

LEAP GOURAVA INDICES OF CERTAIN WINDMILL GRAPHS

V. R. KULLI*

Department of Mathematics, Gulbarga University, Gulbarga - 585 106, India.

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ABSTRACT

Recently, some leap Zagreb indices of a graph based on the second degrees of vertices were introduced. In this paper, we introduce some leap Gourava indices of a graph based on the second degrees of vertices. We also compute the first and second hyper leap Gourava indices, sum connectivity leap Gourava index, product connectivity leap Gourava index, general first and second leap Gourava indices of certain windmill graphs such as Dutch windmill graph, Kulli cycle windmill graph, Kulli path windmill graph and French windmill graph.

Keywords: Leap Gourava indices, sum connectivity leap Gourava index, product connectivity leap Gourava index, windmill graphs.

Mathematics Subject Classification: 05C05, 05C07, 05C12, 05C76.

I. INTRODUCTION

Let G be a finite, simple, connected graph with vertex set $V(G)$ and edge set $E(G)$. For a vertex v , the degree $d_G(v)$ is the number of vertices adjacent to v . The distance $d(u, v)$ between any two vertices u and v is the length of shortest path connecting u and v . For a positive integer k , the open k -neighborhood $N_k(v)$ of a vertex v in a graph G is defined as $N_k(v/G) = \{u \in V(G) : d(u, v) = k\}$. The k -distance degree $d_k(v)$ of a vertex v in G is defined as the number of k neighbors of v in G . We refer to [1] for undefined graph terminology and notation.

A graph index or a topological index is a numerical parameters mathematically derived from the graph structure [2]. It is a graph invariant. The graph indices have their applications in various disciplines of Science and Technology, see [3, 4].

Recently, some Gourava indices were introduced and studied such as hyper Gourava indices [5], sum connectivity Gourava index [6], product connectivity Gourava index [7], general first and second Gourava indices [8], multiplicative Gourava indices [9].

In [5], Kulli defined the first and second Gourava indices of a graph G as

$$GO_1(G) = \sum_{uv \in E(G)} [d_G(u) + d_G(v) + d_G(u)d_G(v)],$$

$$GO_2(G) = \sum_{uv \in E(G)} (d_G(u) + d_G(v))(d_G(u)d_G(v)).$$

The first and second leap Gourava indices of a graph G are defined as [10]

$$LGO_1(G) = \sum_{uv \in E(G)} [d_2(u) + d_2(v) + d_2(u)d_2(v)],$$

$$LGO_2(G) = \sum_{uv \in E(G)} (d_2(u) + d_2(v))(d_2(u)d_2(v)).$$

We introduce the first and second hyper leap Gourava indices of a graph G , defined as

$$HLGO_1(G) = \sum_{uv \in E(G)} [d_2(u) + d_2(v) + d_2(u)d_2(v)]^2,$$

$$HLGO_2(G) = \sum_{uv \in E(G)} [(d_2(u) + d_2(v))d_2(u)d_2(v)]^2.$$

Corresponding Author: V. R. Kulli*

Motivated by the definitions of the sum connectivity Gourava index [6] and product connectivity Gourava index [7] of a graph as follows:

The sum connectivity leap Gourava index of a graph G is defined as

$$SLGO(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{d_2(u) + d_2(v) + d_2(u)d_2(v)}},$$

$$PLGO(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{(d_2(u) + d_2(v))(d_2(u)d_2(v))}}.$$

We continue this generalization and define the general first and second leap Gourava indices of a graph G as

$$LGO_1^a(G) = \sum_{uv \in E(G)} [d_2(u) + d_2(v) + d_2(u)d_2(v)]^a, \quad (1)$$

$$LGO_2^a(G) = \sum_{uv \in E(G)} [(d_2(u) + d_2(v))d_2(u)d_2(v)]^a, \quad (2)$$

where a is a real number.

Recently, some leap indices were studied, for example, in [11, 12, 13, 14, 15, 16, 17].

In this paper, we compute leap Gourava indices, hyper leap Gourava indices, sum connectivity leap Gourava index, product connectivity leap Gourava index, general leap Gourava indices of certain windmill graphs.

