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AN L-FUZZY α- SUPRACONTINUOUS IN α-SUPRATOPOLOGICAL TM- SYSTEM

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ABSTRACT

In 2010, Tamilarasi and Megalai introduced a new class of algebras called as TM-algebras. In this paper, we discuss the notion of An L-Fuzzy α -Supracontinuous in α -Supratopological TM-system.

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Key words: BCK/BCI Algebra, TM-Algebra, Fuzzy set, Fuzzy Topology.

1. INTRODUTION

Recently in 2010, Tamilarasi and Megalai introduced a new class of algebras, called TM-algebras [12]. In their paper they investigated the relationship between TM-algebras and other algebras. They claimed that the TM-algebra is a generalization of BCH/BCK/BCI and Q algebras.

In 1965, L.A.Zadeh [14] introduced the notion of fuzzy sets, to evaluate the modern concept of uncertainty in real physical world. In the notion of fuzzy sets, the boundaries are not crisp or sharp but flexible. In 1967, J.A.Goguen [8] introduced the concept of L- fuzzy sets.

The theory of fuzzy topological spaces is developed by Chang [6], Wong [13], Lowen [9] and others. Mashhour *et al* [10] introduced the concepts of supratopological spaces. In 1987 M.E.Abd El-Monsef and A.E.Ramadan [11] introduced fuzzy supratopological spaces. R.Devi, S.Sampathkumar and M.Caldas [7] introduced supra α - open sets and S α - continuous functions.

In [1], we studied Fuzzy Topological subsystem on a TM-algebra. In [2], we studied L- Fuzzy Topological TM-system. In [3], we studied L- Fuzzy Topological TM-system. In [4], [5] we studied Fuzzy Supratopological TM-system, Fuzzy α -supracontinuous functions. In this paper, we discuss the notion of An L-fuzzy α -supracontinuous in α -supratopological TM-system and investigate some simple properties.

2. PRELIMINARIES

In this section we recall some basic definitions that are required in the sequel.

Definition 2.1: Let X be a non-empty set. A mapping $\mu: X \to L$ is called an L-fuzzy set of X, where L is a complete lattice, with sup 1 and inf 0.

Definition 2.2: Let A and B be any two fuzzy sets in a non-empty set X.

- (1) The union of A and B denoted by, AUB is defined to be the L-Fuzzy set $(AUB)(x)=\mu_A(x)V\mu_B(x)$ for all $x\in X$.
- (2) The intersection of A and B, denoted by, $A \cap B$ is defined to be the L- fuzzy set $(A \cap B)(x) = \mu_A(x) \wedge \mu_B(x)$ for all $x \in X$.
- (3) $A \subset B \Rightarrow A(x) \leq (B)x$ for all $x \in X$.
- (4) The Complement of A is defined to be A' (x) = 1- A(x) for all $x \in X$

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Definition 2.3: A lattice is a partially ordered set in which any two elements have a least upper bound and a greatest lower bound.

Definition 2.4: A lattice L is called a complete lattice if every subset $A=\{a_{\alpha}\}$ of L has a sup denoted by $1 \equiv Va_{\alpha}$ and an inf denoted by $0 \equiv \Lambda a_{\alpha}$

Definition 2.5: A TM-Algebra (X, *, 0) is a non-empty set X with a constant 0 and a binary operation * satisfying the following axioms;

- (1) x*0 = x
- (2) (x*y)*(x*z)=z*y for all x, y, $z \in X$

Definition 2.6: L – Fuzzy TM-Subalgebra

Let L be a complete lattice with sup 1 and inf 0. An L-fuzzy subset μ of a TM-Algebra (X, *, 0) is called an L –fuzzy TM-Subalgebra of X if, for all $x,y \in X$, $\mu(x*y) \ge \mu(x) \land \mu(y)$.

Definition 2.7: Let (X, *) be a TM Algebra. X is said to be a fuzzy Supratopological TM-system if there is a family T of fuzzy subalgebras in X which satisfies the following conditions.

