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BIPOLAR INTERVAL VALUED MULTI FUZZY GENERALIZED SEMIPRECLOSED MAPPINGS

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ABSTRACT

In this paper, bipolar interval valued multi fuzzy generalized semi-preclosed mappings and bipolar interval valued multi fuzzy generalized semi-preopen mappings are defined and introduced. Using this definitions, some theorems are introduced.

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Keywords: *Bipolar interval valued multi fuzzy subset, bipolar interval valued multi fuzzy topological space, bipolar interval valued multi fuzzy interior, bipolar interval valued multi fuzzy closure, bipolar interval valued multi fuzzy continuous mapping, bipolar interval valued multi fuzzy generalized semi-preclosed set, bipolar interval valued multi fuzzy generalized semi-preopen set, bipolar interval valued multi fuzzy generalized semi-preclosed mapping, bipolar interval valued multi fuzzy generalized semi-preopen mapping.*

INTRODUCTION

The concept of a fuzzy subset was introduced and studied by L.A.Zadeh [19] in the year 1965, the subsequent research activities in this area and related areas have found applications in many branches of science and engineering. The following papers have motivated us to work on this paper C.L.Chang [4] introduced and studied fuzzy topological spaces in 1968 as a generalization of topological spaces many researchers like, and many others have contributed to the development of fuzzy topological spaces. Andrijevic [1] has introduced semipreclosed sets and Dontchev [5] has introduced generalized semipreclosed sets in general topology. After that the set was generalized to fuzzy topological spaces by saraf and khanna [15]. Tapas kumar mondal and S.K.Samantha [12] have introduced the topology of interval valued fuzzy sets. The interval valued fuzzy set has been extended into the bipolar interval valued multi fuzzy topological spaces. R.Selvam *et.al* [16] have defined and introduced the bipolar interval valued multi fuzzy generalized semipreclosed sets. In this paper, we introduce bipolar interval valued multi fuzzy generalized semi-preclosed mappings and bipolar interval valued multi fuzzy generalized semi-preopen mappings and some properties are investigated.

1. PRELIMINARIES

Definition 1.1[19]: Let X be a non-empty set. A **fuzzy subset A** of X is a function A: X → [0, 1].

Definition 1.2[14]: A **multi fuzzy subset** A of a set X is defined as an object of the form $A = \{\langle x, A_1(x), A_2(x), A_3(x), \ldots, A_n(x) \rangle / x \in X\}$, where $A_i : X \to [0, 1]$ for all i and i = 1, 2, …, n.

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Definition 1.3[19]: Let X be any nonempty set. A mapping A: $X \to D[0, 1]$ is called an interval valued fuzzy subset (briefly, IVFS) of X, where $D[0,1]$ denotes the family of all closed subintervals of [0, 1].

Definition 1.4[19]: A interval valued multi fuzzy subset A of a set X with degree n is defined as an object of the form $A = \{ \langle x, A_1(x), A_2(x), A_3(x), \ldots, A_n(x) \rangle / x \in X\}$, where $A_i : X \to D[0, 1]$ for all i and i = 1, 2, …, n.

Definition 1.5[10]: A bipolar valued fuzzy set A in X is defined as an object of the form $A = \{\langle x, M(x), N(x) \rangle / x \in X\}$, where $M: X \to [0, 1]$ and $N: X \to [-1, 0]$. The positive membership degree M(x) denotes the satisfaction degree of an element x to the property corresponding to a bipolar valued fuzzy set A and the negative membership degree N(x) denotes the satisfaction degree of an element x to some implicit counter-property corresponding to a bipolar valued fuzzy set A.

Example 1.6: $A = \{\langle a, 0.8, -0.6\rangle, \langle b, 0.6, -0.7 \rangle, \langle c, 0.3, -0.9 \rangle\}$ is a bipolar valued fuzzy subset of X = {a, b, c}.

Definition 1.7[10]: A bipolar valued multi fuzzy set A in X with degree n is defined as an object of the form $A = \{\langle x, M_1(x), M_2(x), M_3(x), \ldots, M_n(x), N_1(x), N_2(x), N_3(x), \ldots, N_n(x) \rangle / x \in X\}$, where $M_i : X \to [0, 1]$ and $N_i : X \to [-1, 0]$ for all i and i = 1, 2, …, n.. The positive membership degrees $M_i(x)$ denotes the satisfaction degrees of an element x to the property corresponding to a bipolar valued multi fuzzy set A and the negative membership degrees $N_i(x)$ denotes the satisfaction degrees of an element x to some implicit counter-property corresponding to a bipolar valued multi fuzzy set A.

Example 1.8: $A = \{\langle a, 0.6, 0.4, 0.7, -0.4, -0.5, -0.8\rangle, \langle b, 0.8, 0.7, 0.3, -0.3, -0.5, -0.6\rangle, \langle c, 0.5, 0.3, 0.4, -0.5, -0.7, -0.9 \rangle\}$ is a bipolar valued multi fuzzy subset of X = { a, b, c } with degree 3.

