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NOTES ONINTUITIONISTICMULTIFUZZY GRAPH

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ABSTRACT

In this paper, some properties of intuitionisticmultifuzzy graph are studied and proved. Fuzzy graph is the generalization of the crisp graph, intuitionisticfuzzy graph is the generalization of fuzzy graph and intuitionisticmulti fuzzy graph is the generalization of intuitionisticfuzzy graph. A new structure of an intuitionisticmulti fuzzy graph is introduced.

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Key Words: *Intuitionisticfuzzy subset, intuitionisticmulti fuzzy relation, Strong intuitionisticmulti fuzzy relation, intuitionisticmulti fuzzy graph, intuitionisticmulti fuzzy loop, intuitionisticmulti fuzzy pseudo graph, intuitionisticmulti fuzzy spanning subgraph, intuitionisticmulti fuzzy induced subgraph, intuitionisticmulti fuzzy underling graph, Level set, Degree of intuitionisticmultifuzzy vertex, order of the intuitionisticmultifuzzy graph, size of the intuitionisticmultifuzzy graph, intuitionisticmultifuzzy regular graph, intuitionisticmulti fuzzy strong graph, intuitionisticmulti fuzzy complete graph.*

INTRODUCTION

In 1965, Zadeh [13] introduced the notion of fuzzy set as a method of presenting uncertainty. Since complete information in science and technology is not always available. Thus we need mathematical models to handle various types of systems containing elements of uncertainty. After that Rosenfeld [10] introduced fuzzy graphs. Fuzzy graphs are useful to represent relationships which deal with uncertainty and it differs greatly from classical graph. It has numerous applications to problems in computer science, electrical engineering system analysis, operations research, economics, networking routing, transportation, etc. Nagoor Gani.A [7, 8] introduced a fuzzy graph and regular fuzzy graph. Multi fuzzy set was introduced by Sabu Sebastian, T.V.Ramakrishnan[11].After that the fuzzy sets have been generalized with fuzzy loop and fuzzy multiple edges, this type of concepts was introduced by K.Arjunan and C.Subramani[1,2]. The intuitionistic fuzzy graph with multiple edges and selfloops has been introduced by K.Arjunan and C.Subramani[3]. In this paper a new structure is introduced that is intuitionistic multi fuzzy graph with selfloop and multiple edges and some results of intuitionistic multi fuzzy graph are stated and proved.

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1. PRELIMINARIES

Definition 1.1[4]: An intuitionistic fuzzy set (IFS)A in X is defined as an object having the form A ={(x, $\mu_A(x)$, $\gamma_A(x)$) /x $\in$ X},where μ_A: X $\rightarrow$ [0,1] and γ_A: X $\rightarrow$ [0,1]define the degree of membership and the degree of non-membership of the element x $\in$ X respectively and every x in X satisfying $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$.

Example 1.2: An intuitionisticsubset A= {(a, 0.3, 0.6), (b, 0.3, 0.5), (c, 0.2, 0.5)} of a set X= {a, b, c}.

Definition 1.3[12]: An**intuitionisticmulti fuzzy subset** A of a set X is defined as an object of the form A = {(x, $\mu_{A1}(x)$, $\mu_{A2}(x)$, .., $\mu_{An}(x)$, $\gamma_{A1}(x)$, $\gamma_{A2}(x)$,..,$\gamma_{An}(x)$)/ x$\in$X}, where μ_{Ai} : X$\rightarrow$[0, 1] and γ_{Ai} : X$\rightarrow$[0, 1] for all i, define the degrees of membership and the degrees of non-membership of the element x $\in$ X respectively and every x in X satisfying $0 \leq \mu_{Ai}(x) + \gamma_{Ai}(x) \leq 1$ for all i. It is denoted as A = (μ_A, γ_A), where μ_A = (μ_{A1}, μ_{A2}… μ_{An}) and γ_A= (γ_{A1}, γ_{A2},…,γ_{An}).

Example 1.4: Let X = {a, b, c} be a set. Then A = {(a, (0.4, 0.3, 0.2), (0.2, 0.4, 0.4)), (b, (0.2, 0.4, 0.3), (0.3, 0.5, 0.7)), (c, (0.4, 0.1, 0.5), (0.5, 0.6, 0.4)} is a intuitionisticmulti fuzzy subset of X with the dimension three.

Definition 1.5: Let A and B be any two intuitionisticmulti fuzzy subset of X. We define the following relations and operations:

(i) A $\subseteq$ B if and only if $\mu_{Ai}(x) \leq \mu_{Bi}(x)$ and $\gamma_{Ai}(x) \geq \gamma_{Bi}(x)$ for all x $\in$ X and for all i.
(ii) A = B if and only if $\mu_{Ai}(x) = \mu_{Bi}(x)$ and $\gamma_{Ai}(x) = \gamma_{Bi}(x)$ for all x $\in$ X and for all i.
(iii) A$\cap$B if and only if (A$\cap$B) (x) = {min{$\mu_{Ai}(x)$, $\mu_{Bi}(x)$},max{$\gamma_{Ai}(x)$, $\gamma_{Bi}(x)$}}for all x $\in$ X and for all i.
(iv) A$\cup$B if and only if (A$\cup$B) (x) = {max{$\mu_{Ai}(x)$, $\mu_{Bi}(x)$},min{$\gamma_{Ai}(x)$, $\gamma_{Bi}(x)$}}for all x $\in$ X and for all i.

Definition 1.6: Let A be aintuitionisticmulti fuzzy subset in a set S, the **strongest intuitionistic multi fuzzy relation** on S, that is an intuitionisticmulti fuzzy relation with respect to A given by $\mu_{Vi}(x,y)$ = min {$\mu_{Ai}(x)$, $\mu_{Ai}(y)$} and $\gamma_{Vi}(x, y)$ = max{$\gamma_{Ai}(x)$, $\gamma_{Ai}(y)$} for all x and y in S and for all i.

Definition 1.7: Let V be any nonempty set, E be any set and f: E $\rightarrow$ V×V be any function. Then A is aintuitionistic multi fuzzy subset of V, Sis anintuitionisticmulti fuzzy relation on V with respect to A and B is an intuitionisticmulti fuzzy subset of E such that $\mu_{B_i}(e) \leq \underset{e\in f^{-1}(x,y)}{\mu_{S_i}(x,y)}$ and $\gamma_{B_i}(e) \geq \underset{e\in f^{-1}(x,y)}{\gamma_{S_i}(x,y)}$ for all i. Then the ordered triple F= (A, B, f) is called an **intuitionistic multi fuzzy graph**, where the elements of A are called **intuitionisticmultifuzzy points** or **intuitionistic multifuzzy vertices** and the elements of B are called **intuitionistic multifuzzy lines** or **intuitionistic multi fuzzy edges** of the intuitionisticmulti fuzzy graph F. Iff(e) = (x, y), then the intuitionistic multi fuzzy points (x, $\mu_A(x)$, $\gamma_A(x)$), (y, $\mu_A(y)$, $\gamma_A(y)$) are called **intuitionistic multi fuzzyadjacent points** and intuitionisticmulti fuzzy points (x, $\mu_A(x)$, $\gamma_A(x)$), intuitionisticmulti fuzzy line (e, $\mu_B(e)$, $\gamma_B(e)$) are called **incident** with each other. If two district intuitionisticmulti fuzzy lines (e_1, $\mu_B(e_1)$,$\gamma_B(e_1)$) and (e_2, $\mu_B(e_2)$, $\gamma_B(e_2)$) are incident with a common intuitionisticmulti fuzzy point, then they are called **intuitionisticmultifuzzy adjacent lines.**

Definition 1.8: Anintuitionistic multi fuzzy line joining anintuitionisticmulti fuzzy point to itself is called an **intuitionistic multi fuzzy loop**.

Definition 1.9: Let F= (A, B, f) be an intuitionisticmulti fuzzy graph. If more than one intuitionisticmulti fuzzy line joining two intuitionisticmulti fuzzy vertices is allowed, then the intuitionisticmulti fuzzy graph F is called an **intuitionisticmultifuzzy pseudo graph**.

Definition 1.10: F= (A, B, f) is called an**intuitionisticmulti fuzzy simple graph** if it has neitherintuitionisticmulti fuzzy multiple lines nor intuitionistic multifuzzy loops.