- (1) ϕ $X \in T$
- (2) If $A_i \in T$ for each $i \in I$ then $U_I A_i \in T$ where I is an indexing set

If X is a TM system with a fuzzy supratopology T, then (X, T) is called a fuzzy supratopological TM system and any element in T is called a T-fuzzy supraopensubalgebra in X. The complement of T – fuzzy supraopensubalgebra in X is called a T-fuzzy supraclosedsubalgebra.

Definition 2.8: Let f be a function from X to Y. Let σ be a fuzzy set in Y. The inverse of function, f^1 is defined as $\sigma_{f^{-1}}(x) = \sigma(f(x))$ for all x in X

Let μ be a fuzzy set in X, satisfying supremum property. The image of μ is defined as

$$\mu_f(y) = \begin{cases} \sup_{z \in f^{-1}(x)} \{\mu(z)\}, & f^{-1} \text{ is not empty} \\ 0 & \text{otherwise} \end{cases} \forall y \in Y$$

Definition 2.9: Let $(X,^*)$ be a TM Algebra. Let (X,T) be a fuzzy supratopological TM-system. Consider a fuzzy supraopensubalgebra μ in (X,T). The fuzzy suprainterior TM-system of μ is the union of all fuzzy supraopensubalgebras contained in μ . It is denoted by $SI(\mu)$.

Definition 2.10: Let (X,*) be a TM Algebra. Let (X,T) be a fuzzy supratopological TM-system. Consider a fuzzy supraopensubalgebra μ in (X,T). The fuzzy supraclosure TM-system of μ is the intersection of all fuzzy supraclosedsubalgebras contained in μ . It is denoted by $SC(\mu)$.

Definition 2.11: [7] Let (X,T) be a fuzzy supratopological TM – system. If the L- fuzzy supratopological μ in (X,T) satisfies the condition $\mu \subseteq SI(SC(SI((\mu))))$ then it is called as fuzzy α – supratopensubalgebra in (X,T)

Definition 2.12: [5] Let (X, T) be a fuzzy supratopological TM – system. If every fuzzy supratopensubalgebra in (X, T) is a fuzzy α –supratopological TM – system and it is denoted by (X, T_{α}) .

3. AN L- FUZZY α - SUPRACONTINUOUS IN α -SUPRATOPOLOGICAL TM-SYSTEM

In this section we introduced the notion of an L-fuzzy α -supracontinuous in α -supratopological TM-system.

Definition 3.1: Let (X, *) be a TM Algebra. Let (X, L_T) be an L-fuzzy supratopological TM-system. Consider an L-fuzzy supraopensubalgebra μ in (X, L_T) . An L – fuzzy suprainterior TM-system of μ is the union of all an L-fuzzy supraopensubalgebra contained in μ . It is denoted by $LSI(\mu)$.

Definition 3.2: Let (X, *) be a TM – Algebra. Let (X, L_T) be an L- fuzzy supratopological TM – system. Consider an L-fuzzy supraopensubalgebra μ in (X, L_T) . An L – fuzzy supraclosure TM – system of μ is the intersection of all an L-fuzzy supraclosedsubalgebra containing μ . It is denoted by LSC(μ).

Definition 3.3: Let (X,L_T) be an L – fuzzy supratopological TM – system. If the L- fuzzy supraopensubalgebra μ in (X,L_T) satisfies the condition $\mu \subseteq LSI(LSC(LSI((\mu)))$ then it is called as L-fuzzy α – supraopensubalgebra in (X,L_T) .

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Definition 3.4: Let (X, L_T) be an L- fuzzy supratopological TM – system. If every L – fuzzy supraopensubalgebra in (X, L_T) is an L – fuzzy α – supraopensubalgebra then (X, L_T) is called an L – fuzzy α – supratopological TM – system and it is denoted by $(X, L_{T\alpha})$.

Example 3.5: Consider the set $X = \{0, 1, 2, 3\}$ with the following cayley table

*	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	1	0	3
3	3	2	1	0

Then (X, *) is a TM-algebra. Let L be a complete lattice with $\sup(L) \equiv 1$ and $\inf(L) \equiv 0$.