Definition 1.9[16]: A bipolar interval valued fuzzy set A in X is defined as an object of the form $A = \{\langle x, M(x), N(x) \rangle / x \in X\}$, where $M: X \to D[0, 1]$ and $N : X \to D[-1, 0]$. The positive membership interval degree M(x) denotes the satisfaction degree of an element x to the property corresponding to a bipolar interval valued fuzzy set A and the negative membership interval degree N(x) denotes the satisfaction degree of an element x to some implicit counter-property corresponding to a bipolar interval valued fuzzy set A.

Example 1.10: $A = \{\langle a, [0.6, 0.9], [-0.6, -0.4] \rangle, \langle b, [0.8,0.9], [-0.7, -0.5] \rangle, \langle c, [0.5, 0.8], [-0.8, -0.6] \rangle\}$ is a bipolar interval valued fuzzy subset of X ={ a, b, c }.

Definition 1.11[16]: A bipolar interval valued multi fuzzy set A in X with degree n is defined as an object of the form $A = \{\langle x, M_1(x), M_2(x), M_3(x), \ldots, M_n(x), N_1(x), N_2(x), N_3(x), \ldots, N_n(x)\rangle / x \in X\}$, where $M_i : X \to D[0, 1]$ and $N_i : X \to D[-1, 0]$ for all i and i = 1, 2, …, n.. The positive membership degrees $M_i(x)$ denotes the satisfaction degrees of an element x to the property corresponding to a bipolar interval valued multi fuzzy set A and the negative membership degrees $N_i(x)$ denotes the satisfaction degrees of an element x to some implicit counter-property corresponding to a bipolar interval valued multi fuzzy set A.

Example 1.12: $A = \{\langle a, [0.6, 0.8], [0.4, 0.6], [0.7, 0.9], [-0.4, -0.2], [-0.5, -0.3], [-0.8, -0.6] \rangle, \langle b, [0.8, 0.9], [0.7, 0.9], [0.3, 0.5], [-0.3, -0.2], [-0.5, -0.3], [-0.6, -0.4] \rangle, \langle c, [0.5, 07], [0.3, 0.5], [0.4, 0.6], [-0.5, -0.3], [-0.7, -0.5], [-0.9, -0.7] \rangle\}$ is a bipolar interval valued multi fuzzy subset of X = {a, b, c} with degree 3.

Definition 1.13[16]: Let $A = \langle M_i, N_i\rangle$ and $B = \langle O_i, P_i\rangle$ be any two bipolar interval valued multi fuzzy subsets of a set X with degree n. We define the following relations and operations:

(i) $A \subseteq B$ if and only if $M_i(x) \le O_i(x)$ and $N_i(x) \ge P_i(x)$ for all x in X and for all i.

(ii) A =B if and only if $M_i(x) = O_i(x)$ and $N_i(x) = P_i(x)$ for all x in X and for all i.

(iii) $(A)^c = \{\langle x, (M_i)^c(x), (N_i)^c(x) \rangle / x \in X\}$.

(iv) $A \cap B = \{\langle x, \text{rmin}\{ M_i(x), O_i(x)\}, \text{rmax}\{N_i(x), P_i(x)\}\rangle / x \in X\}$.

(v) $A \cup B = \{\langle x, \text{rmax} \{ M_i(x), O_i(x)\}, \text{rmin}\{N_i(x), P_i(x)\}\rangle / x \in X\}$.

Remark 1.14: $\overline{0} = \{\langle x, [0, 0], [0, 0], \ldots [0, 0] \rangle : x \in X\}$ and $\overline{1} = \{\langle x, [1, 1], [1, 1], .., [1, 1], [-1, -1], [-1, -1], \ldots, [-1, -1] \rangle : x \in X \}$.

Definition 1.15[16]: Let X be a set and $\mathfrak{I}$ be a family of bipolar interval valued multi fuzzy subsets of X. The family $\mathfrak{I}$ is called a bipolar interval valued multi fuzzy topology (BIVMFT) on X if $\mathfrak{I}$ satisfies the following axioms

(i) $\overline{0}, \overline{1} \in \mathfrak{I}$

(ii) (ii) If { A_i ; i∈I } ⊆ $\Im$, then $\bigcup_{i=1}^{\infty} A_i \in \Im$

(iii) If $A_1, A_2, A_3, \ldots A_n \in \Im$, then $\bigcap_{i=1}^{n} A_i \in \Im$.

The pair (X, $\Im$) is called a bipolar interval valued multi fuzzy topological space (BIVMFTS). The members of $\Im$ are called bipolar interval valued multi fuzzy open sets (BIVMFOS) in X. An bipolar interval valued multi fuzzy subset A in X is said to be bipolar interval valued multi fuzzy closed set (BIVMFCS) in X if and only if $(A)^c$ is a BIVMFOS in X.