Example 1.11: F = (A, B, f), where V ={v_1,v_2,v_3,v_4,v_5}, E={a, b, c, d, e, h, g} and f:E $\rightarrow$ V×V is defined by f(a) = (v_1, v_2), f(b) = (v_2, v_2), f(c) = (v_2, v_3), f(d)= (v_3, v_4), f(e) = (v_3, v_4), f(h) = (v_4, v_5), f(g)= (v_1, v_5). An Intuitionisticfuzzy subsetA = {(v_1, (0.3, 0.2,0.5), (0.4,0.2,0.3)), (v_2, (0.2,0.2,0.4),(0.3,0.1,0.2)), (v_3,(0.3,0.2,0.5),(0.3,0.2,0.2)),(v_4,(0.3,0.3,0.5),(0.4,0.2,0.3)),(v_5,(0.3,0.2,0.5),(0.3,0.2,0.2))}of V. An intuitionistic multi fuzzy relation S={ ((v_1,v_1), (0.3, 0.2, 0.5), (0.4, 0.2, 0.3)),((v_1,v_2),(0.2, 0.2, 0.4), (0.4, 0.2, 0.3)),((v_1,v_3),(0.3, 0.2, 0.5), (0.4, 0.2, 0.3)),((v_1,v_4),(0.3,0.2,0.5),(0.4,0.2,0.3)),((v_1,v_5),(0.3,0.2,0.5),(0.4,0.2,0.3)),((v_2,v_1),(0.2,0.2,0.4),(0.4,0.2,0.3)),((v_2,v_2),(0.2,0.2,0.4),(0.3,0.1,0.2)),((v_2,v_3),(0.2,0.2,0.4),(0.3,0.2,0.2)),((v_2,v_4),(0.2,0.2,0.4),(0.4,0.2,0.3)), ((v_2,v_5),(0.2,0.2,0.4), (0.3,0.2,0.2)), ((v_3,v_1), (0.3,0.2,0.5), (0.4,0.2,0.3)), ((v_3,v_2), (0.2,0.2,0.4), (0.3,0.2,0.2)), ((v_3,v_3),(0.3,0.2,0.5),(0.3,0.2,0.2)),((v_3,v_4),(0.3,0.2,0.5),(0.4,0.2,0.3)),((v_3,v_5),(0.3,0.2,0.5),(0.3,0.2,0.2)), ((v_4,v_1), (0.3,0.2,0.5), (0.4,0.2,0.3)),((v_4,v_2),(0.2,0.2,0.4),(0.4,0.2,0.3)), ((v_4,v_3),(0.3,0.2,0.5),(0.4,0.2,0.3)), ((v_4, v_4), (0.3,0.3,0.5), (0.4,0.2,0.3)), ((v_4, v_5),(0.3,0.2,0.5),(0.4,0.2,0.3)), ((v_5,v_1),(0.3,0.2,0.5),(0.4,0.2,0.3)),((v_5, v_2), (0.2,0.2,0.4), (0.3,0.2,0.2)), ((v_5,v_3),(0.3,0.2,0.5),(0.3,0.2,0.2)), ((v_5, v_4),(0.3,0.2,0.5),(0.4,0.2,0.3)), ((v_5, v_5), (0.3,0.2,0.5), (0.3,0.2,0.2))} on V with respect to A and an intuitionisticmulti fuzzy subset B = {(a,(0.1, 0.2, 0.3), (0.5, 0.3, 0.6)), (b, (0.1, 0.2, 0.3), (0.3, 0.2, 0.2)),(c,(0.1,0.2,0.3),0.4,0.3,0.3)),(d,(0.1,0.2,0.3),(0.4,0.2,0.3)), (e,(0.2,0.1,0.2), (0.4,0.3,0.3)), (h,(0.2,0.2,0.3),(0.4,0.2,0.3)),(g,(0.2,0.3,0.3),(0.4,0.3,0.4))}of E.

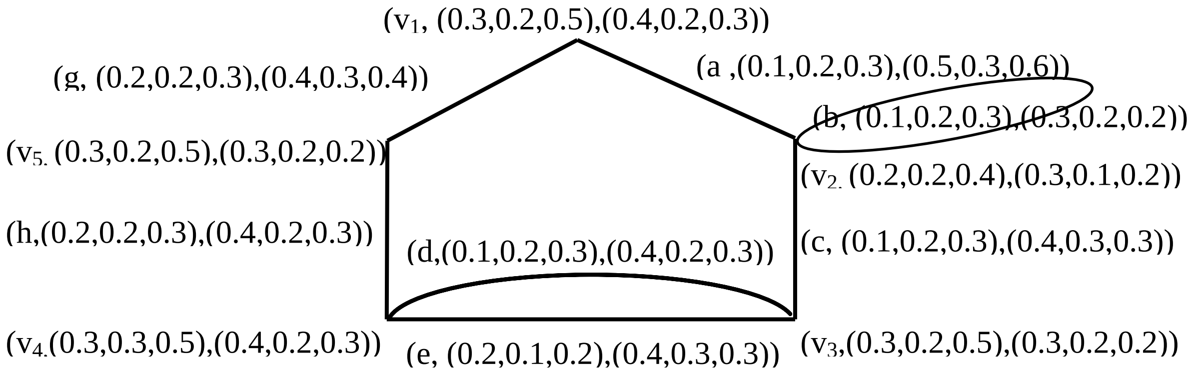

Fig.-1.1

In figure 1.1, (i) (v_1, (0.3,0.2,0.5),(0.4,0.2,0.3)) is an intuitionisticmulti fuzzy point.(ii) (a,(0.1,0.2,0.3),(0.5,0.3,0.6)) is an intuitionisticmulti fuzzy edge.(iii) (v_1, (0.3, 0.2, 0.5), (0.4, 0.2, 0.3)) and (v_2,(0.2,0.2,0.4), (0.3,0.1,0.2)) are intuitionisticmulti fuzzy adjacent points.(iv) (a, (0.1, 0.2, 0.3), (0.5, 0.3, 0.6)) join with (v_1, (0.3, 0.2, 0.5), (0.4, 0.2, 0.3)) and (v_2, (0.2,0.2,0.4),0.3,0.1,0.2) and therefore it is incident with (v_1, (0.3, 0.2, 0.5), (0.4, 0.2, 0.3)) and (v_2, (0.2, 0.2, 0.4), (0.3, 0.1, 0.2)).(v) (a, (0.1, 0.2, 0.3), (0.5, 0.3, 0.6)) and (g,(0.2,0.2,0.3),(0.4,0.3,0.4)) are intuitionisticmulti fuzzy adjacent lines. (vi) (b,(0.1,0.2,0.3),(0.3,0.2,0.2)) is an intuitionisticmulti fuzzy loop.(vii) (d,(0.1,0.2,0.3),(0.4,0.2,0.3)) and (e,(0.2,0.1,0.2),(0.4,0.3,0.3)) are intuitionisticmulti fuzzy multiple edges. (viii) It is not an intuitionisticmultifuzzy simple graph.(ix)It is an intuitionisticmulti fuzzy pseudo graph.

Definition 1.12: The multi fuzzy graph H= (C, D, f) where C = <μ_C ,γ_C> and D = <μ_D,γ_D> is called an **intuitionisticmultifuzzy subgraph** of F = (A, B, f) if C$\subseteq$ A and D$\subseteq$ B.

Definition 1.13: The intuitionisticmultifuzzy subgraph H = (C, D, f) is said to be an **intuitionisticmultifuzzy spanning subgraph** of F= (A, B, f) if C= A.

Definition 1.14: The intuitionisticmulti fuzzy subgraph H = (C, D, f) is said to be an**intuitionisticmulti fuzzy induced sub graph** of F = (A, B, f) if H is the maximal intuitionisticmulti fuzzy subgraph of F with intuitionisticmulti fuzzy point set C.

Definition 1.15: Let F = (A, B, f) be anintuitionisticmulti fuzzy graph with respect to the sets V and E. Let C be an intuitionistic multifuzzy subset of V, the intuitionisticmulti fuzzy subset D of E is defined as $\mu_{Di}(e) = \min\{\mu_{Ci}(u), \mu_{Ci}(v), \mu_{Bi}(e)\}$, $\gamma_{Di}(e) = \max\{\gamma_{Ci}(u),\gamma_{Ci}(v),\gamma_{Bi}(e)\}$for all i, where f(e)= (u, v)for all e in E. Then H=(C, D, f) is called **intuitionisticmulti fuzzy partial subgraph** of F.

Definition 1.16: Let F = (A, B, f) be anintuitionisticmulti fuzzy graph. Let A is a intuitionisticmulti fuzzy sub graph of F obtained by removing the intuitionisticmulti fuzzy point $(x, \mu_A(x), \gamma_A(x))$ and all the intuitionisticmulti fuzzy lines incident with $(x, \mu_A(x), \gamma_A(x))$ is called the intuitionisticmulti fuzzy subgraph obtained by the removal of the intuitionistic multi fuzzy point $(x, \mu_A(x), \gamma_A(x))$ and is denoted $F-(x, \mu_A(x), \gamma_A(x))$. Thus if $F-(x, \mu_A(x), \gamma_A(x)) = (C, D, f)$ then $C = A-\{(x, \mu_A(x),\gamma_A(x))\}$ and $D=\{(e, \mu_B(e), \gamma_B(e))/ (e, \mu_B(e),\gamma_B(e))\in B$ and$(x, \mu_A(x), \gamma_A(x))$ is not incident with $(e, \mu_B(e), \gamma_B(e))\}$. Clearly $F-(x, \mu_A(x), \gamma_A(x))$ is **intuitionisticmulti fuzzy induced subgraph** of F. Let $(e, \mu_B(e), \gamma_B(e))\in B$. Then $F-(e, \mu_B(e), \gamma_B(e)) = (A, D, f)$ is called intuitionisticmulti fuzzy sub graph of F obtained by the removal of the intuitionistic multifuzzy line $(e, \mu_B(e), \gamma_B(e))$, where $D =B-\{(e, \mu_B(e),\gamma_B(e))\}$. Clearly $F-(e, \mu_B(e), \gamma_B(e))$ is an intuitionisticmulti fuzzy spanning sub graph of F which contains all the lines of F except $(e, \mu_B(e),\gamma_B(e))$. Here $\mu_B(e) = (\mu_{B1}(e), \mu_{B2}(e),\ldots, \mu_{Bn}(e))$ and $\gamma_B(e) = (\gamma_{B1}(e), \gamma_{B2}(e),\ldots,\gamma_{Bn}(e))$.