Let t_1 , t_2 , t_3 , t_4 , $t_5 \in L$ such that $0 \le t_1 \le t_2 \le t_3 \le t_4 \le t_5 \le 1$. Let the L – fuzzy subalgebras $\mu_i : X \to L$, i = 1, 2, 3, 4, 5, 6 be given by

$$\mu_{1}(x) = \begin{cases} 1 & \text{if } x = 0, 1 \, \mu_{2}(x) = \\ t_{4} & \text{if } x = 2, 3 \end{cases} \begin{cases} 1 & \text{if } x = 0, 1 \, \mu_{3}(x) = \\ t_{5} & \text{if } x = 2, 3 \end{cases} \begin{cases} 1 & \text{if } x = 0 \\ t_{3} & \text{if } x = 1 \\ t_{1} & \text{if } x = 2, 3 \end{cases}$$

$$\mu_{4}(x) = \begin{cases} 1 & \text{if } x = 0, 1 \\ 0 & \text{if } x = 2, 3 \, \mu_{5}(x) = \begin{cases} 0 & \text{if } x = 0, 1, 2, 3 \\ 0 & \text{if } x = 2, 3 \end{cases} \end{cases}$$

$$\mu_{6}(x) = \begin{cases} 1 & \text{if } x = 0 \\ t_{2} & \text{if } x = 1 \\ 0 & \text{if } x = 2, 3 \end{cases}$$

Then an L-fuzzy supratopology on X is $L_T = \{ \phi, X, \mu_1, \mu_2, \mu_3, \mu_4, \mu_5, \mu_6 \}$

Hence (X, L_T) is an L-fuzzy supratopological TM – system.

L-Fuzzy α – Supraopensubalgebras:

 $\mu_1 \subset LSI(LSC(LSI(\mu_1))) = \mu_2 [LSI(\mu_1) = \mu_3, LSC(\mu_3) = \mu_5 LSI(\mu_5) = \mu_2]$

 $\mu_2 \subset LSI(LSC(LSI(\mu_2))) = \mu_2 \ [LSI(\mu_2) = \mu_1, LSC(\mu_1) = \mu_5 \ LSI(\mu_5) = \mu_2]$

 $\mu_3 \subset LSI(LSC(LSI(\mu_3))) = \mu_2 [LSI(\mu_3) = \mu_6, LSC(\mu_6) = \mu_5, LSI(\mu_5) = \mu_2]$

 $\mu_4 \subseteq LSI(LSC(LSI(\mu_4))) = \mu_2 [LSI(\mu_4) = \mu_6, LSC(\mu_6) = \mu_5, LSI(\mu_5) = \mu_2]$

 $\mu_5 \subseteq LSI(LSC(LSI(\mu_5))) = \mu_5 \ [LSI(\mu_5) = \mu_5, LSC(\mu_5) = {\mu'}_2, LSI({\mu'}_2) = \mu_5]$

 $\mu_6 \subset LSI(LSC(LSI(\mu_6))) = \mu_2 [LSI(\mu_6) = \mu_6, LSC(\mu_6) = \mu_5, LSI(\mu_5) = \mu_2]$

Thus every L-fuzzy supraopensubalgebra in X is also an L – fuzzy α – superaopensubalgebra.

Hence an L-fuzzy supratopological TM-system (X, L_T) becomes an L-fuzzy α -supratopological TM-system $(X, L_{T\alpha})$.

Definition 3.6: Let $(X, L_{T\alpha})$ be an L- fuzzy α-supratopological TM-system. The complement of an L-fuzzy α-supraopensubalgebra in $(X, L_{T\alpha})$ is called an L- fuzzy α-supraclosed subalgebra of X.

Definition 3.7: Let $(X, L_{T\alpha})$, $(Y, L_{U\alpha})$ be any two L- fuzzy α-supratopological TM-system. A mapping $f:(X, L_{T\alpha}) \to (Y, L_{U\alpha})$ is called an L- fuzzy α-supracontinuous if the inverse image of each L- fuzzy α - supraopensubalgebra in Y is an L- fuzzy α-supraopensubalgebra in X.