Definition 1.16[16]: Let (X, $\Im$) be a BIVMFTS and A be a BIVMFS in X. Then the bipolar interval valued multi fuzzy interior and bipolar interval valued multi fuzzy closure are defined by $bivmfint(A) = \cup\{G : G$ is a BIVMFOS in X and $G \subseteq A\}$, $bivmfcl(A) = \cap\{K : K$ is a BIVMFCS in X and $A \subseteq K\}$. For any BIVMFS A in (X, $\Im$), we have $bivmfcl(A^c) = (bivmfint(A))^c$ and $bivmfint(A^c) = (bivmfcl(A))^c$.

Definition 1.17[16]: A BIVMFS A of a BIVMFTS (X, $\Im$) is said to be a

(i) bipolar interval valued multi fuzzy regular closed set (BIVMFRCS for short) if $A = bivmfcl(bivmfint(A))$

(ii) bipolar interval valued multi fuzzy semiclosed set (BIVMFSCS for short) if $bivmfint(bivmfcl(A)) \subseteq A$

(iii) bipolar interval valued multi fuzzy preclosed set (BIVMFPCS for short) if $bivmfcl(bivmfint(A)) \subseteq A$

(iv) bipolar interval valued multi fuzzy α closed set (BIVMF**α**CS for short) if $bivmfcl\left(bivmfint(bivmfcl(A))\right) \subseteq A$

(v) bipolar interval valued multi fuzzy β closed set (BIVMF**β**CS for short) if $bivmfint\left(bivmfcl(bivmfint(A))\right) \subseteq A$.

Definition 1.18[16]: A BIVMFS A of a BIVMFTS (X, $\Im$) is said to be a

(i) bipolar interval valued multi fuzzy generalized closed set (BIVMFGCS for short) if bivmfcl(A) ⊆ U, whenever A⊆ U and U is a BIVMFOS

(ii) bipolar interval valued multi fuzzy regular generalized closed set (BIVMFRGCS for short) if $bivmfcl(A) \subseteq U$, whenever A⊆ U and U is a BIVMFROS.

Definition 1.19[16]: A BIVMFS A of a BIVMFTS (X, $\Im$) is said to be a

(i) bipolar interval valued multi fuzzy semipreclosed set (BIVMFSPCS for short) if there exists a BIVMFPCS B such that $bivmfint(B) \subseteq A \subseteq B$

(ii) bipolar interval valued multi fuzzy semipreopen set (BIVMFSPOS for short) if there exists a BIVMFPOS B such that $B \subseteq A \subseteq bivmfcl(B)$.

Definition 1.20[16]: Two BIVMFSs A and B are said to be not q-coincident if and only if $A \subseteq B^c$.

Definition 1.21[16]: Let A be a BIVMFS in a BIVMFTS (X, $\Im$). Then the bipolar interval valued multi fuzzy semipre interior of A ($bivmfspint(A)$ for short) and the bipolar interval valued multi fuzzy semipre closure of A ($bivmfspcl(A)$ for short) are defined by $bivmfspint(A) = \cup\{G : G$ is a BIVMFSPOS in X and $G \subseteq A\}$, $bivmfspcl(A) = \cap\{K : K$ is a BIVMFSPCS in X and $A \subseteq K\}$. For any BIVMFS A in $(X, \Im)$, we have $bivmfspcl(A^c) = (bivmfspint(A))^c$ and $bivmfspint(A^c) = (bivmfpcl(A))^c$.

Definition 1.22[16]: A BIVMFS A in BIVMFTS (X, $\Im$) is said to be a bipolar interval valued multi fuzzy generalized semipreclosed set (BIVMFGSPCS for short) if $bivmfspcl(A) \subseteq U$ whenever $A \subseteq U$ and U is a BIVMFOS in (X,$\Im$).

Example 1.23: Let $X = \{a, b\}$ and $\Im = \{\bar{0}, G, \bar{1}\}$ is a BIVMFT on X, where G = {⟨ a, [0.5, 0.5], [0.6, 0.6], [0.4, 0.4], [–0.4, –0.4], [–0.5, –0.5], [–0.3, –0.3] ⟩, ⟨ b, [0.4, 0.4], [0.5, 0.5], [0.3, 0.3], [–0.3, –0.3], [–0.4, –0.4], [–0.2, –0.2] ⟩}. And the BIVMFS A = {⟨ a, [0.4, 0.4], [0.5, 0.5], [0.3, 0.3], [–0.3, –0.3], [–0.4, –0.4], [–0.2, –0.2] ⟩, ⟨ b, [0.2, 0.2], [0.3, 0.3], [0.1, 0.1], [–0.1, –0.1], [–0.2, –0.2], [–0.05, –0.05] ⟩ } is a BIVMFGSPCS in (X,$\Im$).

Definition 1.24[16]: The complement A^c of a BIVMFGSPCS A in a BIVMFTS (X, $\Im$) is called a bipolar interval valued multi fuzzy generalized semi-preopen set (BIVMFGSPOS) in X.

Definition 1.25[16]: A BIVMFTS (X, $\Im$) is called a bipolar interval valued multi fuzzy semi-pre $T_{1/2}$ space ($BIVMFSPT_{1/2}$), if every BIVMFGSPCS is a BIVMFSPCS in X .

