International Journal of Mathematical Archive-10(12), 2019, 21-24
Available online through www.ijma.info ISSN 2229 – 5046

SOME RESULTS ON DOUBLE SEQUENCE THEOREMS IN METRIZABLE SPACES

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(Received On: 11-11-19; Revised & Accepted On: 09-12-19)

ABSTRACT

In this paper, we have discussed some generalized results in double sequence theorems on metrizable spaces and also some new concepts of generalized metric spaces.

Keywords: *metric space, locally-finite, covering space, vector space,*

AMS Mathematics Subject Classification (2000): *47H04, 47H10.*

INTRODUCTION

The discussion of some equivalence metrication theorems, modified single sequence theorem and modified double sequence theorems has been studied by Nigata [1]. Here we also defined metric topologies, Before that, however, we want to give a name to those topological spaces whose topologies are metric topologies. For the sake of compactness we have studied and the proof given by Mattin [3]. Some of authors studied various types of theorems in metric spaces. In this paper, we have establish and generalized results which is observed in results in [2]. Further we have added some illustrative examples and results.

Definition of T_1 spaces: A T_1- space is a topological space in which given any pair of disjoint points, each has a neighbourhood which does not contain the other.

It is obvious that any subspace of T_1-space is also a T_1-space.

MAIN RESULTS

In the present section, clearly, it validates the equivalences of metrization conditions of Bing's Theorem, Nagata Theorem, and Double sequence Theorems. Symmetrically which follows only from Urysohn's Theorem. Hence therefore we studied the following theorems based on T_1-spaces.

Theorem 1: If a topological space τ then

(i). Is a T_1-space
(ii). is a regular space
(iii) . has a $\sigma -$ discrete base then τ is a metrizablespace

Theorem 2: If a topological space τ then

(iv). is a T_1-space
(v). is a regular space
(vi).has a $\sigma -$ locally finite base then τ is a metrizablespace.

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Theorem 3: If a topological space τ then

(vii). is a T_1-space

(viii). there exists a sequence $u_1, u_2, u_3....$ of open covering such that

(ix). $u_1 > u_2^* > u_2 > u_3^* > ......can \quad be\ repaced\ as\ u_1 > u_1^{\Delta} > u_2 > u_2^{\Delta}....$

(x). $\{s(p,u_n): n = 1,2,3.....\}$ is a neighbourhood basis at each point of "p" of R then R is a metrizable.

Proof:

Theorem (1) $\Rightarrow$ Theorem (2) is obvious thus implies.

Theorem (3) $\Rightarrow$ Theorem (2) have been proved by Nagata[1]

It is enough to show that conditions of theorem 3 imply the conditions of theorem 1. which we do now.

We assume theorem 3.

$$\text{(vii)} \Rightarrow \text{(i)} \tag{1}$$

We now prove (viii)

Let $N(p)$ be neighbourhood of a point p, There exists from (x) and $n \in N$ such that

$$S(p,u_n) \subset N(p) \tag{2}$$

Now consider $S(p,u_{n+1})$ then $S(p,u_n)$ is closed neighbourhood of p.

Let $$b \in \overline{S(p,u_{n+1})}. \tag{3}$$

Every G_{n+1} such that $b \in G_{n+1}$. $G_{n+1} \in u_{n+1}$ call them as $G_{n+1}(b)$ which meets $S(p,u_{n+1})$, i.e meets to G_{n+1} which contains "p". Call it as $G_{n+1}(p)$.

$$G_{n+1}(p) \cup G_{n+1}(b) \subset u_{n+1}^* \subset u_n \tag{4}$$

There exists a set u_n which contains "p" for every b.

Now $S(p,u_n)$ examples contain p and b.

$$b \in S(p,u_n) \tag{5}$$

$$\overline{S(p,U_{n+1})} \in S(p,u_n) \subset N(p). \tag{6}$$

Which established (viii).

We prove (ix) in a series of steps.

STEP-1: The following results are obvious

$$\{q \in S_n(p)\} \Rightarrow \{p \in S_n(q)\} \tag{7}$$

If $q \in S(p,u_{n+1})$, then from (viii)(ix)(x)

$$S(p,u_{n+1}) \cup S(q,u_{n+1}) \subset S(p,u_n) \tag{8}$$

STEP-2: Let u be an open cover in R such that $U \in u$.

Let $$U_n = \{x \in U : x \neq S(p,u_n), p \in (R-U)\} \tag{9}$$

We assert that if $q \notin U_{n+1}$ (10)

and $x \notin U_{n+1}$ (11)

Further

$$x \notin S_{n+1}(q) \tag{12}$$

For otherwise if $x \in S_{n+1}(q) = S(q, u_{n+1})$.

Then for some value of "p" in (R-U), $q \in S(p, u_{n+1})$ and $S(p, u_{n+1}) \cap U_{n+1} = \phi$. Which contradiction, Hence the proof of equation 12.

Now let us consider u is an open cover. Well defined order of u by a relation λ.

Let us consider $U, V \in u$, then

$$V \lambda\ U \tag{13}$$

Or

$$U\ \lambda\ V \tag{14}$$

Set

$$V_n^{\ *} = V_n - \cup U_{n+1} \tag{15}$$

For all $U \in u$ such that $V\ \lambda\ U$.

Therefore

$$U_n^{\ *} = U_n - \cup V_{n+1} \tag{16}$$

For all $V\ \lambda\ U$

Then

$$V_n^{\ *} \subset R - \cup U_{n+1}\ \lambda\ R - U_{n+1} \tag{17}$$

Since $U_{n+1} \subset U_n$,

Thus

$$V_n^{\ *} \cap U_n^{\ *} = \phi \tag{18}$$

For all the values of n=1,2,3......

Let $p \in V_n^{\ *}$ (19)

Then $p \in V_n - \cup U_{n+1}, U\ \lambda\ V$.

$$p \notin U_{n+1} \tag{20}$$

Therefore

$$S_{n+1}(p) \cap U_n^{\ *} = \phi \tag{21}$$

If $p \in U_n^{\ *}$ (22)

Similarly we get

$$S_{n+1}(p) \cap V_n^{\ *} = \varphi \tag{23}$$

For each $p \in U_n^*$, consider the set points for x,

CONCLUSION

In the present paper, we gave generalized concepts of Nagata and Bing, further the sequence theorems and some double sequence theorems will play an important rule on T_1-spaces.

ACKNOWLEDGEMENTS

I extend my sincere thanks to the editors and referees for their valuable suggestions, which leads to improve the presentation of my research article.

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Source of support: Nil, Conflict of interest: None Declared.