Example 3.8: Consider the set $X=\{0, 1, 2, 3, 4\}$ with the following Cayley table

*	0	1	2	3	4
0	0	4	3	2	1
1	1	0	4	3	2
2	2	1	0	4	3
3	3	2	1	0	4
4	4	3	2	1	0

Then (X, *) is a TM-algebra. Let L be a complete lattice with $\sup(\mathbb{Z}L) \equiv 1$ and $\inf(\mathbb{Z}L) \equiv 0$.

Let $t_1, t_2, t_3, t_4 \in L$ such that $0 \le t_1 \le t_2 \le t_3 \le t_4 \le 1$.

Let the L- fuzzy subalgebras $u_i: X \rightarrow L$, i=1,2,3,4,5,6 be given by

$$\mu_{1}(x) = \begin{cases} 1 & \text{if } x = 0 \\ t_{2} & \text{if } x = 1 \\ 0 & \text{if } x = 2,3,4 \end{cases} \begin{cases} 0 & \text{if } x = 0,1,2,3,4 \\ \mu_{3}(x) = \begin{cases} 1 & \text{if } x = 0 \\ t_{3} & \text{if } x = 1 \\ t_{1} & \text{if } x = 2,3,4 \end{cases}$$

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$$\mu_4(x) = \begin{cases} 1 & \text{if } x = 0 \\ 0 & \text{if } x = 1,2,3,4 \\ \mu_5(x) = \end{cases} \begin{cases} 1 & \text{if } x = 0 \\ t_4 & \text{if } x = 1,2,3,4 \\ \mu_6(x) = \begin{cases} t_2 & \text{if } x = 0 \\ 0 & \text{if } x = 1,2,3,4 \end{cases}$$

Then the collection $L_{T\alpha} = \{\phi, X, \mu_1, \mu_2, \mu_3, \mu_4, \mu_5, \mu_6\}$ is an L-fuzzy α -Supratopology on X. Hence $(X, L_{T\alpha})$ is an L-fuzzy α -Supratopological TM-system.

Consider the set $Y = \{0,a,b,c\}$ with the following Cayley table

*	0	a	b	С
0	0	a	b	С
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

Then (Y, *) is a TM-algebra. Let L be a complete lattice with $\sup(L) \equiv \inf(L) \equiv 0$.

Let $t_1, t_2, t_3, t_4 \in L$ such that $0 \le t_1 \le t_2 \le t_3 \le t_4 \le 1$.

