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COMPARATIVE NATURE OF SOLUTION FOR O. D. E. WITH VARIABLE COEFFICIENTS BY APPLYING LAPLACE TRANSFORM & ELZAKI TRANSFORM

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ABSTRACT

We know that Sumudu transform and Elzaki transforms are useful in solving ordinary differential equations with constant and variable coefficients.[3,4,6] Hassan Eltayeb and Adem Kilicman has proved some relation regarding existence of L.T. and S.T. [5]. He has solved the proposed equation by applying Laplace transform and obtained solution in complex form. In this paper we will solve ordinary differential equation with variable coefficients by applying Elzaki transform and proved relation regarding existence of Laplace transform and Elzaki transform by comparing their solution in nature.

Keywords: *Elzaki Transform, Laplace transform, ordinary differential equations with variable coefficients.*

1.1 INTRODUCTION

There are various applications of integral transforms in applied Mathematics & in engineering field [1, 2]. We know that Laplace transform is an integral transform which is widely used in solving linear ordinary differential equations with constant and variable coefficients [1]. There is a contribution of Laplace transform in evaluating some complicated definite integrals. [10]. Laplace transform is one of the oldest and commonly used integral transform available in literature. Laplace transform technique was developed by the French Mathematician Pierre Simon de Laplace in 1779 [1]. It is a very powerful tool applied in various areas like Engineering and other Sciences. In 1990 Gamage K. Watugala has introduced a new transform namelySumudu transform which is similar to Laplace transform [3]. The meaning of Sumudu is smooth and this is Sinhala word.Sumudu transform is theoretical dual of the Laplace transform There is one integral transform namely Elzaki trans form which is similar to Laplace transform. Elzaki transform which is introduced by Tarig M.Elzaki in 2010 [9].There is a deep connection between L.T. and E.T.

1.2 SOME USEFUL DEFINITIONS AND THEOREMS

Def.1.2.1: Elzaki Transform: The Elzaki transform of f(x) denoted by E[f(x)] and is defined by

$$\mathrm{E}[f\ (x)] = \mathrm{F}(w) = \mathrm{w}\int_0^\infty f\ (x) e^{\frac{-x}{w}}\, dx\ , x \geq 0,$$

Definition 1.2.2: The sumudu transform of $f(x)$ is defined by

$\mathrm{G}\ (\mathrm{w}) = \int_0^\infty \frac{1}{w} e^{\frac{-x}{w}} \mathrm{f}\ (x)\ \mathrm{d}x$ over the set B of functions defined by

$\mathrm{B} = \{f(x)$ such that $\exists$ N,$x_1, x_2 > 0, |f(x)| < \mathrm{N} e^{\frac{|x|}{x_j}}, x \in (-1)^j \mathrm{X}[0,\infty)\}$

Definition 1.2.3: If G (w) is the Sumudu transform of $f(x)$ then the inverse Sumudu transform of G (w) is $f(x)$ and we write $\mathrm{S}^{-1}(\mathrm{G\ (w)}) = f\ (x)$.

Theorem 1.2.4: If F(w) is the Elzaki transform of $f(x\ )$ then

(a) $\mathrm{E}\{x f'(x)\} = \left\{w^2 \frac{\mathrm{d}}{\mathrm{dw}}\left[\frac{\mathrm{F(w)}}{w} - \mathrm{w\ f(0)}\right] - w\left[\frac{\mathrm{F(w)}}{\mathrm{w}} - w\ f(0)\right]\right\}$

(b) $\mathrm{E}\{x f''(x)\} = \mathrm{w}^2 \frac{d}{dw}\left[\frac{\mathrm{F(w)}}{\mathrm{w}^2} - f(0) - \mathrm{w}\ f'(0)\right] - w\left[\frac{\mathrm{F(w)}}{\mathrm{w}^2} - f(0) - \mathrm{w}\ f'(0)\right]$

(c) $\mathrm{E}\{x^2 f'(x)\} = w^4 \frac{\mathrm{d}^2}{\mathrm{dw}^2}\left\{\frac{\mathrm{F(w)}}{\mathrm{w}} - \mathrm{w}\ f(0)\right\}$