Definition 1.17: By deleting from a intuitionistic multi fuzzy graph F all intuitionistic multifuzzy loops and in each collection of intuitionisticmulti fuzzy multiple edges all intuitionisticmulti fuzzy edge but one intuitionisticmulti fuzzy edge in the collection we obtain an intuitionisticmulti fuzzy simple spanning subgraph F, called **intuitionisticmultifuzzy underling simple graph of F.**

Example 1.18:

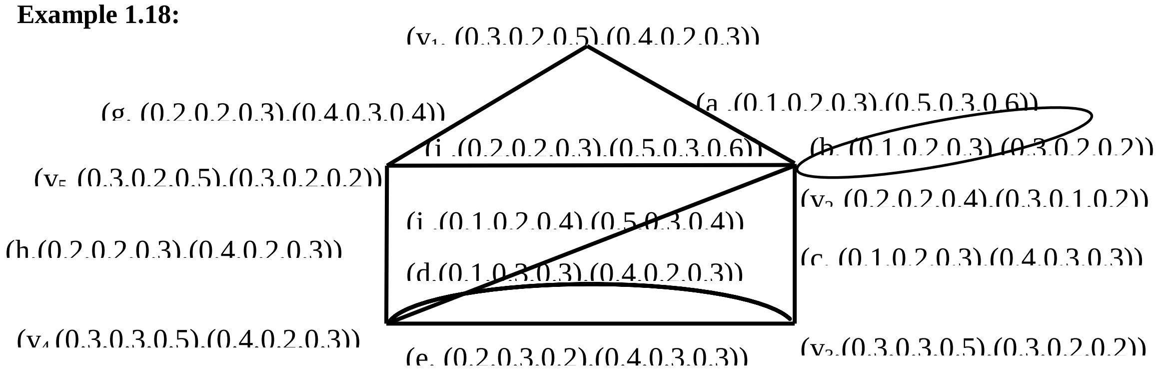

Fig.-1.2: Anintuitionisticmulti fuzzy pseudo graph F = (A, B, f)

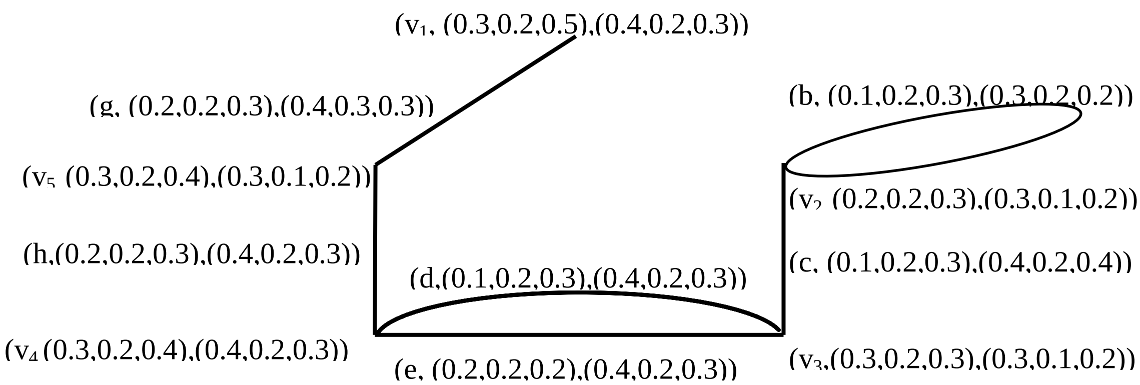

Fig.-1.3: An intuitionisticmulti fuzzy subgraph of F

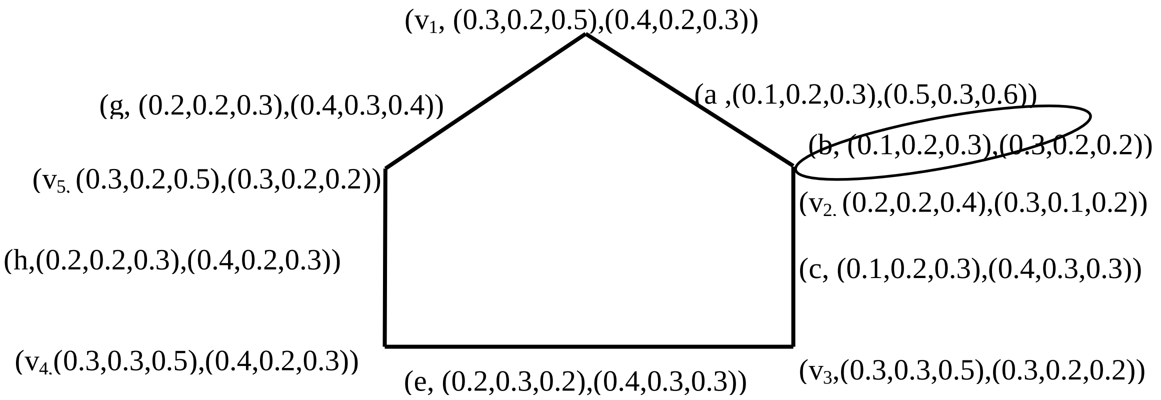

Fig.-1.4: An intuitionisticmulti fuzzy spanning subgraph of F

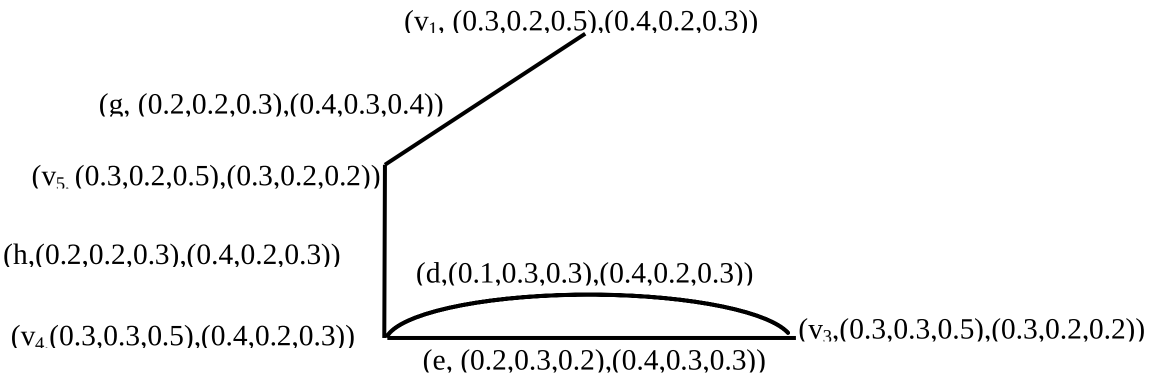

Fig.-1.5: An intuitionisticmultifuzzy subgraph Induced by P = {v_1, v_3, v_4,v_5}

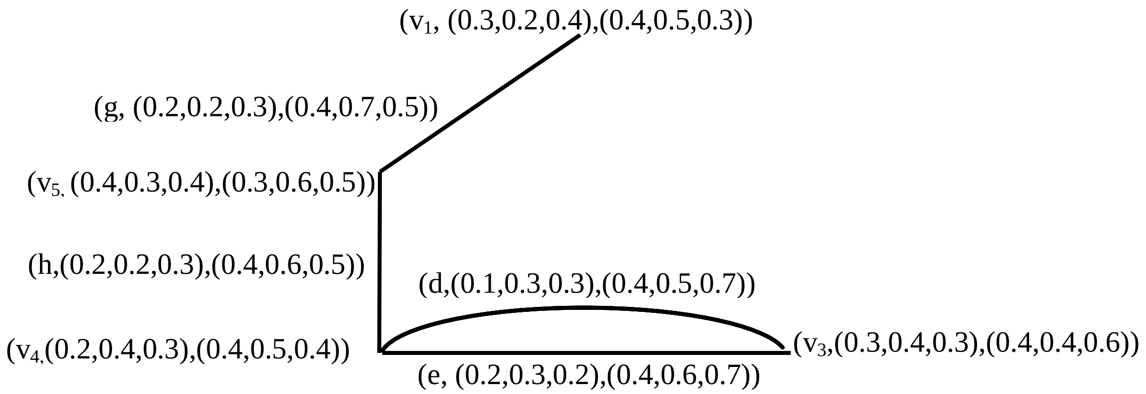

Fig.-1.6: A partialintuitionistic multi fuzzy subgraph induced by C,

where C(v_1) = ((0.3, 0.2, 0.4), (0.4, 0.5, 0.3)), C(v_3) = ((0.3, 0.4, 0.3), (0.4, 0.4, 0.6)), C(v_4) = ((0.2, 0.4, 0.3), (0.4, 0.5, 0.4)), C(v_5) = ((0.4, 0.3, 0.4), (0.3, 0.6, 0.5)).