Let the fuzzy subalgebras σ_i : Y \rightarrow L, i=1, 2, 3, 4, 5 be given by

$$\sigma_{1}(\mathbf{y}) = \begin{cases} 1 & \text{if } y = 0 \\ t_{4} & \text{if } y = a, b, c \\ \sigma_{2}(\mathbf{y}) = \begin{cases} 1 & \text{if } y = 0 \\ t_{2} & \text{if } y = b \\ 0 & \text{if } y = a, c \end{cases} \\ \sigma_{3}(\mathbf{y}) = \begin{cases} 1 & \text{if } y = 0 \\ t_{2} & \text{if } y = 0 \\ 0 & \text{if } x = a, b, c \end{cases} \\ \sigma_{4}(\mathbf{y}) = \begin{cases} 0 & \text{if } y = 0, a, b, c \\ \sigma_{5}(\mathbf{y}) = \begin{cases} 1 & \text{if } y = 0 \\ t_{3} & \text{if } y = b \\ t_{1} & \text{if } y = a, c \end{cases} \\ \sigma_{4}(\mathbf{y}) = \begin{cases} 0 & \text{if } y = 0, a, b, c \\ \sigma_{5}(\mathbf{y}) = \begin{cases} 1 & \text{if } y = 0 \\ t_{3} & \text{if } y = b \\ t_{1} & \text{if } y = a, c \end{cases} \\ \sigma_{5}(\mathbf{y}) = \begin{cases} 0 & \text{if } y = 0, a, b, c \\ 0 & \text{if } y = a, b, c \end{cases} \\ \sigma_{5}(\mathbf{y}) = \begin{cases} 0 & \text{if } y = 0, a, b, c \\ 0 & \text{if } y = a, b, c \end{cases} \\ \sigma_{5}(\mathbf{y}) = \begin{cases} 0 & \text{if } y = 0, a, b, c \\ 0 & \text{if } y = a, b, c \end{cases} \\ \sigma_{5}(\mathbf{y}) = \begin{cases} 0 & \text{if } y = 0, a, b, c \\ 0 & \text{if } y = a, b, c \end{cases} \\ \sigma_{5}(\mathbf{y}) = \begin{cases} 0 & \text{if } y = 0, a, b, c \\ 0 & \text{if } y = a, b, c \end{cases} \\ \sigma_{5}(\mathbf{y}) = \begin{cases} 0 & \text{if } y = a, b, c \\ 0 & \text{if } y = a, b, c \end{cases} \\ \sigma_{5}(\mathbf{y}) = \begin{cases} 0 & \text{if } y = a, b, c \\ 0 & \text{if } y = a, b, c \end{cases} \\ \sigma_{5}(\mathbf{y}) = \begin{cases} 0 & \text{if } y = a, b, c \\ 0 & \text{if } y = a, b, c \end{cases} \\ \sigma_{5}(\mathbf{y}) = \begin{cases} 0 & \text{if } y = a, b, c \\ 0 & \text{if } y = a, b, c \end{cases} \\ \sigma_{5}(\mathbf{y}) = \begin{cases} 0 & \text{if } y = a, b, c \\ 0 & \text{if } y = a, b, c \end{cases} \\ \sigma_{5}(\mathbf{y}) = \begin{cases} 0 & \text{if } y = a, b, c \\ 0 & \text{if } y = a, b, c \end{cases} \\ \sigma_{5}(\mathbf{y}) = \begin{cases} 0 & \text{if } y = a, b, c \\ 0 & \text{if } y = a, b, c \end{cases} \\ \sigma_{5}(\mathbf{y}) = \begin{cases} 0 & \text{if } y = a, b, c \\ 0 & \text{if } y = a, b, c \end{cases} \\ \sigma_{5}(\mathbf{y}) = \begin{cases} 0 & \text{if } y = a, b, c \\ 0 & \text{if } y = a, b, c \end{cases} \\ \sigma_{5}(\mathbf{y}) = \begin{cases} 0 & \text{if } y = a, b, c \\ 0 & \text{if } y = a, b, c \end{cases} \\ \sigma_{5}(\mathbf{y}) = \begin{cases} 0 & \text{if } y = a, b, c \\ 0 & \text{if } y = a, b, c \end{cases} \\ \sigma_{5}(\mathbf{y}) = \begin{cases} 0 & \text{if } y = a, b, c \\ 0 & \text{if } y = a, b, c \end{cases} \\ \sigma_{5}(\mathbf{y}) = \begin{cases} 0 & \text{if } y = a, b, c \\ 0 & \text{if } y = a, b, c \end{cases} \\ \sigma_{5}(\mathbf{y}) = \begin{cases} 0 & \text{if } y = a, b, c \\ 0 & \text{if } y = a, b, c \end{cases} \\ \sigma_{5}(\mathbf{y}) = \begin{cases} 0 & \text{if } y = a, b, c \\ 0 & \text{if } y = a, b, c \end{cases} \\ \sigma_{5}(\mathbf{y}) = \begin{cases} 0 & \text{if } y = a, b, c \\ 0 & \text{if } y = a, b, c \end{cases} \\ \sigma_{5}(\mathbf{y}) = \begin{cases} 0 & \text{if } y = a, b, c \\ 0 & \text{if } y = a, b, c \end{cases} \\ \sigma_{5}(\mathbf{y}) = \begin{cases} 0 & \text{if } y = a, b, c \\ 0$$

Then the collection $L_{U\alpha} = \{ \phi, X, \sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5 \}$ is an L-fuzzy α -Supratopology on Y. Hence $(Y, L_{U\alpha})$ is an L-fuzzy α -Supratopological TM-system.