Definition 1.26: Let (X, $\Im$) and (Y, σ) be BIVMFTSs. Then a map h: X→Y is called a

(i) bipolar interval valued multi fuzzy continuous (BIVMF continuous) mapping if h^{-1}(B) is BIVMFOS in X for all BIVMFOS B in Y.
(ii) a bipolar interval valued multi fuzzy closed mapping (BIVMFC mapping) if h(A) is a BIVMFCS in Y for each BIVMFCS A in X.
(iii) bipolar interval valued multi fuzzy semi-closed mapping (BIVMFSC mapping) if $h(A)$ is a BIVMFSCS in Y for each BIVMFCS A in X.
(iv) bipolar interval valued multi fuzzy preclosed mapping (BIVMFPC mapping) if $h(A)$ is a BIVMFPCS in Y for each BIVMFCS A in X.
(v) bipolar interval valued multi fuzzy semi-open mapping (BIVMFSO mapping) if $h(A)$ is a BIVMFSOS in Y for each BIVMFOS A in X.
(vi) bipolar interval valued multi fuzzy generalized semi-preopen mapping (BIVMFGSPO mapping) if $h(A)$ is a BIVMFGSPOS in Y for each BIVMFOS A in X.
(vii) bipolar interval valued multi fuzzy generalized semi-preclosed mapping (BIVMFGSPC mapping) if $h(A)$ is a BIVMFGSPCS in Y for each BIVMFCS A in X.

2. SOME PROPERTIES

Theorem 2.1: [16] Every BIVMFCS in (X,$\Im$) is a BIVMFGSPCS in (X,$\Im$).

Theorem 2.2: Every BIVMFC mapping is a BIVMFGSPC mapping.

Proof: Let X and Y be two BIVMFTSs. Assume that h : X→Y is a BIVMFC mapping. Let A be a BIVMFCS in X. Then h(A) is a BIVMFCS in Y. Since every BIVMFCS is a BIVMFGSPCS (by Theorem 2.1), h(A) is a BIVMFGSPCS in Y and hence h is a BIVMFGSPC mapping.

Remark 2.3: The converse of the above theorem 2.2 need not be true from the following example: Let X = {a, b}, Y = {u, v} and K_1 = {⟨a, [0.6, 0.6], [0.7, 0.7], [0.5, 0.5], [–0.5, –0.5], [–0.6,–0.6], [–0.4, –0.4] ⟩, ⟨b, [0.7, 0.7],[0.8, 0.8], [0.6, 0.6], [–0.6, –0.6], [–0.7, –0.7], [–0.5, –0.5] ⟩ }, L_1 = { ⟨ u, [0.5, 0.5], [0.6, 0.6], [0.4, 0.4], [–0.4, –0.4], [–0.5, –0.5], [–0.3, –0.3]⟩, ⟨v, [0.4, 0.4], [0.5, 0.5], [0.3, 0.3], [–0.3, –0.3], [–0.4, –0.4], [–0.2, –0.2]⟩}. Then $\Im = \{\overline{0}_X, K_1, \overline{1}_X\}$ and $\sigma = \{\overline{0}_Y, L_1, \overline{1}_Y\}$ are BIVMFT on X and Y respectively. Define a mapping h: $(X,\Im) \to (Y,\sigma)$ by h(a) = u and h(b) = v. Then h is a BIVMFGSPC mapping but not a BIVMFC mapping, since $K_1^{\,c}$ is a BIVMFCS in X but h($K_1^{\,c}$) = {⟨ u, [0.4, 0.4], [0.3, 0.3], [0.5, 0.5], [–0.5, –0.5], [–0.4, –0.4], [–0.6, –0.6] ⟩, ⟨v, [0.3, 0.3], [0.2, 0.2], [0.4, 0.4], [–0.4, –0.4], [–0.3, –0.3], [–0.5, –0.5] ⟩} is not a BIVMFCS in Y, because bivmfcl $(h(K_1^{\,c})) = L_1^{\,c} \neq h(K_1^{\,c})$.

Theorem 2.4: [16] Every BIVMFαCS in (X,$\Im$) is a BIVMFGSPCS in (X,$\Im$).

Theorem 2.5: Every BIVMFαC mapping is a BIVMFGSPC mapping.

Proof: Let X and Y be two BIVMFTSs. Assume that h: X→Y is a BIVMFαC mapping. Let A be a BIVMFCS in X. Then h(A) is a BIVMFαCS in Y. Since every BIVMFαCS is a BIVMFGSPCS (by Theorem 2.4), $h(A)$ is a BIVMFGSPCS in Y and hence h is a BIVMFGSPC mapping.