2. RESULTS FOR DUTCH WINDMILL GRAPHS

The Dutch windmill graph D_n^m , $m \geq 2$, $n \geq 5$ is the graph obtained by taking m copies of the cycle C_n with a vertex in common [18]. The graph D_n^m is shown in Figure 1. The Dutch windmill graph D_3^m is called a friendship graph.

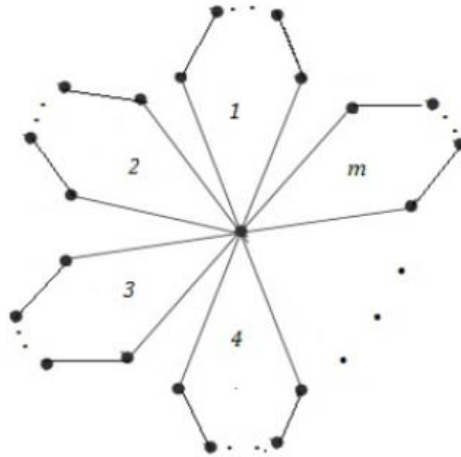


Figure-1: Dutch windmill graph D_n^m

Let G be a Dutch windmill graph D_n^m with $1 + m(n - 1)$ vertices and mn edges, $m \geq 2$, $n \geq 2$. Then G has three types of 2-distance degree of edges as given in Table 1.

$d_2(u), d_2(v) \setminus uv \in E(D)$	$(2m, 2m)$	$(2m, 2)$	$(2, 2)$
Number of edges	$2m$	$2m$	$m(n - 4)$

Table-1: 2-distance degree edge partition of D_n^m

Theorem 1: Let D be the graph of a Dutch windmill graph D_n^m . Then

$$LGO_1^a(D_n^m) = (4m + 4m^2)2m + (6m + 2)^a 2m + 8^a m(n - 4). \quad (3)$$

Proof: From equation (1) and by using Table 1, we deduce

$$\begin{aligned} LGO_1^a(D_n^m) &= \sum_{uv \in E(D)} [d_2(u) + d_2(v) + d_2(u)d_2(v)]^a \\ &= (2m + 2m + 2m \times 2m)^a 2m + (2m + 2 + 2m \times 2)^a 2m + (2 + 2 + 2 \times 2)^a m(n - 4) \\ &= (4m + 4m^2)^a 2m + (6m + 2)^a 2m + 8^a m(n - 4). \end{aligned}$$

We establish the following results by using Theorem 1.

Corollary 1.1: The first leap Gourava index of D_n^m is given by

$$LGO_1(D_n^m) = 8mn + 8m^3 + 20m^2 - 28m.$$

Corollary 1.2: The first hyper leap Gourava index of D_n^m is given by

$$HLGO_1(D_n^m) = 64mn + 32m^5 + 64m^4 + 104m^3 + 48m^2 - 248m.$$

Corollary 1.3: The sum connectivity leap Gourava index of D_n^m is given by

$$SLGO(D_n^m) = \frac{m}{\sqrt{m+m^2}} + \frac{2m}{\sqrt{6m+2}} + \frac{m(m-4)}{\sqrt{8}}$$

Proof: Put $a = 1, 2, -1/2$ in equation (3), we get the desired results.

Theorem 2: Let D be the graph of a Dutch windmill graph D_n^m . Then

$$LGO_2^a(D_n^m) = (16m^3)^a 2m + [8m(m+1)]^a 2m + 16^a m(n-4). \quad (4)$$

Proof: By using equation (2) and Table 1, we derive

$$\begin{aligned} LGO_2^a(D_n^m) &= \sum_{uv \in E(D)} [(d_2(u) + d_2(v))(d_2(u)d_2(v))]^a \\ &= [(2m+2m)(2m \times 2m)]^a 2m + [(2m+2)(2m \times 2)]^a 2m + [(2+2)(2 \times 2)]^a m(n-4) \\ &= (16m^3)^a 2m + [8m(m+1)]^a 2m + 16^a m(n-4). \end{aligned}$$

We obtain the following results by using Theorem 2.