Let $f: X \to Y$ be the function given by, f(0) = 0, f(1) = b, f(2) = a, f(3) = c, f(4) = a. $\sigma_{f^{-1}}(x) = \sigma(f(x))$ for all x in X for any fuzzy subalgebra σ in Y.

$$(\sigma_1)_f^{-1}(x) = \mu_5(x), \ (\sigma_2)_f^{-1}(x) = \mu_1(x), \ (\sigma_3)_f^{-1}(x) = \mu_6(x), \ (\sigma_4)_f^{-1}(x) = \mu_2(x), \ (\sigma_5)_f^{-1}(x) = \mu_3(x), \ x \in X.$$

Hence the inverse image of each L-fuzzy α -supraopensubalgebra in Y isan L- fuzzy α -supraopensubalgebra in X.

Hence the function f is L-fuzzy α -supracontinuous.

Theorem 3.9: Let $(X, L_{T\alpha})$, $(Y, L_{U\alpha})$ be any two L-fuzzy α -supratopological TM-systems. Every L-fuzzy α -supracontinuous mapping from X to Y is an L- fuzzy supracontinuous mapping.

Proof: Let $(X, L_{T\alpha})$, $(Y, L_{U\alpha})$ be any two L-fuzzy α -supratopological TM-systems.

Let $f:(X, L_{T\alpha}) \to (Y, L_{U\alpha})$ be an L- fuzzy α -supracontinuous.

Let $\sigma \in L_{U\alpha}$ be the subalgebra of L- fuzzy α – supraopensubalgebra in Y. Then $f^1(\sigma)$ is an L- fuzzy α -supraopensubalgebra (X, $L_{T\alpha}$).

Thus $f^{-1}(\sigma)$ is an L- fuzzy α - supraopensubalgebra in (X, L_T) . Hence f is an L- fuzzy supracontinuous mapping.

Theorem 3.10: Let $(X, L_{T\alpha})$, $(Y, L_{U\alpha})$ be any two L- fuzzy α -supratopological TM-systems. $f: (X, L_{T\alpha}) \to (Y, L_{U\alpha})$ is an L- fuzzy α -supracontinuous mapping if and only if the inverse image of every L-fuzzy α – supraclosed subalgebra in Y is an L- fuzzy α – supraclosed subalgebra in X.

Proof: Suppose the function f is an L- fuzzy α -supracontinuous mapping. That is the inverse image of each L- fuzzy α -supraopensubalgebra in Y is an L-fuzzy α – supraopensubalgebra in X.

Let $L_{U'_{\alpha}}$ be the subalgebra of an L-fuzzy α – supraclosed subalgebra in Y. Then L_{σ} is a fuzzy α – supraclosed subalgebra in U'_{α} .

By definition 2.9,
$$L_{\sigma_{f^{-1}(U_{\alpha}')}}(x) = L_{\sigma_{U_{\alpha}'}}(f(x)) = L_{1-\sigma_{U_{\alpha}}}(f(x))$$

$$= L_{1-\sigma_{f^{-1}(U_{\alpha})}}(x) = L_{\sigma_{f^{-1}(U_{\alpha})}}(x)$$

 $\Rightarrow L_{f^{-1}(U_{\alpha})} = L_{\{f^{-1}(U_{\alpha})\}'}$, for all x in X.

Since f is an L-fuzzy α -supracontinuous mapping, the inverse image of each L-fuzzy α – supraclosed subalgebra in Y is an L-fuzzy α – supraclosed subalgebra in X.

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Conversely, let $L_{\sigma} \in L_{U\alpha}$ be the subalgebra of an L- fuzzy α – supraopensubalgebra in Y. Then by definition 2.9, $L_{\sigma_{f^{-1}(U)}}(x) = L_{\sigma_{U}}(f(x))$ for all x in X.

Since the inverse image of each L- fuzzy α – supraclosedsubalgebra in Y is an L- fuzzy α – supraclosedsubalgebra in X, the inverse image of each L-fuzzy α – supraopensubalgebra in Y is an L-fuzzy α – supraopensubalgebra in X. Therefore the function f is L- fuzzy α – supracontinuous.