Remark 2.6: The converse of the above theorem 2.5 need not be true from the following example: Let X = {a, b}, Y = {u, v} and Then $\Im = \{\overline{0}_X, K_1, \overline{1}_X\}$ and $\sigma = \{\overline{0}_Y, L_1, \overline{1}_Y\}$ are BIVMFT on X and Y respectively. K_1 = {⟨a, [0.6, 0.6], [0.7, 0.7], [0.5, 0.5], [–0.5, –0.5], [–0.6, –0.6], [–0.4, –0.4] ⟩, ⟨ b, [0.7, 0.7], [0.8, 0.8], [0.6, 0.6], [–0.6, –0.6], [–0.7, –0.7], [–0.5, –0.5] ⟩ }, L_1 = { ⟨u, [0.5, 0.5] ,[0.6, 0.6], [0.4, 0.4], [–0.4, –0.4], [–0.5, –0.5], [–0.3, –0.3] ⟩, ⟨v, [0.4 ,0.4], [0.5, 0.5], [0.3, 0.3], [–0.3, –0.3], [–0.4, –0.4], [–0.2, –0.2] ⟩ }. Define a mapping $h:(X,\Im) \to (Y,\sigma)$ by $h(a) = u$ and $h(b) = v$. Then g is a BIVMFGSPC mapping but not a BIVMFαC mapping, since $K_1^{\,c}$ is a BIVMFCS in X but h($K_1^{\,c}$) = {⟨u, [0.4, 0.4], [0.5, 0.5], [0.3, 0.3], [–0.3, –0.3], [–0.4, –0.4], [–0.2, –0.2] ⟩, ⟨ v, [0.2, 0.2], [0.3, 0.3], [0.1, 0.1], [–0.1, –0.1], [–0.2, –0.2], [–0.05, –0.05] ⟩ }is not a BIVMFαCS in Y, because bivmbcl (bivmfint (bivmfcl $(h(K_1^{\,c})))) = L_1^c \not\subset h(K_1^{\,c})$.

Theorem 2.7[16]: Every BIVMFSCS in (X,$\Im$) is a BIVMFGSPCS in (X,$\Im$).

Theorem 2.8: Every BIVMFSC mapping is a BIVMFGSPC mapping.

Proof: Assume that h: X→Y is a BIVMFSC mapping, where X and Y be two BIVMFTSs. Let A be a BIVMFCS in X. Then $h(A)$ is a BIVMFSCS in Y. Since every BIVMFSCS is a BIVMFGSPCS (by Theorem 2.7), $h(A)$ is a BIVMFGSPCS in Y and hence h is a BIVMFGSPC mapping.

Remark 2.9: The converse of the above theorem 2.8 need not be true from the following example: Let X= {a, b}, Y= {u, v} and Then $\Im = \{\overline{0}_X, K_1, \overline{1}_X\}$ and $\sigma = \{\overline{0}_Y, L_1, \overline{1}_Y\}$ are BIVMFT on X and Y respectively. K_1 = { ⟨ a, [0.6, 0.6], [0.7, 0.7], [0.5, 0.5], [–0.5, –0.5], [–0.6, –0.6], [–0.4, –0.4] ⟩, ⟨ b, [0.7, 0.7], [0.8, 0.8], [0.6, 0.6], [–0.6, –0.6], [–0.7, –0.7], [–0.5, –0.5] ⟩ }, L_1 = { ⟨ u, [0.5, 0.5] ,[0.6, 0.6], [0.4, 0.4], [–0.4, –0.4], [–0.5, –0.5], [–0.3, –0.3]⟩, ⟨v, [0.4 ,0.4], [0.5, 0.5], [0.3, 0.3], [–0.3, –0.3], [–0.4, –0.4], [–0.2, –0.2]⟩}. Define a mapping $h:(X,\Im)\to(Y,\sigma)$ by $h(a)=u$ and $h(b)=v$. Then h is a BIVMFGSPC mapping but not a BIVMFSC mapping, sinceK_1^c is a BIVMFCS in X but h(K_1^c) = { ⟨ u, [0.4, 0.4], [0.5, 0.5], [0.3, 0.3], [–0.3, –0.3], [–0.4, –0.4], [–0.2,–0.2] ⟩, ⟨ v, [0.2, 0.2], [0.3, 0.3], [0.1, 0.1], [–0.1, –0.1], [–0.2, –0.2], [–0.05, –0.05] ⟩ } is not a BIVMFSCS in Y, because bivmfint (bivmfcl $(h(K_1{}^c))) = bivmfint(L_1{}^c) = L_1 \not\subset h(K_1{}^c)$.

Theorem 2.10[16]: Every BIVMFPCS in (X,$\Im$) is a BIVMFGSPCS in (X,$\Im$).

Theorem 2.11: Every BIVMFPC mapping is a BIVMFGSPC mapping.

Proof: Assume that h: X→Y is a BIVMFPC mapping, where X and Y be two BIVMFTSs. Let A be a BIVMFCS in X. Then $h(A)$ is a BIVMFPCS in Y. Since every BIVMFPCS is a BIVMFGSPCS (by Theorem 2.10), $h(A)$ is a BIVMFGSPCS in Y and hence h is a BIVMFGSPC mapping.

