International Journal of Mathematical Archive-10(10), 2019, 21-25
IJMA Available online through www.ijma.info ISSN 2229 – 5046

ON THE RISING SUN OFFSET PEARSON TYPE II MODEL

S. V. S. GIRIJA*
Department of Mathematics, Hindu College, Guntur, India.

A. J. V. RADHIKA
Department of Mathematics,
University College of Engineering and Technology,
Acharya Nagarjuna University, Guntur, India.

P. SRINIVASULU
Department of Statistics, Sri Chandra Reddy Degree College, Nellore, India.

A. V. DATTATREYA RAO
Department of Statistics, Acharya Nagarjuna University, Guntur, India.

(Received On: 18-09-19; Revised & Accepted On: 08-10-19)

ABSTRACT

The circular models based on the Rising Sun function are motivated by purely mathematical considerations as a smoothing function and possible application. This work takes a further step in this direction using several mathematical tools such as Real Analysis along with MATLAB and are applied to enlarge the horizon of Mathematical Statistics. Here an attempt is made to construct new circular model using the Rising Sun function on the Offset Pearson Type II model and both linear and circular representations of graphs of pdf are plotted using MATLAB.

***Mathematics Subject Classification 2000**: 60E10.*

***Keywords:** Circular model, trigonometric moments, Rising Sun function, characteristic function, Offset Pearson Type II model.*

1. INTRODUCTION

The available methods of generating circular models are wrapping a linear model, offsetting a bivariate linear model and applying stereographic projection on a linear model. The Rising Sun function [Van Rooij and Schikoff (1982), p.10] smoothens the existing curve and many bumps disappear. This may lead to the effect of increasing the smoothing the curve in density estimation. Girija (2010) proposed a new method of generating circular model by using the Rising Sun function (**RSF**) and Radhika.*et.al* (2013) derived circular models based on the Rising Sun function motivated by the mathematical significance of the Rising Sun function behind the construction of circular models, here an attempt is made to construct a new circular model named The Rising Sun Offset Pearson Type II Model and various properties are discussed.

2. CONSTRUCTION OF CIRCULAR MODELS USING THE RISING SUN FUNCTION

The Circular Distribution [Jammalamadaka and Sengupta (2001)] is defined as under
In the continuous case $g:[0,\ 2\pi)\to\mathbb{R}$ is the probability density function of a circular distribution if and only if *g* has the following basic properties

- $g(\theta)\geq 0,\ \ \forall\theta$ (2.1)
- $\int_0^{2\pi} g(\theta)\, d\theta = 1$ (2.2)

Corresponding Author: S. V. S. Girija*
Department of Mathematics, Hindu College, Guntur, India.

- $g(\theta) = g(\theta + 2k\pi)$ (2.3)

for any integer k (i.e., g is periodic) (Mardia,1972)

It may be noted that the circular distribution is a probability distribution whose total probability is concentrated on the unit circle $\{(\cos\theta, \sin\theta) / 0 \le \theta < 2\pi\}$ in the plane which satisfies the properties (2.1) through (2.3).

If $G(\theta)$ denotes the cdf of the r.v., the characteristic function of the circular model is given by

$$\varphi_\theta(t) = E\left(e^{it\theta}\right) = \int_0^{2\pi} e^{it\theta} dG(\theta) = \rho_t e^{i\mu_t} \quad t \in \mathbb{R} \tag{2.4}$$

It is known that whenever $\varphi(t) \ne 0$, $e^{2\pi it} = 1$ (Mardia 1972 p. 41). This suggests that the function $\varphi(t)$ should only be defined for integer values of *t*. accordingly the characteristic function $\varphi(p) = \varphi_p$ is defined by

$$\varphi_\theta(p) = E\left(e^{ip\theta}\right) = \int_0^{2\pi} e^{ip\theta} dF(\theta) = \rho_p e^{i\mu_p} \qquad p \in \mathbb{Z}$$

(2.5)

Clearly, $\varphi_0 = 1$, $\overline{\varphi}_p = \varphi_{-p}$.