Corollary 2.1: The second leap Gourava index of D_n^m is given by

$$LGO_2(D_n^m) = 16mn + 32m^4 + 16m^3 + 16m^2 - 64m.$$

Corollary 2.2: The second hyper leap Gourava index of D_n^m is given by

$$HLGO_2(D_n^m) = 256mn + 512m^7 + 128m^5 + 256m^4 + 128m^3 - 1024m.$$

Corollary 2.3: The product connectivity leap Gourava index of D_n^m is given by

$$PLGO(D_n^m) = \frac{1}{2\sqrt{m}} + \frac{m}{\sqrt{2m(m+1)}} + \frac{m(n-4)}{4}$$

Proof: put $a = 1, 2, -1/2$ in equation (4), we obtain the desired results.

3. RESULTS FOR KULLI CYCLE WINDMILL GRAPHS

The Kulli cycle windmill graph [19] is the graph obtained by taking m copies of the graph $K_1 + C_n$ for $n \geq 3$ with a vertex K_1 in common and it is denoted by C_{n+1}^m . This graph is presented in Figure 2.

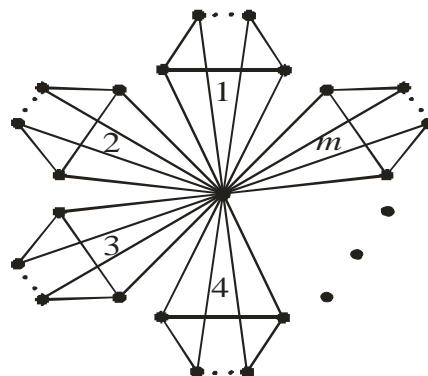


Figure-2: Kulli cycle windmill graph C_{n+1}^m

Let $C = C_{n+1}^m$ be a Kulli cycle windmill graph with $mn+1$ vertices and $2mn$ edges, $m \geq 2$, $n \geq 5$. Then C has two types of 2-distance degree of edges as given in Table 2.

$d_2(u), d_2(v) \setminus uv \in E(C)$	$(0, mn-2)$	$(mn-2, mn-2)$
Number of edges	mn	mn

Table-2: 2-distance degree edge partition of C_{n+1}^m

Theorem 3: The general first leap Gourava index of C_{n+1}^m is given by

$$LGO_1^a(C_{n+1}^m) = (mn-2)^a mn(1+m^a n^a). \quad (5)$$

Proof: Let $C = C_{n+1}^m$. By using equation (1) and Table 2, we obtain

$$\begin{aligned} LGO_1^a(C_{n+1}^m) &= \sum_{uv \in E(C)} [d_2(u) + d_2(v) + d_2(u)d_2(v)]^a \\ &= [0 + mn - 2 + 0 \times (mn - 2)]^a mn + [(mn - 2) + (mn - 2) + (mn - 2)(mn - 2)]^a mn \\ &= (mn - 2)^a mn + [2(mn - 2) + (mn - 2)^2]^a mn \\ &= (mn - 2)^a mn(1 + m^a n^a) \end{aligned}$$

We obtain the following results by using Theorem 3.

Corollary 3.1: The first leap Gourava index of C_{n+1}^m is

$$LGO_1(C_{n+1}^m) = (mn - 2)(1 + mn)mn.$$

Corollary 3.2: the first hyper leap Gourava index of C_{n+1}^m is

$$HLGO_1(C_{n+1}^m) = (mn - 2)^2 (1 + m^2 n^2)mn.$$

Corollary 3.3: The sum connectivity leap Gourava index of C_{n+1}^m is

$$SLGO(C_{n+1}^m) = \frac{mn}{\sqrt{mn-2}} \times \left(1 + \frac{1}{\sqrt{mn}}\right).$$

Proof: Put $a = 1, 2, -1/2$ in equation (5), we obtain the desired results.