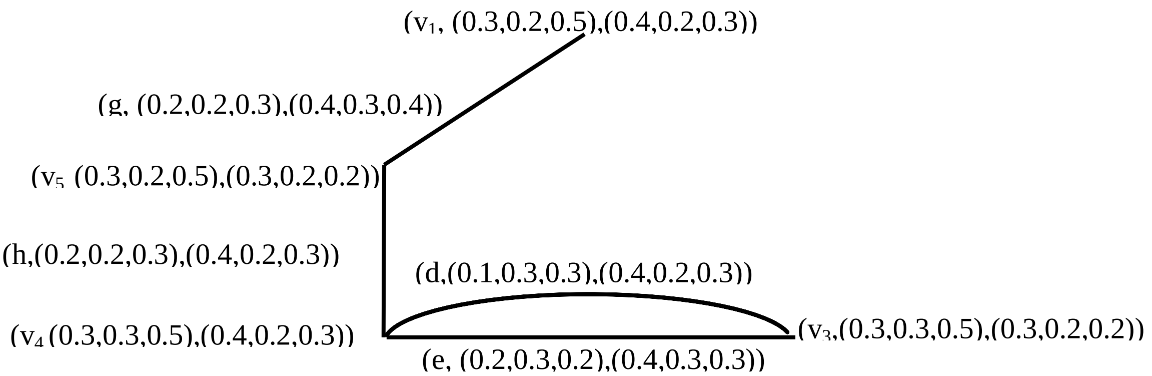

Fig. -7: F − ((v_2 , 0.2,0.2, 0.4), 0.3, 0.1, 0.2))

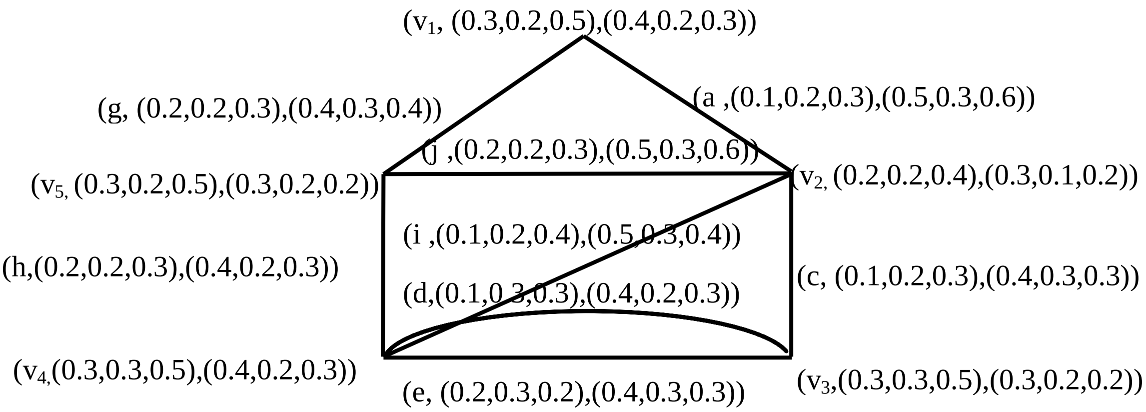

Fig.-1.8: F− (b ,(0.1, 0.2, 0.3), (0.3, 0.2, 0.2))

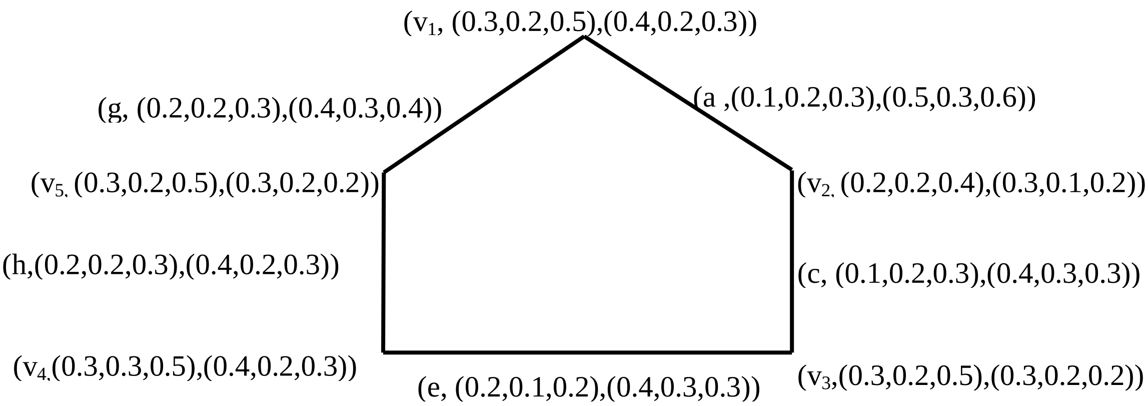

Fig.-1.9: Underling intuitionisticmultifuzzy simple graph of F.

Definition 1.19: Let A be a intuitionisticmulti fuzzy subset of X then the **level subset** or **(α,β)-cut** of A is $A_{(\alpha,\beta)} = \{x \in A / \mu_{Ai}(x) \geq \alpha_i$ and $\gamma_{Ai}(x) \leq \beta_i\}$, where $\alpha_i, \beta_i \in [0,1]$ for all i and $\alpha_i + \beta_i \leq 1$. Here α means $(\alpha_1, \alpha_2, ..., \alpha_n)$.

Note: αmeans $(\alpha_1, \alpha_2, ..., \alpha_n)$ and β means $(\beta_1, \beta_2, ..., \beta_n)$.

Theorem 1.20: Let F = (A,B, f) be aintuitionistic multi fuzzy graph with respect to the set V and E.Letα, β, λ,$\eta \in [0,1]$ and $\alpha \leq \beta$ and$\lambda \geq \eta$.Then $(A_{(\beta,\eta)}, B_{(\beta,\eta)}, f)$ is a subgraphof $(A_{(\alpha,\lambda)}, B_{(\alpha,\lambda)}, f)$.

Proof: The proof follows from definition 1.19.

Theorem 1.21: Let F = (A, B, f) be a intuitionistic multi fuzzy graph with respect to the set V and E, the level subsets $A_{(\alpha,\lambda)}, B_{(\alpha,\lambda)}$ of A and B subset of V and E respectively. Then $F_{(\alpha,\lambda)} = (A_{(\alpha,\lambda)}, B_{(\alpha,\lambda)}, f)$ is a subgraph of G = (V, E, f).

Proof: The proof follows from definition 1.19 and Theorem 1.20.

Theorem 1.22: Let H = (C, D, f) be a intuitionistic multi fuzzy subgraph of F = (A, B, f) and α, λ ∈ [0,1]. Then $H_{(\alpha,\lambda)} = (C_{(\alpha,\lambda)}, D_{(\alpha,\lambda)}, f)$ is a subgraphof $F_{(\alpha,\lambda)} = (A_{(\alpha,\lambda)}, B_{(\alpha,\lambda)}, f)$.

Proof: Let H = (C, D, f) be a intuitionistic multi fuzzy subgraph of F = (A, B, f). Therefore $\mu_{Ci}(v) \leq \mu_{Ai}(v)$ and $\gamma_{Ci}(v) \geq \gamma_{Ai}(v)$ for all u in V and for all i. $\mu_{Di}(e) \leq \mu_{Bi}(e)$ and $\gamma_{Di}(e) \geq \gamma_{Bi}(e)$ for all e in E and for all i. We have to prove that $(C_{(\alpha,\lambda)}, D_{(\alpha,\lambda)}, f)$ is a subgraph of $(A_{(\alpha,\lambda)}, B_{(\alpha,\lambda)}, f)$. It is enough to prove that $C_{(\alpha,\lambda)} \subseteq A_{(\alpha,\lambda)}$ and $D_{(\alpha,\lambda)} \subseteq B_{(\alpha,\lambda)}$. Let $u \in C_{(\alpha,\lambda)} \Longrightarrow \mu_{Ci}(u) \geq \alpha_i$ and $\gamma_{Ci}(u) \leq \lambda_i \Longrightarrow \mu_{Ai}(u) \geq \mu_{Ci}(u) \geq \alpha_i$ and $\gamma_{Ai}(u) \leq \gamma_{Ci}(u) \leq \lambda_I \Longrightarrow \mu_{Ai}(u) \geq \alpha_i$ and $\gamma_{Ai}(u) \leq \lambda_I$ for all i $\Longrightarrow u \in A_{(\alpha,\lambda)}$. Therefore $C_{(\alpha,\lambda)} \subseteq A_{(\alpha,\lambda)}$. Let $e \in D_{(\alpha,\lambda)} \Longrightarrow \mu_{Di}(e) \geq \alpha_i$ and $\gamma_{Di}(e) \leq \lambda_I \Longrightarrow \mu_{Bi}(e) \geq \mu_{Di}(e) \geq \alpha_i$ and $\gamma_{Bi}(e) \leq \gamma_{Di}(e) \leq \lambda_i \Longrightarrow \mu_{Bi}(e) \geq \alpha_i$ and $\gamma_{Bi}(e) \leq \lambda_i \Longrightarrow e \in B_{(\alpha,\lambda)}$. Therefore $D_{(\alpha,\lambda)} \subseteq B_{(\alpha,\lambda)}$. Hence $H_{(\alpha,\lambda)}$ is a subgraph of $F_{(\alpha,\lambda)}$.