Trigonometric moments [Jammalamadaka and Sengupta (2001)]

The value of the characteristic function φ_p at an integer *p* is called the p^{th} trigonometric moment of θ. The real and the imaginary parts of φ_p are denoted by α_p and β_p respectively. We can also view these trigonometric moments in terms of

$$\alpha_p = E(\cos p\theta),\ \beta_p = E(\sin p\theta),\ p \in \mathbb{Z}$$

The Rising Sun function (RSF) of a bounded function $f : [a,b] \to \mathbb{R}$ is defined by

$$f_\Theta(x) = \sup\{f(t) : x \le t \le b\} \tag{2.6}$$

It is easy to show that

- when f is nonnegative then f_Θ is nonnegative
- when f is continuous then f_Θ is continuous
- f_Θ is monotonically decreasing , hence $f_\Theta = f$ when f is decreasing and f_Θ is the smallest monotonically decreasing function such that $f_\Theta = f$.

Imagine the Rising Sun on x – axis. Then $\left\{(x,y) \in \mathbb{R}^2 : y \ge f_\Theta(x)\right\}$ is illuminated by the sun whereas $\left\{(x,y) \in \mathbb{R}^2 : y < f_\Theta(x)\right\}$ is covered by darkness. The set $\left\{(x, f(x)) : f(x) = f_\Theta(x)\right\}$ is the collection of those points of the graph of f that receive light from the sun.

If f is continuous on [a,b] then for any k in the range of f_Θ, $S = \left\{x \in [a,b] : f_\Theta(x) = k\right\}$ is a closed and bounded interval **[**Van Rooij and Schikoff (1982)].

The Rising Sun Lemma 2.1: [Van Rooij and Schikoff (1982)]: Let $f : [a,b] \to \mathbb{R}$ be a continuous function. Then $E = \left\{x \in (a,b) : f_\Theta(x) > f(x)\right\}$ is open. If (α, β) is a component of *E* then

$f_\Theta(\beta) = f(\beta)$ and $f_\Theta(\alpha) = f(\alpha)$ for $\alpha \ne a$, and f_Θ is constant in that interval.

The well known Lebesgue theorem is also proved using the Rising Sun function.

A new construction procedure of a class of Circular Models using RSF is obtained in the following theorem and an illustration is also included in this section. These distributions are named as 'Rising Sun Circular models'.

Theorem 2.2: [Girija (2010)]: If g is the pdf and *G* is the cdf of a random variable of a circular distribution then the Rising Sun function g_Θ, gives rise to the pdf g_c of a circular model. The distribution function of g_c is given by

$$G_c = \begin{cases} \frac{1}{\mathrm{K}}\left[\theta_1\, g(\theta_1) + G(\theta) - G(\theta_1)\right] & \text{for } \theta_1 < \theta \\ \frac{1}{\mathrm{K}}\left[\theta\, \mathrm{g}(\theta_1)\right] & \text{for } \theta_1 \geq \theta \end{cases} \tag{4.2.2}$$

Analogous to Theorem 2.1 the Rising Sun lemma for circular data is presented as follows

The Circular Rising Sun Lemma 2.3 [Radhika et al (2013)]: Let $g:[0,2\pi]\to\mathbb{R}$ be a continuous function. Then $E=\{\theta\in(0,2\pi): g_\Theta(\theta)>g(\theta)\}$ is open. If (α,β) is a component of *E* then $g_\Theta(\beta)=g(\beta)$ and $g_\Theta(\alpha)=g(\alpha)$ for $\alpha\neq 0,$ and g_Θ is constant in that interval. g_c is the normalized function of g_Θ, hence β represents the mode of the circular model at which both g_Θ and g_c have maximum value. Hence component of g_c as well as g_Θ is (0, β) and g_c is the pdf of circular model known as Rising Sun circular model.

3. THE RISING SUN OFFSET PEARSON TYPE II MODEL

The Offset Pearson Type II (OP-II) distribution is derived from the Bivariate Pearson Type II distribution [Balakrishnan and Chin (2008), p. 371]. Radhika *et.al* (2013) derived the Offset Pearson Type II model.