Remark 2.12: The converse of the above theorem 2.11 need not be true from the following example:

Let X = {a, b}, Y = {u, v} and Then $\Im = \{\overline{0}_X, K_1, \overline{1}_X\}$ and $\sigma = \{\overline{0}_Y, L_1, \overline{1}_Y\}$ are BIVMFT on X and Y respectively. K_1 = {⟨ a, [0.5, 0.5], [0.6, 0.6], [0.4, 0.4], [–0.4, –0.4], [–0.5, –0.5], [–0.3, –0.3] ⟩, ⟨ b, [0.3, 0.3], [0.4, 0.4], [0.2, 0.2], [–0.2, –0.2], [–0.3, –0.3], [–0.1, –0.1] ⟩}, L_1 = { ⟨ u, [0.5, 0.5] ,[0.6, 0.6], [0.4, 0.4], [–0.4, –0.4], [–0.5, –0.5], [–0.3, –0.3] ⟩,⟨ v, [0.6 ,0.6], [0.7, 0.7], [0.5, 0.5], [–0.5, –0.5], [–0.6, –0.6], [–0.4, –0.4 ⟩}. Define a mapping $h:(X,\Im)\to(Y,\sigma)$ by $h(a)=u$ and $h(b)=v$. Then h is a BIVMFGSPC mapping but not a BIVMFPC mapping, since $K_1{}^c$ is a BIVMFCS in X but h(K_1^c) = {⟨ u, [0.5, 0.5], [0.6, 0.6], [0.4, 0.4], [–0.4, –0.4], [–0.5, –0.5], [–0.3,–0.3] ⟩, ⟨v, [0.7, 0.7], [0.8, 0.8], [0.6, 0.6], [–0.6, –0.6], [–0.7, –0.7], [–0.5, –0.5] ⟩ } is not a BIVMFPCS in Y, because bivmfcl (bivmfint $(h(K_1{}^c))) = bivmfcl(L_1{}^c) \not\subset h(K_1{}^c)$.

Theorem 2.13: Let h: X→Y be a BIVMFGSPC mapping between two BIVMFTSs X and Y. Then for every BIVMFS A of X, $h(bivmfcl(A))$ is a BIVMFGSPCS in Y.

Proof: Let A be any BIVMFS in X. Then $bivmfcl(A)$ is a BIVMFCS in X. By hypothesis $h(bivmfcl(A))$ is a BIVMFGSPCS in Y.

Theorem 2.14: Let A be a BIVMFGCS in X. If a mapping h: X→Y from a BIVMFTS X onto a BIVMFTS Y is both BIVMF continuous and a BIVMFGSPC, then $h(A)$ is a BIVMFGSPCS in Y.

Proof: Let $h(A)\subseteq U$ where U is a BIVMFOS in Y. Then $A\subseteq h^{-1}(h(A))\subseteq h^{-1}(U)$, where $h^{-1}(U)$ is a BIVMFOS in X, by hypothesis. Since A is a BIVMFGCS, $bivmfcl(A)\subseteq h^{-1}(U)$ in X. This implies $h(bivmfcl(A))\subseteq h(h^{-1}(U)=U$. But $h(bivmfcl(A))$ is a BIVMFGSPCS in Y, since $bivmfcl(A)$ is a BIVMFCS in X and by hypothesis. Therefore $bivmfspcl\ (h(bivmfcl(A)))\subseteq U$. Now $bivmfspcl(h(A))$ $\subseteq bivmfspcl\ (h(bivmfcl(A)))\subseteq U$. Hence $h(A)$ is a BIVMFGSPCS in Y.

Theorem 2.15: A bijective mapping h: X→Y from a BIVMFTS X into a BIVMFTS Y is a BIVMFGSPC mapping if and only if for every BIVMFS B of Y and for every BIVMFOS U containing $h^{-1}(B)$, there is a BIVMFGSPOS A of Y such that $B \subseteq A$ and $h^{-1}(A) \subseteq U$.

Proof: Necessity. Let B be any BIVMFS in Y. Let U be a BIVMFOS in X such that $h^{-1}(B) \subseteq U$. Then U^c is a BIVMFCS in X. By hypothesis $h(U^c)$ is a BIVMFGSPCS in Y. Let $A = (h(U^c))^c$. Then A is a BIVMFGSPOS in Y and $B \subseteq A$. Now $h^{-1}(A) = h^{-1}(h(U^c))^c = (h^{-1}(h(U^c)))^c \subseteq U$.

Sufficiency. Let A be any BIVMFCS in X. Then A^c is a BIVMFOS in X and $h^{-1}(h(A^c)) \subseteq A^c$ where $h(A)$ is a BIVMFS in Y. By hypothesis, there exists a BIVMFGSPOS B in Y such that $h(A^c) \subseteq B$ and $h^{-1}B \subseteq A^c$. Therefore $A \subseteq (h^{-1}(B))^c$. Hence $B^c \subseteq h(A) \subseteq h(h^{-1}(B))^c \subseteq B^c$. This implies that $h(A) = B^c$. Since $\overline{B}^c$ is a BIVMFGSPCS in Y, $h(A)$ is a BIVMFGSPCS in Y. Hence h is a BIVMFGPC mapping.

Theorem 2.16: Let X, Y and Z be BIVMFTSs. If h: X→Y is a BIVMFC mapping and g: Y→Z is a BIVMFGSPC mapping, then g•h is a BIVMFGSPC mapping.