Definition 1.23: Let A be a intuitionistic multi fuzzy subset of X. Then the **strong level subset** or **strong(α, β)-cut** of A is $A_{(\alpha+,\beta+)} = \{x \in A / \mu_{Ai}(x) > \alpha_I$ and $\gamma_{Ai}(x) < \beta_i\}$ for all i and $\alpha_i + \beta_i < 1$ where $\alpha_i, \beta_i \in [0,1]$ for all i.

Theorem 1.24: Let F=(A,B,F) be a intuitionisticmulti fuzzy graph with respect to the set V and E. Let α, β, λ,η ∈ [0,1] and $\alpha \leq \beta$ and $\lambda \geq \eta$ then $(A_{(\beta+,\eta+)}, B_{(\beta+,\eta+)}, f)$ is a subgraph of $(A_{(\alpha+,\lambda+)}, B_{(\alpha+,\lambda+)}, f)$.

Proof: The proof follows from definition 1.23 and Theorem 1.22.

Theorem 1.25: Let F = (A, B, f) be a intuitionisticmulti fuzzy subgraph with respect to the set V and E, the level subsets $A_{(\alpha+,\lambda+)}, B_{(\alpha+,\lambda+)}$ of A and B subset of V and E respectively. Then $F_{(\alpha+,\lambda+)} = (A_{(\alpha+,\lambda+)}, B_{(\alpha+,\lambda+)}, f)$ is a subgraph of G = (V, E, f).

Proof: The proof follows from definition 1.23 and Theorem 1.24.

Theorem 1.26: Let H = (C, D, f) be a intuitionisticmulti fuzzy subgraph of F = (A, B, f) and α,λ ∈ [0,1]. Then $H_{\alpha+} = (C_{\alpha+}, D_{\alpha+}, f)$ is a subgraph of $F_{(\alpha+,\lambda+)} = (A_{(\alpha+,\lambda+)}, B_{(\alpha+,\lambda+)}, f)$.

Proof: The proof follows from definition 1.23 and Theorem 1.25.

Theorem 1.27: Let F = (A, B, f) be a intuitionisticmulti fuzzy subgraph with respect to the set V and E, let α, β, λ,η ∈ [0,1] and $F_{(\alpha,\lambda)}$ and $F_{(\beta,\eta)}$ be two subgraphs of G. Then (i) $F_{(\alpha,\lambda)} \cap F_{(\beta,\eta)}$ is a subgraph of G. (ii) $F_{(\alpha,\lambda)} \cup F_{(\beta,\eta)}$is a subgraph of G.

Proof: Since $A_{(\alpha,\lambda)}$ and $A_{(\beta,\eta)}$ are subset of V. Clearly $F_{(\alpha,\lambda)} \cap F_{(\beta,\eta)}$ is a subgraph of G. Also $F_{(\alpha,\lambda)} \cup F_{(\beta,\eta)}$ is a subgraph of G.

Definition 1.28: Let F = (A, B, f) be an intuitionisticmulti fuzzy graph. Then the **degree of an intuitionisticmulti fuzzy vertex** is defined by $d(v) = (d_\mu(v), d_\gamma(v))$ where

$$d_\mu(v) = \sum_{e \in f^{-1}(u,v)} \mu_B(e) + 2 \sum_{e \in f^{-1}(v,v)} \mu_B(e) \text{ and } d_\gamma(v) = \sum_{e \in f^{-1}(u,v)} \gamma_B(e) + 2 \sum_{e \in f^{-1}(v,v)} \gamma_B(e).$$

Defination 1.29: The **minimum degree** of the intuitionisticmulti fuzzy graph F=(A, B, f) is $\delta(F) = (\delta_\mu(F), \delta_\gamma(F))$ where $\delta_\mu(F) = \wedge\{d_\mu(v)/v \in V\}$ and $\delta_\gamma(F) = \wedge\{d_\gamma(v)/ v \in V\}$ and the **maximum degree** of F is $\Delta(F) = (\Delta_\mu(F), \Delta_\gamma(F))$ where $\Delta_\mu(F) = \vee\{d_\mu(v)/v \in V\}$ and $\Delta_\gamma(F) = \vee\{d_\gamma(v)/v \in V\}$.

Definition 1.30: Let F = (A, B, f) be an intuitionisticmulti fuzzy graph. Then the **order of intuitionisticmultifuzzy graph** F is defined to be o(F) = (o_μ(F),o_γ(F)) where $o_\mu(F) = \sum_{v\in V} \mu_A(v)$ *and* $o_\gamma(F) = \sum_{v\in V} \gamma_A(v)$.

Definition 1.31: Let F= (A, B, f) be an intuitionistic multi fuzzy graph. Then the **size of theintuitionisticmulti fuzzy graph F** is defined to be S(F) = (S_μ(F), S_γ(F)) where $S_\mu(F) = \sum_{e\in f^{-1}(u,v)} \mu_B(e)$ *and* $S_\gamma(F) = \sum_{e\in f^{-1}(u,v)} \gamma_B(e)$.

Example 1.32:

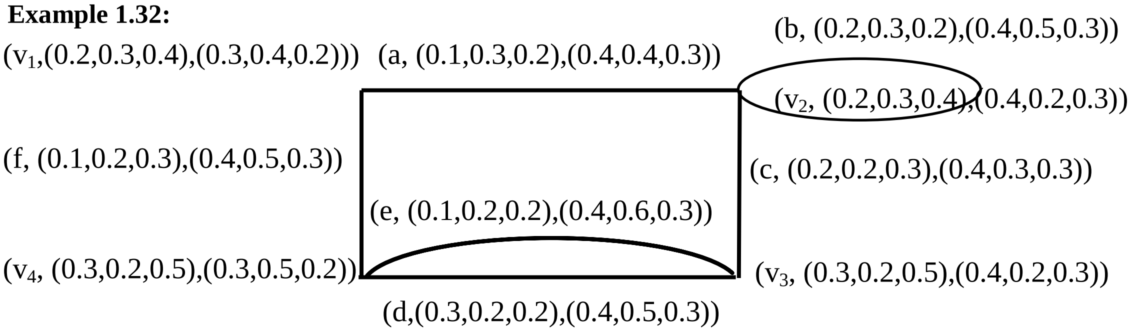

Fig.-1.10: Intuitionisticmulti fuzzy graph F

Here d(v_1) = ((0.2, 0.5, 0.5), (0.8, 0.9, 0.6)), d(v_2) = ((0.7, 1.1, 0.9), (1.6, 1.7, 1.2)), d(v_3) = ((0.6, 0.6, 0.7), (1.2, 1.4, 0.9)), d(v_4)=((0.5, 0.6, 0.7), (1.2, 1.6, 0.9)),δ(F) = ((0.2, 0.5, 0.5), (0.8, 0.9, 0.6)),Δ(F) = ((0.7, 1.1, 0.9), (1.6, 1.7, 1.2)), o(F) = ((1.0, 1.0, 1.8), (1.4, 1.3, 1.0)),S(F) = ((1.0, 1.4, 1.4), (2.4, 2.8, 1.8)).

Theorem 1.34:

(i)The sum of the degree of membership value of all intuitionisticmulti fuzzy vertices in an intuitionisticmulti fuzzy graph is equal to twice the sum of the membership value of all intuitionisticmulti fuzzy edges .i.e., $\sum_{v\in V} d_\mu(v) = 2S_\mu(F)$.(ii)The sum of the degree of non membership value of all intuitionisticmulti fuzzy vertices in an intuitionisticmulti fuzzy graph is equal to twice the sum of the non membership value of all intuitionisticmulti fuzzy edges. i.e., $\sum_{v\in V} d_\gamma(v) = 2S_\gamma(F)$.