The pdf $g(\theta)$ and cdf $G(\theta)$ of the Offset Pearson Type II model for the Bivariate Pearson Type II distribution with parameters *q*> 1 and ρ where $|\rho|<1$ are respectively given by

$$g(\theta) = \frac{\sqrt{1-\rho^2}}{2\pi\,(1-\rho\sin 2\theta)} \quad \text{where } \theta\in[0,2\pi) \tag{3.1}$$

and

$$G(\theta) = \begin{cases} \frac{1}{2\pi}\left\{\tan^{-1}\left(\frac{\tan\theta-\rho}{\sqrt{1-\rho^2}}\right) - \tan^{-1}\left(\frac{-\rho}{\sqrt{1-\rho^2}}\right)\right\}, \theta\in\left[0,\frac{\pi}{2}\right) \\ \frac{1}{2\pi}\left\{\pi+\tan^{-1}\left(\frac{\tan\theta-\rho}{\sqrt{1-\rho^2}}\right) - \tan^{-1}\left(\frac{-\rho}{\sqrt{1-\rho^2}}\right)\right\}, \theta\in\left(\frac{\pi}{2},\frac{3\pi}{2}\right) \\ \frac{1}{2\pi}\left\{2\pi+\tan^{-1}\left(\frac{\tan\theta-\rho}{\sqrt{1-\rho^2}}\right) - \tan^{-1}\left(\frac{-\rho}{\sqrt{1-\rho^2}}\right)\right\}, \theta\in\left(\frac{3\pi}{2},2\pi\right) \end{cases} . \tag{3.2}$$

The Rising Sun function of the Offset Pearson Type II distribution is defined as

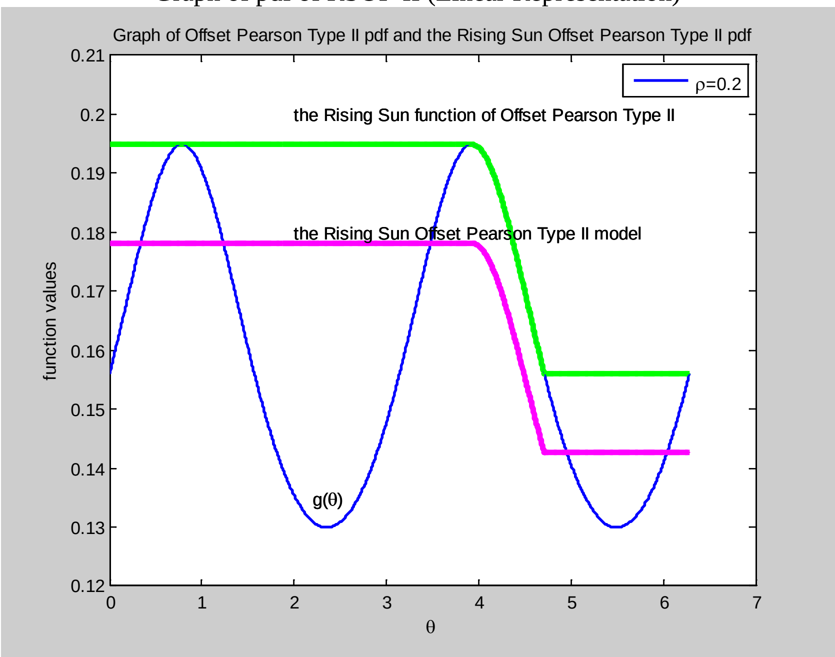
$$\begin{aligned} g_\Theta(\theta) &= \mathrm{Sup}\left(g(t):\theta\le t<2\pi\right) \\ &= \mathrm{Sup}\left(\frac{1}{2\pi}\ \frac{\sqrt{1-\rho^2}}{(1-\rho\sin 2t)} \quad \text{where } \theta\in[0,2\pi)\ :\theta\le t<2\pi\right) \end{aligned} \tag{3.3}$$

Normalizing this function with the constant $K_1 = \int_0^{2\pi} g_\Theta(\theta)\ d\theta$ the pdf of the Rising Sun Offset Pearson Type II distribution (RSOP-II) is obtained.