Proof: Let A be a BIVMFCS in X. Then h(A) is a BIVMFCS in Y, by hypothesis. Since g is a BIVMFGSPC mapping, g(h(A)) is a BIVMFGSPCS in Z. Therefore g•h is a BIVMFGSPC mapping.

Theorem 2.17: Let h: X→Y be a bijection from a BIVMFTS X to a BIVMFSPT$_{1/2}$ space Y. Then the following statements are equivalent:
- (i) h is a BIVMFGSPC mapping,
- (ii) bivmfspcl $(h(A)) \subseteq h(bivmfcl(A))$ for each BIVMFS A of X,
- (iii) $h^{-1}(bivmfspcl(B)) \subseteq$ bivmfcl $(h^{-1}(B))$ for every BIVMFS B of Y.

Proof: $(i) \Rightarrow (ii)$ Let A be a BIVMFS in X. Then $bivmfcl(A)$ is a BIVMFCS in X. (i) implies that $h(bivmfcl(A))$ is a BIVMFGSPC in Y. Since Y is a BIVMFSPT$_{1/2}$ space, $h(bivmfcl(A))$ is a BIVMFSPCS in Y. Therefore $bivmfspcl(h(bivmfcl(A))) = h(bivmfcl(A))$. Now Hence $bivmspcl(h(A)) \subseteq h(bivmfcl(A))$ for each BIVMFS A of X. $(ii) \Rightarrow (i)$ Let A be any BIVMFCS in X. Then BIVMFcl $(A) = A$. (ii) implies that $bivmfspcl\ (h(A)) \subseteq h(bivmfcl(A)) = h(A)$. But $h(A) \subseteq bivmfspcl(h(A))$. Therefore $bivmfspcl(h(A)) = h(A)$. This implies $h(A)$ is a BIVMFSPC in Y. Since every BIVMFSPCS is a BIVMFGSPCS, $h(A)$ is a BIVMFGSPCS in Y. Hence h is a BIVMFGSPC mapping. $(ii) \Rightarrow (iii)$ Let B be a BIVMFS in Y. Then $h^{-1}(B)$ is a BIVMFS in X. Since g is onto, $bivmfspcl(B) = bivmfspcl(h(h^{-1}(B)))$ and (ii) implies $bivmfspcl\ (h(h^{-1}(B))) \subseteq h(bivmfcl(h^{-1}(B)))$. Therefore we have $bivmfspcl\ (B) \subseteq h(bivmfcl\ (h^{-1}(B)))$. Now $h^{-1}(bivmfspcl(B)) \subseteq h^{-1}(h(bivmfcl(h^{-1}(B)))) = bivmfcl(h^{-1}(B))$, since h is one to one. Hence $h^{-1}(bivmfspcl(B)) \subseteq bivmfcl(h^{-1}(B))$.
$(iii) \Rightarrow (ii)$ Let A be any BIVMFS in X. Then $h(A)$ is a BIVMFS in Y. Since h is one to one, (iii) implies that $h^{-1}(bivmfspcl\ (h(A)) \subseteq bivmfcl(h^{-1}(h(A))) = bivmfcl(A)$. Therefore $h(h^{-1}(bivmfspcl(h(A)))) \subseteq h(bivmfcl(A))$. Since h is onto $bivmfspcl(h(A)) = h(h^{-1}(bivmfspcl(h(A)))) \subseteq h(bivmfcl(A))$.

Theorem 2.18: Let h: X→Y be bijective mapping, where X is a BIVMFTS and Y is a BIVMFSPT$_{1/2}$ space. Then the following statements are equivalent:
- (i) h is a BIVMFGSPC mapping,
- (ii) h is a BIVMFGSPO mapping,
- (iii) $h(bivmfint(B)) \subseteq bivmfcl(bivmfint(bivmfcl(h(B))))$ for every BIVMFS B in X.

Proof: $(i) \Leftrightarrow (ii)$ is obvious.

$(ii) \Rightarrow (iii)$ Let B be a BIVMFS in X. Then $bivmfint(B)$ is a BIVMFOS in X. By hypothesis $h(bivmfint(B))$ is a BIVMFGSPOS in Y. Since Y is a BIVMFSPT$_{1/2}$ space, $h(bivmfint(B))$ is a BIVMFSPOS in Y. Therefore $h(bivmfint\ (B)) \subseteq bivmfcl(bivmfint(bivmfcl(h(bivmfint(B))))) \subseteq bivmfcl(bivmfint(bivmfcl(h(B))))$.
$(iii) \Rightarrow (i)$ Let A be a BIVMFCS in X. Then A^c is a BIVMFOS in X. By hypothesis, $h(bivmfint(A^c)) = h(A^c) \subseteq bivmfcl(bivmfint(bivmfcl(h(A^c))))$.

That is $bivmfint(bivfcl(bivmfint(h(A)))) \subseteq h(A)$. This implies $h(A)$ is a BIVMFβCS in Y and hence a BIVMFGSPCS in Y. Therefore h is a BIVMFGSPC mapping.

