or B ⊂ P.

Definition 1.15: An ideal B of a non-associative Γ-semi sub near-field space of a Γ-near-field space over near-field N **(NA-Γ-SSNFS-Γ-NFS-NF)** called strictly essential if B ∩ K ≠ {0} for every non-zero N-Γ-semi sub near-field spaces of a Γ-near-field space over near-field K of N.

Definition 1.16: An element y in non-associative Γ-semi sub near-field space of a Γ-near-field space over near-field N is said to be singular if there exists a non-zero strictly essential left ideal A in N such that Ay = {0}.

SECTION 2: INTRODUCTION AND SOME RESULTS ON NON-ASSOCIATIVE Γ-SEMI SUB NEAR-FIELD SPACE OF A Γ-NEAR-FIELD SPACE OVER NEAR-FIELD.

Several other analogous results existing in near-rings, semi near-rings, near-fields can also be defined for Thurumella non-associative Γ-semi sub near-field spaces (T means Thurumella). The definition of T-NA-Γ-semi sub near-field space of a Γ-near-field space over near-field I we assumed (N, +) is a T-Γ-semi sub near-field space of a Γ-near-field space over near-field so we have (P, +) for a suitable proper Γ-semi sub near-field P to be a Γ-near-field space and since (N, .) is a T-groupoid we have (Q, +) is a Γ-semi sub near-field space, if P = Q we see N has P to be a Γ-near-field space. This has forced us to define T-NA-Γ-semi sub near-field space I of type B.

It is pertinent to mention here that by no means have we assumed or state that the definitions give some what conditionally equivalent conditions or the Thurumella structures gives way for such concepts i.e. a non-associative algebraic structure containing completely an associative structure or more richer algebraic structure.

Definition 2.1: Let (N, + ,.) be a non-associative Γ-semi sub near-field space of a Γ-near-field space over near-field. N is said to be a non-associative Γ-semi sub near-field space of level 1 (T-NA-Γ-SSNFS-Γ-NFS-NF 1) if
(i) (N, +) is a T-Γ-semi sub near-field space,
(ii) (N,.) is a T-groupoid and
(iii) (a + b) .c = a.c + b.c for all a, b, c ∈ N.

Definition 2.2: Let (N,+,.) be a non-associative Γ-semi sub near-field space of a Γ-near-field space over near-field. N is said to have a T-Γ-semi sub near-field space P ⊂ N if P is itself a T-Γ-semi sub near-field space of a Γ-near-field space over near-field.

Definition 2.3: Let (N,+,.) be a non-associative Γ-semi sub near-field space of a Γ-near-field space over near-field. An additive T-Γ-semi sub near-field space A of N is called the Thurumella N-Γ-semi sub near-field space of a Γ-near-field space over near-field (T-N-Γ-SSNFS-Γ-NFS-NF)(right N-Γ-SSNFS-Γ-NFS-NF) if NA ⊂ N (AN ⊂ A) where NA = { Na / n ∈ N, a ∈ A }.

Definition 2.4: A non-empty subset I of N is called Thurumella left ideal (T-left ideal) in N if
1. (I,+) is a normal T-Γ-semi sub near-field space of a Γ-near-field space over near-field Of (N,+)
2. $n(n_1 + i) + nn_1 \in I$ for each i ∈ I, n, n_1 ∈ N.

Definition 2.5: Let N be a NA-Γ- semi sub near-field space of a Γ-near-field space over near-field. A non empty subset I of N is called a Thurumella ideal (T-ideal) in N if (i) I is a T-left ideal (ii) IN ⊂ I.

Definition 2.6: A T- NA-Γ- semi sub near-field space of a Γ-near-field space over near-field N is called Thurumella left bipotent (T-left bipotent) if $Na = Na^2$ for every a ∈ N.

Theorem 2.7: Every T-NA-Γ- semi sub near-field space of a Γ-near-field space over near-field N has a non trivial Γ-semi sub near-field space P if and only if (P, +) is a Γ- semi sub near-field space of a Γ-near-field space over near-field.

Proof: If N has P to be a Γ- semi sub near-field space of a Γ-near-field space over near-field N where $P \subset N$ then (P, +) and (P, .) are - semi sub near-field space of a Γ-near-field spaces. (N, .) is a T-groupoid which is always possible. Converse is straightforward. This completes the proof of the theorem.

Theorem 2.8: Every T-NA-Γ-semi sub near-field space of a Γ-near-field space over near-field N has a subset P which is a Γ- semi sub near-field space if and only if (P, +) is a Γ-near-field space and (P, .) is a Γ-semi sub near-field space.

Proof: This is possible when N is a T-NA-Γ-semi sub near-field space of a Γ-near-field space over near-field as (N, +) is a T-Γ-semi sub near-field space, so has a Γ-near-field space w.r.t. + and (N, .) is T-groupoid so has a Γ-semi sub near-field space which is a T-Γ-semi sub near-field space. If the same subset N happens to be near-field space under '+' and Γ-semi sub near-field space '.' Then we have theorem to be true. This completes the proof of the theorem.

Definition 2.9: Let (N, +, .) be a NA-Γ-semi sub near-field space of a Γ-near-field space over near-field. N is said to be a Thurumella Γ-semi sub near-field space I of type A (T-Γ-semi sub near-field space I of type A) if N has a proper Γ-semi sub near-field space such that (P , +) is an associative Γ-semi sub near-field space.

Note 2.10: By no means that two definitions T-NA-Γ-semi sub near-field space of a Γ-near-field space over near-field I and T-NA-Γ-semi sub near-field space of a Γ-near-field space over near-field I of type A are equivalent.

Definition 2.11: Let N be a NA-Γ-semi sub near-field space of a Γ-near-field space over near-field. N is said to be a Thurumella NA-Γ-semi sub near-field space I of type B (T-NA-Γ-semi sub near-field space I of type B) if N has a proper Γ-semi sub near-field space P where P is a Γ-near-field space over near-field.

Example 2.12: Let (N, +, .) be a Γ-semi sub near-field space of a Γ-near-field space over near-field and M be a groupoid. The groupoid Γ-semi sub near-field space of a Γ-near-field space NM is a non-associative Γ-semi sub near-field space of a Γ-near-field space over near-field. NM is a T-NA-Γ-semi sub near-field space of a Γ-near-field space over near-field if and only if N is a T-Γ-semi sub near-field space of a Γ-near-field space over near-field.

Definition 2.13: Let M_1 and M_2 be two T-Γ-semi sub near-field spaces of a Γ-near-field space over near-field. A map $\phi: M_1 \rightarrow M_2$ is a Thurumella non-associative -Γ-semi sub near-field space homomorphism(T-NA-Γ-SSNFS) if $\phi(x + y) = \phi(x) + \phi(y)$; $\phi(x . y) = \phi(x) . \phi(y)\ \forall\ x, y \in N$.

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