Theorem 4: The general second leap Gourava index of C_{n+1}^m is given by

$$LGO_2^a(C_{n+1}^m) = [2(mn-2)^3]^a mn. \quad (6)$$

Proof: Let $C = C_{n+1}^m$. By using equation (2) and Table 2, we deduce

$$\begin{aligned} LGO_2^a(C_{n+1}^m) &= \sum_{uv \in E(C)} [(d_2(u) + d_2(v))(d_2(u)d_2(v))]^a \\ &= [(0 + mn - 2)0 \times (mn - 2)]^a mn + [(mn - 2 + mn - 2)(2mn - 2)(mn - 2)]^a mn \\ &= [2(mn - 2)^3]^a mn. \end{aligned}$$

The following results are obtained by using Theorem 4.

Corollary 4.1: The second leap Gourava index of C_{n+1}^m is

$$LGO_2(C_{n+1}^m) = 2(mn - 2)^3 mn.$$

Corollary 4.2: The second hyper leap Gourava index of C_{n+1}^m is

$$HLGO_2(C_{n+1}^m) = 4(mn - 2)^6 mn.$$

Corollary 4.3: The product connectivity leap Gourava index of C_{n+1}^m is

$$PLGO(C_{n+1}^m) = \frac{mn}{(mn-2)\sqrt{2(mn-2)}}.$$

Proof: Put $a = 1, 2, -1/2$ in equation (6), we get the desired results.

4. RESULTS FOR KULLI PATH WINDMILL GRAPHS

The Kulli path windmill graph [20] is the graph obtained by taking m copies of the graph $K_1 + P_n$ with a vertex K_1 in common and it is denoted by P_{n+1}^m . This graph is shown in Figure 3. The Kulli path windmill graph P_3^m is a friendship graph.

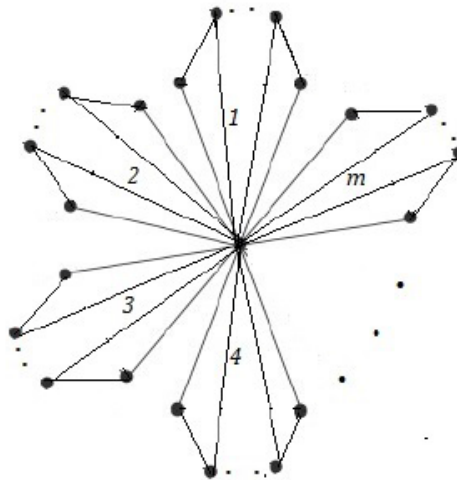


Figure-3: Kulli path windmill graph P_{n+1}^m

Let $P = P_{n+1}^m$, $m \geq 2$, $n \geq 5$. Then P has $mn+1$ vertices and $2mn - m$ edges. The graph P has four types of 2-distance degree of edges as given Table 3.

$d_2(u), d_2(v) \setminus uv \in E(G)$	$(0, mn-2)$	$(0, mn-3)$	$(mn-2, mn-3)$	$(mn-3, mn-3)$
Number of edges	$2m$	$mn-2m$	$2m$	$mn-3m$

Table-3: 2-distance degree edge partition of P_{n+1}^m

Theorem 5: The general first leap Gourava index of P_{n+1}^m is

$$LGO_1^a(P_{n+1}^m) = (mn-2)^2 2m + (mn-3)^a (mn-2m) + (m^2 n^2 - 3mn + 1)^a 2m + (m^2 n^2 - 4mn + 3)^a (mn-3m). \quad (7)$$

Proof: Let $P = P_{n+1}^m$, By using equation (1) and Table 3, we deduce

$$\begin{aligned} LGO_1^a(P_{n+1}^m) &= \sum_{uv \in E(P)} [d_2(u) + d_2(v) + d_2(u)d_2(v)]^a \\ &= [0 + mn - 2 + 0(mn-2)]^a 2m + [0 + mn - 3 + 0(mn-3)]^a (mn-2m) \\ &\quad + [mn-2 + mn-3 + (mn-2)(mn-3)]^a 2m + [mn-3 + mn-3(mn-3)(mn-3)]^a (mn-3m) \\ &= (mn-2)^2 2m + (mn-3)^a (mn-2m) + (m^2 n^2 - 3mn + 1)^a 2m + (m^2 n^2 - 4mn + 3)^a (mn-3m) \end{aligned}$$

We obtain the following results by using Theorem 5.