Theorem 2.19: Let h: X→Y be bijective mapping, where X is a BIVMFTS and Y is a BIVMFSPT$_{1/2}$ space. Then the following statements are equivalent:
(i) h is a BIVMFGSPC mapping,
(ii) $h(B)$ is a BIVMFGSPCS in Y for every BIVMFCS B in X,
(iii) $bivmfint(bivmfcl(bivmfint(h(B)))) \subseteq h(bivmfcl(B))$ for every BIVMFS B in X.

Proof: $(i) \Leftrightarrow (ii)$ is obvious.
$(ii) \Rightarrow (iii)$ Let B be a BIVMFS in X. Then $bivmfcl(B)$ is a BIVMFCS in X. By hypothesis $h(bivmfcl(B))$ is a BIVMFGSPCS in Y. Since Y is a BIVMFSPT$_{1/2}$ space, $h(bivmfcl(B))$ is a BIVMFSPCS in Y. Therefore $h(bivmfcl\ (B)) \supseteq bivmfint(bivmfcl(bivmfint(h(bivmfcl(B))))) \supseteq bivmfint(bivmfcl(bivmfint(h(B))))$.
$(iii) \Rightarrow (i)$ Let A be a BIVMFCS in X. By hypothesis, $h(bivmfcl(A)) = h(A) \supseteq bivmfint(bivmfcl(bivmfint(h(A))))$. This implies $h(A)$ is a BIVMFβCS in Y and hence a BIVMFGSPCS in Y. Therefore h is a BIVMFGSPC mapping.

Theorem 2.20: Let X and Y be BIVMFTSs. A mapping h: X→Y is a BIVMFGSPC mapping if $h(bivmfspint\ (A)) \subseteq bivmfspint(h(A))$ for every $A \subseteq$ X.

Proof: Let A be a BIVMFOS in X. Then $bivmfint(A) = A$. Now $h(A) = h(bivmfint(A)) \subseteq h(bivmfspint(A)) \subseteq bivmfspint\ (h(A))$, by hypothesis. But $bivmfspint\ (h(A)) \subseteq h(A)$. Therfore $h(A)$ is a BIVMFSPOS in Y. That is $h(A)$ is a BIVMFGSPOS in Y. Hence h is a BIVMFGSPC mapping, by theorem 2.18.

Theorem 2.21: Let X be a BIVMFTS and Y be a BIVMFSPT$_{1/2}$ space. Let h: X→Y be bijection. Then the following statements are equivalent:
(i) h is a BIVMFGSPC mapping,
(ii) $h(bivmfint(A) \subseteq bivmfspint(h(A))$ for each BIVMFS A of X,
(iii) $bivmfint(h^{-1}(B)) \subseteq h^{-1}(bivmfspint(B))$ for every BIVMFS B of Y.

Proof: $(i) \Rightarrow (ii)$ Let h be a BIVMFGSPC mapping. Let A be any BIVMFS in X. Then $bivmfint(A)$ is a BIVMFOS in X. Now $h(bivmfint(A))$ is a BIVMFGSPOS in Y, by theorem 2.18. Since Y is a BIVMFSPT$_{1/2}$ space, $h(bivmfint(A))$ is a BIVMFSPOS in Y. Therefore $bivmfspint(h(bivmfint(A))) = h(bivmfint(A))$.
Now $h(bivmfint(A)) = bivmfspint(h(bivmfint(A))) \subseteq bivmfspint(h(A))$.
$(ii) \Rightarrow (iii)$ Let B be a BIVMFS in Y. Then $h^{-1}(B)$ is a BIVMFS in X. By (ii), $h(bivmfint(h^{-1}(B))) \subseteq bivmfspint(h(h^{-1}(B))) \subseteq bivmfspint(B)$ .Now $bivmfint(h^{-1}(B)) \subseteq h^{-1}(h(bivmfint(h^{-1}(B)))) \subseteq h^{-1}(bivmfspint(B))$. $(iii) \Rightarrow (i)$ Let A be a BIVMFOS in X. Then $bivmfint(A) = A$ and $h(A)$ is a BIVMFS in Y. By (iii), $bivmfint(h^{-1}(h(A))) \subseteq h^{-1}(bivmfspint(h(A)))$ .Now $A = bivmfint(A) \subseteq bivmfint(h^{-1}(h(A))) \subseteq h^{-1}(bivmfspint(h(A)))$ .Therefore $h(A) \subseteq h(h^{-1}(bivmfspint(h(A)))) \subseteq bivmfspint(h(A)) \subseteq h(A)$. Therefore $bivmfspint(h(A)) = h(A)$ is a BIVMFSPOS in Y and hence a BIVMFGSPOS in Y. Thus h is a BIVMFGSPC mapping, by theorem 2.18.

CONCLUSION

We conclude that, every BIVMF continuous mapping, BIVMFα continuous mapping, BIVMFβ continuous mapping, BIVMFP continuous mapping, BIVMFSP continuous mapping are an BIVMFGSP continuous mapping. Also some equivalent condition Theorems are proved in this paper. Using this concept, we can develop some new theorems and properties.

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