(iii) The sum of the degree of all intuitionisticmulti fuzzy vertices in an intuitionisticmulti fuzzy graph is equal to twice the sum of the all intuitionisticmulti fuzzy edges. i.e., $\sum_{v\in V} d(v) = 2S(F)$.

Proof:

(i) Let F = (A, B, f) be an intuitionistic multi fuzzy graph with respect to the set V and E. Since degree of an intuitionisticmulti fuzzy vertex denote sum of the membership values of all intuitionisticmulti fuzzy edges incident on it. Each intuitionistic multifuzzy edge of F is incident with two intuitionisticmulti fuzzy vertices. Hence membershipvalue of each intuitionisticmulti fuzzy edgecontributes two to the sum of degrees of intuitionisticmulti fuzzy vertices. Hence the sum of the degree of all intuitionisticmulti fuzzy vertices in an intuitionisticmulti fuzzy graph is equal to twice the sum of the membership value of all intuitionistic multifuzzy edges. i.e., $\sum_{v\in V} d_\mu(v) = 2S_\mu(F)$.

(ii) Let F = (A, B, f) be an intuitionisticmulti fuzzy graph with respect to the set V and E. Since degree of an intuitionisticmulti fuzzy vertex denote sum of the non membership values of all intuitionisticmulti fuzzy edges incident on it. Each intuitionisticmulti fuzzy edge of F is incident with two intuitionistic multi fuzzy vertices. Hence non membership value of each intuitionistic multi fuzzy edge contributes two to the sum of degrees of intuitionisticmulti fuzzy vertices. Hence the sum of the degree of all intuitionistic multi fuzzy vertices in an intuitionisticmulti fuzzy graph is equal to twice the sum of the nonmembership value of all intuitionisticmulti fuzzy edges. i.e.,

$$\sum_{v \in V} d_\gamma(v) = 2S_\gamma(F).$$

(iii) From (i) and (ii)

The sum of the degree of all intuitionistic multi fuzzy vertices in an intuitionisticmulti fuzzygraph is equal to twice the sum of the all intuitionisticmulti fuzzy edges.

i.e., $\sum_{v \in V} d(v) = 2S(F)$.

Theorem 1.35: Let F = (A, B, f) be anintuitionisticmulti fuzzy graph with number of intuitionisticmulti fuzzy vertices n, all of whose intuitionistic multifuzzy vertices have degree s=(s_μ, s_γ) or t=(t_μ, t_γ). If F has pintuitionisticmulti fuzzy vertices of degree s and (n–p) intuitionistic multifuzzy vertices of degree t, then 2S(F) = ps + (n–p)t.

Proof: Let V_1 be the set of all intuitionisticmulti fuzzy vertices with degree s. Let V_2 be the set of all intuitionisticmulti fuzzy vertices with degree t. Then $\sum_{v \in V} d(v) = \sum_{v \in V_1} d(v) + \sum_{v \in V_2} d(v)$ which implies that

2S(F) = $\left(\sum_{v \in V_1} d_\mu(v), \sum_{v \in V_1} d_\gamma(v)\right) + \left(\sum_{v \in V_2} d_\mu(v), \sum_{v \in V_2} d_\gamma(v)\right)$ which implies that 2S(F) = p(s_μ , s_γ)+ (n–p)(t_μ, t_γ) which implies that 2S(F) = ps + (n–p)t.

2. INTUITIONISTIC MULTI FUZZYREGULAR GRAPH:

Definition 2.1: An intuitionistic multi fuzzy graph F=(A, B, f) is called **intuitionistic multi fuzzy regular graph** if d(v)=(s, k) for all v in V, where s = (s_1, s_2, ….,s_n) and k = (k_1,k_2, …..,k_n).

Remark 2.2: F is an **intuitionisticmultifuzzy (s, k)-regular graph** if and only if δ(F) = Δ(F) = (s, k).

Example 2.3:

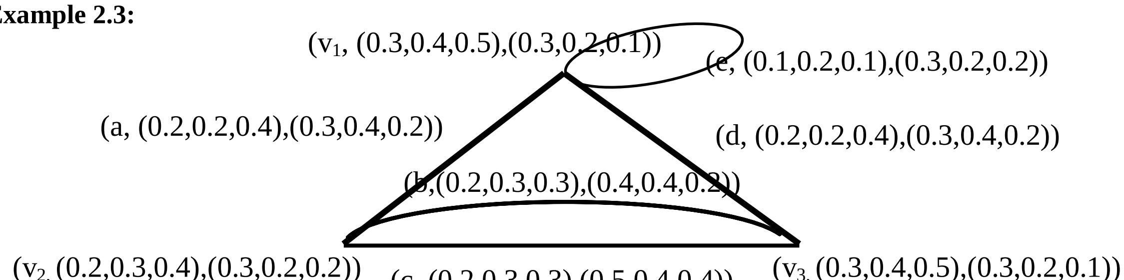

Fig.-2.1

Here d(v_i) = ((0.6, 0.8, 1.0), (1.2, 1.2, 0.8)) for all i, δ(F)=((0.6, 0.8, 1.0), (1.2, 1.2, 0.8)), Δ(F)=((0.6, 0.8, 1.0), (1.2, 1.2, 0.8)). Clearly it is an **intuitionisticmultifuzzy ((0.6, 0.8, 1.0), (1.2, 1.2, 0.8))-regular graph**.

Definition 2.4: An intuitionistic multi fuzzy graph F = (A, B, f) is called an **intuitionisticmultifuzzy complete graph** if every pair of distinct intuitionisticmulti fuzzy vertices are fuzzy adjacent and

$\mu_{Bi}(e) = \underset{e \in f^{-1}(x,y)}{\mu_{s_i}(x,y)}$ *and* $\gamma_{Bi}(e) = \underset{e \in f^{-1}(x,y)}{\gamma_{s_i}(x,y)}$ for all x,y in V and for all i.

Definition 2.5: An intuitionistic multi fuzzy graph F = (A, B, f) is an **intuitionisticmultifuzzy strong graph** if $\mu_{Bi}(e) = \underset{e \in f^{-1}(x,y)}{\mu_{s_i}(x,y)}$ *and* $\gamma_{Bi}(e) = \underset{e \in f^{-1}(x,y)}{\gamma_{s_i}(x,y)}$ for all e in E and for all i.

Example 2.6:

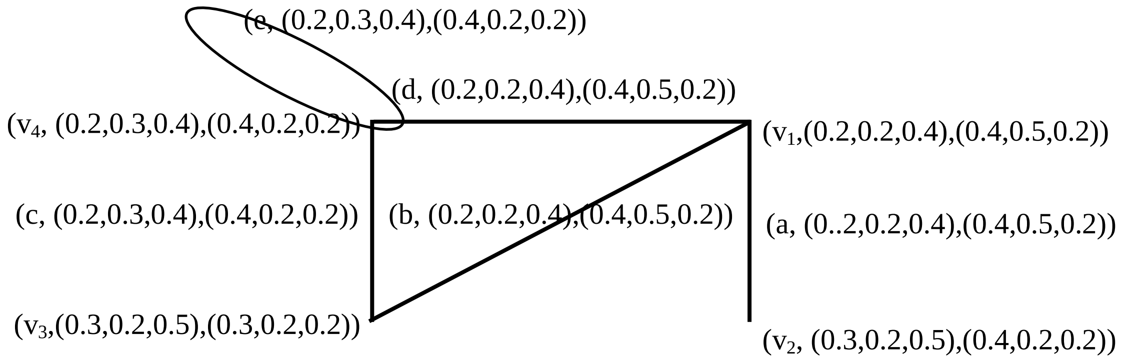

Fig.-2.2: An intuitionistic multi fuzzy strong graph

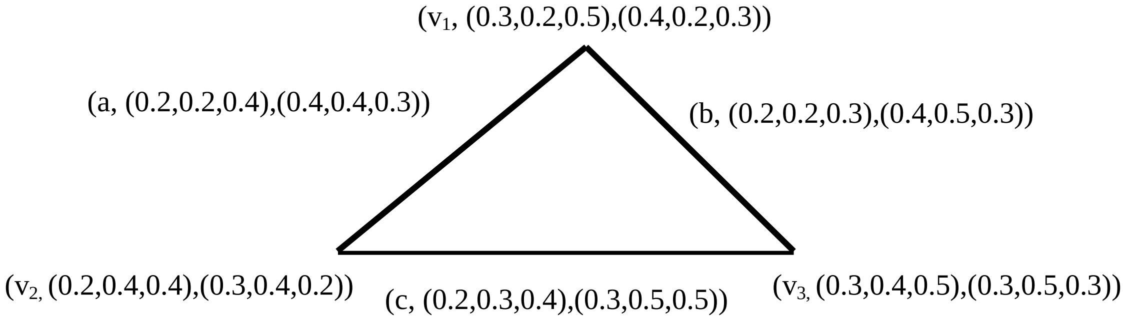

Fig.-2.3: An intuitionisticmulti fuzzycomplete graph

Theorem 2.7: If F is an intuitionisticmulti fuzzy (s, k)-regular graph with p-intuitionisticmulti fuzzy vertices. Then 2S(F) = (ps,pk) .