Corollary 5.1: The first leap Gourava index of P_{n+1}^m is

$$LGO_1(P_{n+1}^m) = m^3 n^3 - m^3 n^2 - 3m^2 n^2 + 6m^2 n - 5m.$$

Corollary 5.2: The first hyper Gourava index of P_{n+1}^m is

$$HLGO_1(P_{n+1}^m) = (mn-2)^2 2m + (mn-3)^2 (mn-2m) + (m^2 n^2 - 3mn + 1)^2 2m + (m^2 n^2 - 4mn + 3)^2 (mn-3m).$$

Corollary 5.3: The sum connectivity leap Gourava index of P_{n+1}^m

$$SLGO(P_{n+1}^m) = \frac{2m}{\sqrt{mn-2}} + \frac{mn-2m}{\sqrt{mn-3}} + \frac{2m}{\sqrt{m^2 n^2 - 3mn + 1}} + \frac{mn-3m}{\sqrt{m^2 n^2 - 4mn + 3}}.$$

Proof: Put $a = 1, 2, -1/2$ in equation (7), we get the desired results.

Theorem 6: The general second leap Gourava index of P_{n+1}^m is

$$LGO_2^a(P_{n+1}^m) = [(2mn-5)(mn-2)(mn-3)]^a 2m + [2(mn-3)^3]^a (mn-3m). \quad (8)$$

Proof: Let $P = P_{n+1}^m$. From equation (2) and by using Table 3, we obtain

$$\begin{aligned} LGO_2^a(P_{n+1}^m) &= \sum_{uv \in E(P)} [(d_2(u) + d_2(v))(d_2(u)d_2(v))]^a \\ &= [(0 + mn - 2)0(mn - 2)]^a 2m + [(0 + mn - 3)0(mn - 3)]^a (mn - 2m) \\ &\quad + [(mn - 2 + mn - 3)(mn - 2)(mn - 3)]^a 2m + [(mn - 3 + mn - 3)(mn - 3)(mn - 3)]^a (mn - 3m) \\ &= [(2mn - 5)(mn - 2)(mn - 3)]^a 2m + [2(mn - 3)^3]^a (mn - 3m). \end{aligned}$$

Corollary 6.1: The second leap Gourava index of P_{n+1}^m is

$$LGO_2(P_{n+1}^m) = (2mn - 5)(mn - 2)(mn - 3)2m + 2(mn - 3)^3(mn - 3)$$

Corollary 6.2: The second hyper leap Gourava index of P_{n+1}^m is

$$HLGO_2(P_{n+1}^m) = (2mn - 5)^2(mn - 2)^2(mn - 3)^2 2m + 4(mn - 3)^6(mn - 3m).$$

Corollary 6.3: The product connectivity leap Gourava index of P_{n+1}^m is

$$PLGO(P_{n+1}^m) = \frac{2m}{\sqrt{(2mn-5)(mn-2)(mn-3)}} + \frac{mn-3m}{(mn-3)\sqrt{2(mn-3)}}.$$

Proof: Put $a = 1, 2, -\frac{1}{2}$ in equation (8), we get the desired results.

5. RESULTS FOR FRENCH WINDMILL GRAPHS

The French windmill graph F_n^m is the graph obtained by taking $m \geq 3$ copies of K_n , $n \geq 3$ with a vertex in common [18]. The graph F_n^m is presented in Figure 4. The French windmill graph F_3^m is called a friendship graph.

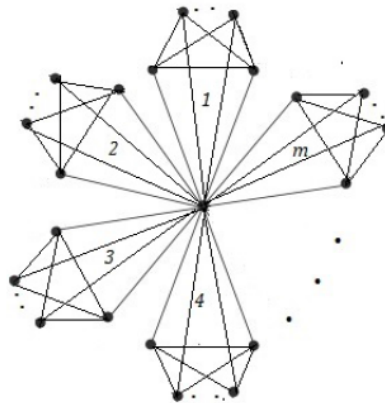


Figure-4: French windmill graph F_n^m

Let F be a French windmill graph F_n^m . Then F has $1 + m(n - 1)$ vertices and $\frac{1}{2}mn(n - 1)$ edges, $m \geq 2, n \geq 2$. In F , there are two types of the 2-distance degree of edges as given in Table 4.