Theorem 3.11: Let $(X, L_{T\alpha})$, $(Y, L_{U\alpha})$ be any two L-fuzzy α -supratopological TM-systems. If the function $f: (X, L_{T\alpha}) \rightarrow (Y, L_{U\alpha})$ is an L-fuzzy α -supracontinuous, then $\alpha - LSC(f^{-1}(L_{\sigma})) = f^{-1}(\alpha - LSC(L_{\sigma}))$ for every $L_{\sigma} \in L_{U\alpha}$.

Proof: Let $(X, L_{T\alpha})$, $(Y, L_{U\alpha})$ be any two L-fuzzy α -supratopological TM-systems. Let $L_{\mu} \in L_{T_{\alpha}}$, $L_{\sigma} \in L_{U_{\alpha}}$ be an L-fuzzy α -supraopen subalgebras in X,Y respectively. Then $L_{\mu'}$, $L_{\sigma'}$ are subalgebras of an L-fuzzy α -supraclosed subalgebras in X,Y respectively.

Since the function f: $(X, L_{T\alpha}) \rightarrow (Y, L_{U\alpha})$ is an L- fuzzy α -supracontinuous, then the inverse image of each L-fuzzy α -supraopen subalgebra in Y is an L-fuzzy α -supraopen subalgebra in X.

That is $f^{-1}(L_{\sigma}) = L_{\mu}$.

Hence
$$\alpha - LSC(f^{-1}(L_{\sigma})) = \alpha - LSC(L_{\mu})$$

$$= L_{\mu'}$$

$$= f^{-1}(L_{\sigma'})$$

$$= f^{-1}(\alpha - LSC(L_{\sigma})).$$
Hence $\alpha - LSC(f^{-1}(L_{\sigma})) = f^{-1}(\alpha - LSC(L_{\sigma}))$ for every $L_{\sigma} \in L_{U_{\alpha}}$.

Theorem 3.12: Let $(X, L_{T\alpha})$, $(Y, L_{U\alpha})$ be any two L- fuzzy α -supratopological TM-systems. If the function $f:(X, L_{T\alpha}) \to (Y, L_{U\alpha})$ is an L- fuzzy α -supracontinuous, then $\alpha - LSC(f(L_{\mu})) = f(\alpha - LSC(L_{\mu}))$.

Proof: Let(X, $L_{T\alpha}$), (Y, $L_{U\alpha}$) be any two L- fuzzy α -supratopological TM-systems. Let $L_{\mu} \in L_{T_{\alpha}}$, $L_{\sigma} \in L_{U_{\alpha}}$ be the L-fuzzy α -supraopen subalgebras in X,Y respectively.

Since the function $f:(X, L_{T\alpha}) \to (Y, L_{U\alpha})$ is an L-fuzzy α -supracontinuous, $f^{-1}(L_{\sigma}) = L_{\mu} \Rightarrow f(L_{\mu}) = L_{\sigma}$

By theorem 3.11,
$$\alpha - LSC(f^{-1}(L_{\sigma})) = f^{-1}(\alpha - LSC(L_{\sigma})).$$

 $\Rightarrow f(\alpha - LSC(f^{-1}(L_{\sigma}))) = f(f^{-1}(\alpha - LSC(L_{\sigma})).$
 $\Rightarrow f(\alpha - LSC(L_{\mu})) = \alpha - LSC(L_{\sigma}).$
 $\Rightarrow f(\alpha - LSC(L_{\mu})) = \alpha - LSC(f(L_{\mu})).$

Hence the theorem.

Theorem 3.13: Let $(X, L_{T\alpha})$, $(Y, L_{U\alpha})$ and $(Z, L_{V\alpha})$ be any three L- fuzzy α -supratopological TM-systems. If the functions $f:(X, L_{T\alpha}) \rightarrow (Y, L_{U\alpha})$ and $g:(Y, L_{U\alpha}) \rightarrow (Z, L_{V\alpha})$ are an L-fuzzy α -supracontinuous, then $g \circ f:(X, L_{T\alpha}) \rightarrow (Z, L_{V\alpha})$ is an L- fuzzy α -supracontinuous.