Proof: Given that the intuitionisticmulti fuzzy graph is an intuitionisticmulti fuzzy (s, k)-regular graph, so d(v) = (s, k) for all v in V. Here there are p-intuitionisticmulti fuzzy vertices, so $\sum_{v\in V} d(v) = (\sum_{v\in V} s, \sum_{v\in V} k)$ = (ps, pk) which implies that 2S(F) = (ps, pk).

Theorem 2.8: Let F = (A, B, f) be intuitionisticmulti fuzzy complete graph and A= (s, k) is constant function. Then F is an intuitionisticmulti fuzzy regular graph.

Proof: Since A is a constant function, so A(v) =(s, k) (say) for all v in V and F is an intuitionisticmulti fuzzy complete graph, so $\mu_{Bi}(e) = \mu_{s_i}(x,y)_{e\in f^{-1}(x,y)}$ *and* $\gamma_{Bi}(e) = \gamma_{s_i}(x,y)_{e\in f^{-1}(x,y)}$ for all x and y in V, for all I and x≠y. Therefore membership and non membership value of all intuitionisticmulti fuzzy edges are s,k respectively. Hence d(v) = ((p–1) s,(p–1)k) for all v in V.

Theorem 2.9: If F = (A, B, f) is intuitionisticmulti fuzzy complete graph with p-intuitionisticmulti fuzzy vertices and A is constant function then S(F) = ($^pC_2\, \mu_A(v)$,$^pC_2\, \gamma_A(v)$), for all v in V.

Proof: Suppose F is an intuitionisticmulti fuzzy complete graph and A= (μ_A, γ_A) is a constant function.
Let A(v) = (s, k) for all v in V and d(v) = ((p–1)s,(p–1)k) for all v in V. Then
$\sum_{v\in V} d(v) = (\sum_{v\in V}(p-1)s, \sum_{v\in V}(p-1)k) = (p(p-1)s,\ p(p-1)k)$ which implies that

2S(F) = (p(p–1)s,p(p–1)k). Hence S(F) = (${}^{p}C_2$ s,${}^{p}C_2$ k).
i.e., S(F) = (${}^{p}C_2\, \mu_A(v)$,${}^{p}C_2\, \gamma_A(v)$) for all v in V.

Definition 2.10: Let F = (A, B, f) be an intuitionistic multi fuzzy graph. The **total degree of intuitionisticmulti fuzzy vertex** vis defined by $d_T(v)=(d_{T\mu}(v),d_{T\gamma}(v))$

Where

$$d_{T_\mu}(v) = \sum_{e\in f^{-1}(u,v)} \mu_B(e) + 2\sum_{e\in f^{-1}(v,v)} \mu_B(e) + \mu_A(v) = d_\mu(v) + \mu_A(v) \text{ and}$$

$$d_{T_\gamma}(v) = \sum_{e\in f^{-1}(u,v)} \gamma_B(e) + 2\sum_{e\in f^{-1}(v,v)} \gamma_B(e) + \gamma_A(v) = d_\gamma(v) + \gamma_A(v) \text{ for all v in V.}$$

Definition 2.11: An intuitionisticmulti fuzzy graph F is **intuitionisticmulti fuzzy (s, k)-totally regular graph** if each intuitionistic multi fuzzy vertex of F has the same total degree(s, k).

Example 2.12:

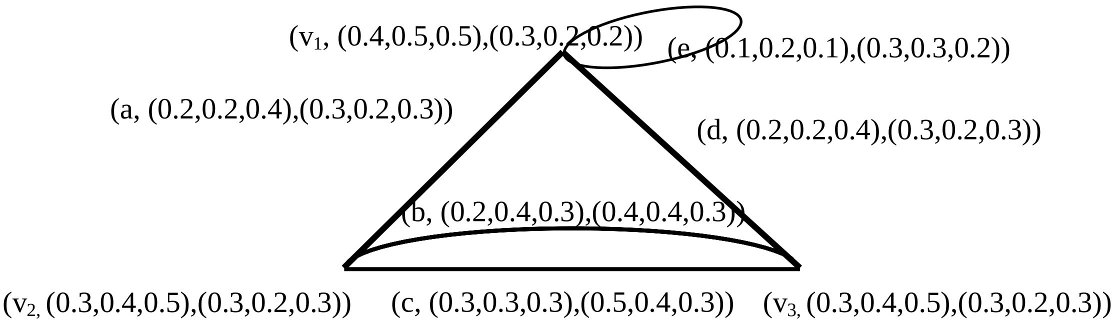

Fig.-2.4

Here $d_T(v_1)$ = ((1.0, 1.3, 1.5),(1.5, 1.2, 1.2)), $d_T(v_2)$ = ((1.0, 1.3, 1.5),(1.5, 1.2, 1.2)),$d_T(v_3)$ = ((1.0, 1.3, 1.5), (1.5, 1.2, 1.2)), it isintuitionisticmulti fuzzy ((1.0, 1.3, 1.5), (1.5, 1.2, 1.2))-totally regular graph.

Example 2.13: Fig.2.1 it is an intuitionisticmulti fuzzy regular graph**,** but it is not an intuitionisticmulti fuzzy totally regular graph since $d_T(v_1)$ = ((0.9, 1.2, 1.5), (1.5, 1.4, 0.9), $d_T(v_2)$ = ((0.8, 1.1, 1.4), (1.5, 1.4, 1.0)) and $d_T(v_1) \neq d_T(v_2)$.

Example 2.14: Fig 2.4, it is an intuitionisticmulti fuzzy totally regular graph but it is not an intuitionisticmulti fuzzy regular graph since $d(v_1)$ = ((0.6, 0.8, 1.0), (1.2, 1.0, 1.0)), $d(v_2)$ = ((0.7, 0.9, 1.0), (1.2, 1.0, 0.9)) and $d(v_1) \neq d(v_2)$.

Example 2.15:

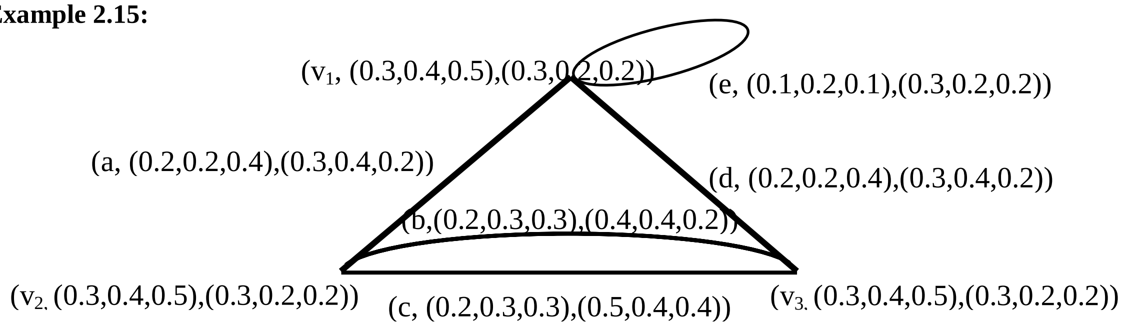

Fig.-2.5

Here $d(v_i)$ = ((0.6, 0.8, 1.0), (1.2, 1.2, 0.8)) for all i,$d_T(v_i)$ = ((0.9, 1.2, 1.5), (1.5, 1.4, 1.0)) for all i. It is both intuitionisticmulti fuzzy regular graph and intuitionisticmulti fuzzy totally regular graph.

Example 2.16:

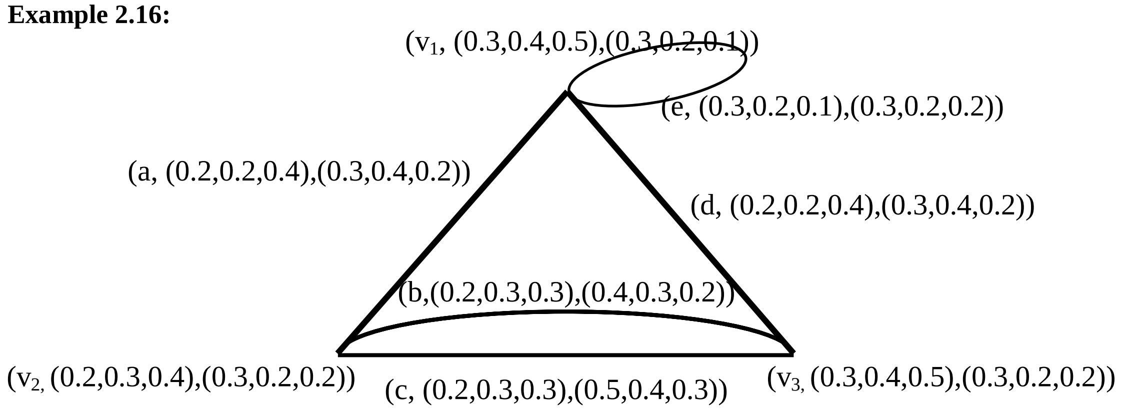

Fig.-2.6

Here $d(v_1)$=((1.0, 0.8, 1.0), (1.2, 1.2, 0.8)), $d(v_2)$ =((0.6, 0.8, 1.0), (1.2, 1.1, 0.7)),$d(v_3)$ = ((0.6, 0.8, 1.0), (1.2, 1.1, 0.7)), $d_T(v_1)$ = ((1.3, 1.2, 1.5), (1.5, 1.4, 0.9)), $d_T(v_2)$ = ((0.8, 1.1, 1.4), (1.5, 1.3, 0.9)),$d_T(v_3)$ = ((0.9, 1.2, 1.5), (1.5, 1.3, 0.9)), it is neither intuitionisticmulti fuzzy regular graph nor intuitionisticmulti fuzzy totally regular graph.