$d_2(u), d_2(v) \setminus uv \in E(F)$	$(0, (n - 1)(m - 1))$	$((n - 1)(m - 1), (n - 1)(m - 1))$
Number of edges	$m(n - 1)$	$\frac{1}{2}m(n - 1)(n - 2)$

Table-4: 2-distance degree edge partition of F .

Theorem 7: The general first leap Gourava index of F_n^m is

$$LGO_1^a(F_n^m) = [(n-1)(m-1)]^a m(n-1) + [2(n-1)(m-1) + (n-1)^2(m-1)^2]^a \frac{1}{2} m(n-1)(n-2). \quad (9)$$

Proof: Let $F = F_n^m$. From equation (1) and by using Table 4, we have

$$\begin{aligned} LGO_1^a(F_n^m) &= \sum_{uv \in E(F)} [d_2(u) + d_2(v) + d_2(u)d_2(v)]^a \\ &= [0 + (n-1)(m-1) + 0(n-1)(m-1)]^a m(n-1) \\ &\quad + [(n-1)(m-1) + (n-1)(m-1) + (n-1)(m-1)(n-1)(m-1)]^a \frac{1}{2} m(n-1)(n-2) \\ &= [(n-1)(m-1)]^a m(n-1) + [2(n-1)(m-1) + (n-1)^2(m-1)^2]^a \frac{1}{2} m(n-1)(n-2). \end{aligned}$$

The following results are obtained by using Theorem 7.

Corollary 7.1: The first leap Gourava index of F_n^m is

$$LGO_1(F_n^m) = m(m-1)(n-1)^3 \left[1 + \frac{1}{2}(m-1)^2(n-2) \right].$$

Corollary 7.2: The first hyper leap Gourava index of F_n^m is

$$HLGO_1(F_n^m) = m(m-1)^2(n-1)^3 + [2(m-1)(n-1) + (m-1)^2(n-1)^2] \frac{1}{2} m(n-1)(n-2).$$

Corollary 7.3: The sum connectivity leap Gourava index of F_n^m is

$$SLGO(F_n^m) = \frac{m(n-1)}{\sqrt{(n-1)(m-1)}} + \frac{m(n-1)(n-2)}{2\sqrt{2(n-1)(m-1) + (n-1)^2(m-1)^2}}.$$

Proof: Put $a = 1, 2, -1/2$ in equation (9), we get the desired results.

Theorem 8: The general second leap Gourava index of F_n^m is

$$LGO_2^a(F_n^m) = [2(n-1)^3(m-1)^3]^a \frac{1}{2} m(n-1)(n-2). \quad (10)$$

Proof: Let $F = F_n^m$. By using equation (2) and Table 4, we obtain

$$\begin{aligned} LGO_2^a(F_n^m) &= \sum_{uv \in E(F)} [(d_2(u) + d_2(v))(d_2(u)d_2(v))]^a \\ &= [(0 + (n-1)(m-1))0(n-1)(m-1)]^a m(n-1) \\ &\quad + [((n-1)(m-1) + (n-1)(m-1))(n-1)(m-1)(n-1)(m-1))]^a \frac{1}{2} m(n-1)(n-2) \\ &= [2(n-1)^3(m-1)^3]^a \frac{1}{2} m(n-1)(n-2). \end{aligned}$$

We obtain the following results by using theorem 8.

Corollary 8.1: The second leap Gourava index of F_n^m is

$$LGO_2(F_n^m) = m(n-1)^4(m-1)^3(n-2).$$

Corollary 8.2: The second hyper leap Gourava index of F_n^m is

$$HLGO_2(F_n^m) = 2m(n-1)^7(m-1)^6(n-2).$$

Corollary 8.3: The product connectivity leap Gourava index of F_n^m is

$$PLGO(F_n^m) = \frac{m(n-2)}{2(m-1)\sqrt{2(n-1)(m-1)}}.$$

Proof: Put $a = 1, 2, -1/2$ in equation (10), we get the desired results.

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