Proof: Let $(X, L_{T\alpha})$, $(Y, L_{U\alpha})$ and $(Z, L_{V\alpha})$ be any three L-fuzzy α -supratopological TM-systems. Let $L_{\mu} \in L_{T\alpha}$, $L_{\sigma} \in L_{U\alpha}$, $L_{\nu} \in L_{V\alpha}$ be the subalgebras of L-fuzzy α -supraopen subalgebras in X, Y, Z respectively.

Since the functions f, g are L- fuzzy α -supracontinuous, the inverse image of each L- fuzzy α -supraopen subalgebra in Y is an L- fuzzy α -supraopen subalgebra in X. That is $f^{-1}(L_{\alpha}) = L_{\mu}$.

Similarly, the inverse image of each L- fuzzy α -supraopen subalgebra in Z is an L- fuzzy α -supraopen subalgebra in Y, $g^{-1}(L_{\nu}) = L_{\sigma}$.

Y,
$$g^{-1}(L_{\nu}) = L_{\sigma}$$
.
Then $(g \circ f)^{-1}(L_{\nu}) = (f^{-1} \circ g^{-1})(L_{\nu}) = f^{-1}(g^{-1}(L_{\nu}))$
 $= f^{-1}(L_{\sigma})$ [Since g is an L-fuzzy α -supracontinuous].

Since $f^{-1}(L_{\sigma})$ is an L-fuzzy α – supraopensubalgebra in X, $(gof)^{-1}(L_{\nu})$ is an L-fuzzy α – supraopensubalgebra in X. Hence the function gof : $(X, L_{T\alpha}) \to (Z, L_{V_{\alpha}})$ is an L-fuzzy α -supracontinuous.

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REFERENCES

- 1. Annalakshmi.M, Chandramouleeswaran.M, Fuzzy Topological subsystem on a TM-algebra, International Journal of Pure and Applied Mathematics, Vol. 94, No. 3 2014, 439-449
- 2. Annalakshmi.M,Chandramouleeswaran.M, On L-Fuzzy Topological TM-system, International Journal of Mathematical Science and Engineering Applications, Vol. 8, No. IV (July 2014) PP. 135-145.
- 3. Annalakshmi.M, Chandramouleeswaran.M, On L- Fuzzy Topological TM- subsystem, Italian Journal of Pure and Applied Mathematics N. 33-2014 (359-368)
- 4. Annalakshmi.M, Chandramouleeswaran.M, Fuzzy supratopological TM-system, International Journal of Pure and Applied Mathematics, Vol. 98, No. 5 2015, 55-61
- 5. Annalakshmi.M, Chandramouleeswaran.M, Fuzzy α-supracontiuous functions, International Journal of Mathematical Archive -6(5),2015,116-119.
- 6. Chang C.L., Fuzzy Topological Spaces, Journal of Mathematical Analysis and Applications, Vol.24, (1968), pp. 182-190.
- 7. Devi.R, Sampathkumar.S and Caldas.M, On Supra α open sets and α continuous functions, General Mathematics Vol. 16,Nr. 2 (2008), 77-84.
- 8. Goguen J.A., L- fuzzy sets, J.Math.Anal.Appl.18, (1967), 145-174.
- 9. Lowen.R, Fuzzy topological spaces and fuzzy compactness, J. Math. Anal. Appl. 56 (1976), 621-633.
- 10. A.S.Mashhour, A.A.Allam, F.S.Mahmoud and F.H.Khedr, Supratopological spaces, Indian Journal of Pure and Applied Mathematics 14(1984), 592-10.
- 11. M.E.Abd El-Monsef and A.E.Ramadan, Fuzzy Supratopological Spaces, Indian Journal of Pure and Applied Mathematics 18(4):322-329, April 1987.
- 12. Tamilarasi.A and Megalai.K, TM- algebra an introduction, CASCT .,(2010).
- 13. Wong.C.K, FuzzyTopology: Product and quotient theorems, J.Math. Anal. Appl. 45 (1974), 512-521.
- 14. Zadeh.L.A, Fuzzy sets, Inform.control 8 (1965),338-353.

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