Theorem 2.17: Let F= (A, B, f) be intuitionisticmulti fuzzy complete graph and A=(s, k) is constant function. Then F is an intuitionisticmulti fuzzy totally regular graph.

Proof: By theorem 2.8, clearly F is intuitionisticmulti fuzzy regular graph. i.e., d(v) = ((p–1)s, (p–1)k) for all v in V. Also given A is constant function. i.e., A(v) = (s, k) for all v in V. Then $d_T(v) = (d_\mu(v) + \mu_A(v), d_\gamma(v) + \gamma_A(v))$ = ((p–1)s + s,(p–1)k + k) = (ps, pk) for all v in V. Hence F is intuitionisticmulti fuzzy totally regular graph.

Theorem 2.18: Let F = (A, B, f) be an intuitionisticmulti fuzzy regular graph. Then H = (C, B, f) is an intuitionisticmulti fuzzy totally regular graph if

$C(v) = (\sum_{1=1}^{n} \mu_A(v_i), \sum_{1=1}^{n} \gamma_A(v_i)) \leq 1$ for all v_i in V.

Proof: Assume that F = (A, B, f) is an intuitionisticmulti fuzzy (s, k)- regular graph. i.e., $d(v_i)$ = (s, k) for all v_i in V. Given $C(v) = (\sum_{1=1}^{n} \mu_A(v_i), \sum_{1=1}^{n} \gamma_A(v_i)) \leq 1$ for all v_i in V. Then C(v) = (c_1,c_2) (say) for all v_i in V and $d_{T(H)}(v_i) = (d\mu(v_i) + \mu_C(v_i), d_\gamma(v_i) + \gamma_C(v_i))$ = (s + c_1,k+c_2) for all v_i in V. Hence H is intuitionisticmulti fuzzy totally regular graph.

Theorem 2.19: Let F = (A, B, f) be an intuitionisticmulti fuzzy graph and A is a constant function (ie. A(v) = (c_1,c_2) (say) for all $v \in V$)**.** Then F isintuitionisticmulti fuzzy (s, k)-regular graph if and only if F is intuitionisticmulti fuzzy (s+c_1,k+c_2)-totally regular graph.

Proof: Assume that F is an intuitionisticmulti fuzzy (s, k)-regular graph and A(v) = (c_1,c_2) for all v in V, so d(v) = (s, k) for all v in V. Then $d_T(v) = (d_\mu(v) + \mu_A(v), d_\gamma(v) + \gamma_A(v))$ = (s + c_1, k+c_2) for all v in V. Hence F is intuitionistic multifuzzy (s +c_1, k+c_2)-totally regular graph. Conversely, Assume that F is intuitionisticmulti fuzzy (s+c_1,k+c_2)-totally regular graph. ie., $d_T(v)$ = (s + c_1, k+c_2) for all v in V which implies that $(d_\mu(v) + \mu_A(v), d_\gamma(v) + \gamma_A(v))$ = (s + c_1, k+c_2) for all v in V implies that $(\mu_A(v), \gamma_A(v)) = (c_1, c_2)$ for all v in V implies that $d_\mu(v) + c_1 = s + c_1$ and $d_\gamma(v) + c_2 = k+c_2$ for all v in V. Therefore $d_\mu(v)$ = s and $d_\gamma(v)$ =k for all v in V. ie., d(v) = (s, k) for all v in V. Hence F is intuitionisticmulti fuzzy (s, k)-regular graph.

Theorem2.20: If F = (A, B, f) is bothintuitionisticmulti fuzzy regular graph and intuitionisticmulti fuzzy totally regular graph then A is a constant function.

Proof: Assume that F is a both intuitionisticmulti fuzzy regular graph and intuitionisticmulti fuzzy totally regular graph.

Suppose that A is not constant function. Then $\mu_A(u) \neq \mu_A(v)$ or $\gamma_A(u) \neq \gamma_A(v)$ for some u, v in V. Since F is an intuitionisticmulti fuzzy (s, k)-regular graph. Then d(u) = d(v) = (s, k).Then $d_T(u) \neq d_T(v)$ which is a contradiction to our assumption. Hence A is a constant function.

Remark 2.21: Converse of the above theorem need not be true.

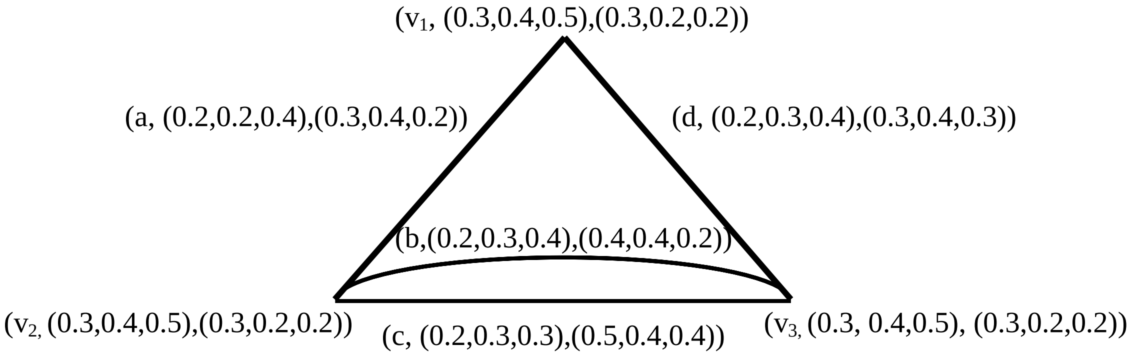

Fig.-2.7

Here $A(v_i)$=((0.3, 0.4, 0.5), (0.3, 0.2, 0.2)) for all i, $d(v_1)$=((0.4, 0.5, 0.8), (0.6, 0.8, 0.5)),$d(v_2)$=((0.6, 0.8, 1.1), (1.2, 1.2, 0.8)), $d(v_3)$ = ((0.6, 0.9, 1.1), (1.2, 1.2, 0.9)), $d_T(v_1)$ = ((0.7, 0.9, 1.3), (0.9, 1.0, 0.7)), $d_T(v_2)$ = ((0.9, 1.2, 1.6), (1.5, 1.4, 1.0)), $d_T(v_3)$ = ((0.9, 1.3, 1.6), (1.5, 1.4, 1.1)). Hence F is neitherintuitionisticmulti fuzzy regular graph nor intuitionisticmulti fuzzy totally regular graph.

Theorem 2.22: If F = (A, B, f) is an intuitionisticmulti fuzzy (c_1, c_2)-totally regular graph with p-intuitionisticmulti fuzzy vertices. Then 2S(F) + o(F) = p(c_1, c_2).

Proof: Assume that F is an intuitionisticmulti fuzzy (c_1, c_2)-totally regular graph with p-intuitionisticmulti fuzzy vertices**.**
Then $d_T(v) = (c_1,c_2)$ for all v in V implies that $(d_\mu(v) + \mu_A(v), d_\gamma(v) + \gamma_A(v)) = (c_1, c_2)$ for all v in V which implies that $(\sum d_\mu(v)+\sum\mu_A(v), \sum d_\gamma(v)+\sum\gamma_A(v))=(\sum c_1,\sum c_2)$ for all v in V which implies that $(2S_\mu(F)+o_\mu(F), 2S_\gamma(F)+o_\gamma(F)) = (pc_1, pc_2)$ for all v in V implies that $(2S_\mu(F), 2S_\gamma(F)) + (o_\mu(F), o_\gamma(F)) = (pc_1,pc_2)$. Hence 2S(F) + o(F) = p(c_1, c_2).

Theorem 2.23: If F = (A, B, f) is both intuitionisticmulti fuzzy k=(s, k)-regular graph and intuitionisticmulti fuzzy c = (c_1, c_2)-totally regular graph with p-intuitionisticmulti fuzzy vertices. Then o(F) = pc–pk.

Proof: Assume that F is intuitionisticmulti fuzzy k-regular graph with p-intuitionisticmulti fuzzy vertices. Then 2S(F) = pk. By theorem 2.22, 2S(F) + o(F) = pc implies that o(F) = pc–pk.

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