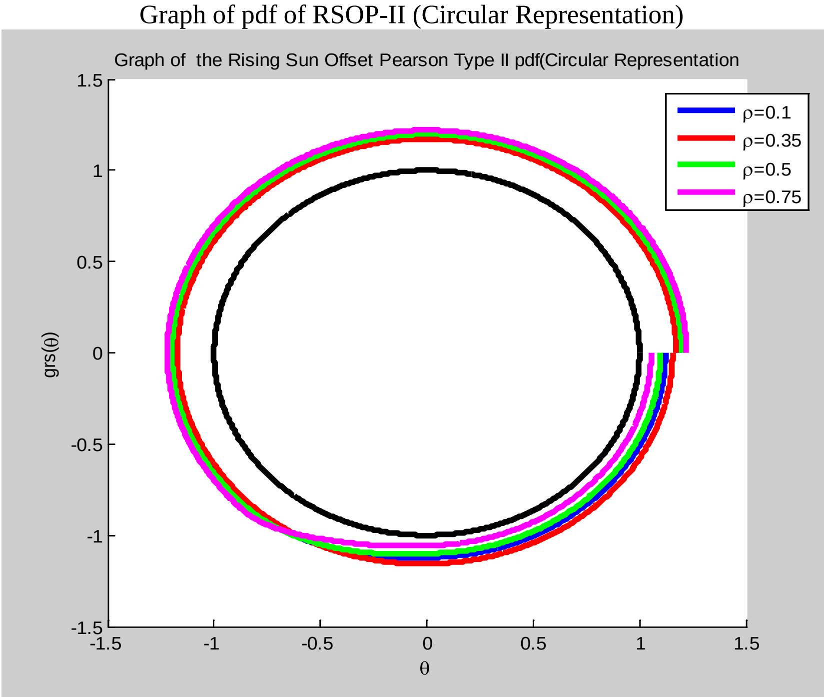
$$g_c(\theta) = \frac{\text{Sup}\left(\frac{1}{2\pi}\frac{\sqrt{1-\rho^2}}{(1-\rho\sin 2t)}\ \text{where}\ \ \theta\in[0,2\pi)\ :\theta\le t<2\pi\right)}{\sqrt{\int_0^{2\pi} g_\Theta(\theta)\ d\theta}} \tag{3.4}$$

The graphs of pdfof the Rising Sun Offset Pearson Type II distribution (RSOP-II) are plotted and population characteristics are studied using MATLAB.

Graph of pdf of RSOP-II (Linear Representation)



Graph of pdf of RSOP-II (Circular Representation)



REFERENCES

1. Balakrishnan, N. And Chin – Diew Lai (2008), Continuous Bivariate Distributions Springer.
2. DattatreyaRao, A.V., Ramabhadra Sarma, I. and Girija, S.V.S. (2007), On Wrapped Version of Some Life Testing Models, *Comm Statist,-Theor.Meth*., 36, issue # 11, pp.2027-2035.
3. Girija, S.V.S., (2010), Construction of New Circular Models, VDM – VERLAG, Germany.
4. Jammalamadaka S. Rao and Sen Gupta, A. (2001), Topics in Circular Statistics, *World Scientific Press, Singapore*.
5. Mardia. K. V. (1972a). *Statistics of Directional Data*. Academic Press, London.
6. Mardia, K.V. and Jupp, P.E. (2000), Directional Statistics, *John Wiley*, *Chichester.*
7. Radhika, A.J.V., Girija, S.V.S. & DattatreyaRao, A.V. (2013a).*On Univariate Offset Pearson Type II Model – Application To Live Data*,International Journal of Mathematics and Statistics Studies, Vol.1, No. 1, 2013, pp. 1-9.
8. Radhika, A.J.V., Girija, S.V.S. & DattatreyaRao, A.V. (2013b). *On Rising Sun von Mises and Rising Sun Wrapped Cauchy Circular Models*, Journal of the Applied Mathematics, Statistics and Informatics (JAMSI), Vol. 9, No. 2, pp. 61 – 67.
9. Van Rooij, A.C.M. and Schikof, W.H. (1982). A Second Course on Real Functions, *Cambridge University Press, Cambridge*

Source of support: Nil, Conflict of interest: None Declared.