(d) $\mathrm{E}\{x^2 f''(x)\} = w^4 \frac{\mathrm{d}^2}{\mathrm{dw}^2}\left\{\frac{\mathrm{F}(w)}{\mathrm{w}^2} - f(0) - \mathrm{w}\ f'(0)\right\}$

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Theorem 12.5: If F(w) is the E.T. of $f(x)$ then

(a) $E\{f'(x)\} = \frac{F(w)}{w} - w f(0)$

(b) $E\{f''(x)\} = \frac{F(w)}{w^2} - f(0) - w f'(0)$

Theorem 1.2.6: [9] Let f (t) be in set B and let G_n (w) denote the S.T. of the nth order derivative, $f^{(n)}$ (t) of f (t) then for n≥1

$$G_n(w) = \frac{G(w)}{w^n} - \sum_{k=0}^{n-1} \frac{f^{(k)}(0)}{w^{n-k}}$$

Theorem 1.2.7: If F(w) is the E.T.of $f(x)$ then

(a) $E\{xf(x)\} = w^2 \frac{d}{dw}[F(w)] - wF(w)$

(b) $E\{x^2 f(x)\} = w^4 \frac{d^2}{dw^2}[F(w)]$

1.3 COMPARATIVE NATURE OF SOLUTION OF O.D.E. WITH VARIABLE COEFFICIENTS BY APPLYING ELZAKI TRANSFORM &LAPLACE TRANSFORM

As stated in the previous chapter [(section (2.7)] that if Laplace transform exists then Sumudu transform exists but not conversely in this section we will prove such result between L.T. & E.T.

Consider the second order O.D.E. with non-constant variables given by

$$xy''(x) - xy'(x) + y(x) = 2 \quad \text{with } y(0) = 2 \;\&\; y'(0) = -1 \tag{1.3.1}$$

Applying E.T. to equation (3.14) we have

$E\{xy''(x)\} - E\{xy'(x)\} + E\{y(x)\} = 2E(1)$ i.e. we have

$w^2 \frac{d}{dw}\left[\frac{F(w)}{w^2} - y(0) - w\,y'(0)\right] - w\left[\frac{F(w)}{w^2} - y(0) - w\,y'(0)\right] - \left\{w^2 \frac{d}{dw}\left[\frac{F(w)}{w} - w\,y(0)\right] - w\left[\frac{F(w)}{w} - w\,y(0)\right]\right\} + F(w) = 2w^2$

Using given initial conditions we have

$w^2 \frac{d}{dw}\left[\frac{F(w)}{w^2} - 2 + w\right] - w\left[\frac{F(w)}{w^2} - 2 + w\right] - \left\{w^2 \frac{d}{dw}\left[\frac{F(w)}{w} - 2w\right] - w\left[\frac{F(w)}{w} - 2w\right]\right\} + F(w) = 2w^2$ i.e. we have

$$w^2 \frac{d}{dw}\left[\frac{F(w)}{w^2}\right] + w^2 \frac{d}{dw}[w] - \frac{F(w)}{w} + 2w - w^2 - w^2 \frac{d}{dw}\left[\frac{F(w)}{w}\right] + 2w^2 \frac{d}{dw}[w] + F(w) - 2w^2 + F(w) = 2w^2$$

$$w^2\left[\frac{w^2 F'(w) - 2wF(w)}{w^4}\right] + w^2 - \frac{F(w)}{w} + 2w - w^2 - w^2\left[\frac{wF'(w) - F(w)}{w^2}\right] = 2w^2$$

$$F'(w) - \frac{2}{w} F(w) - \frac{F(w)}{w} + 2w - wF'(w) + F(w) = 2w^2$$

Simplifying the above equation we obtain

$$(1-w)F'(w) + \left(3 - \frac{3}{w}\right) F(w) = 2w(w-1) \text{ i.e}$$

$$F'(w) - \frac{3}{w} F(w) = -2w$$

This is linear differential equation with integrating factor $\frac{1}{w^3}$

$F(w)\,\frac{1}{w^3} = \int -2w\left(\frac{1}{w^3}\right) dw.$ Therefore we have

$F(w) = 2w^2 + cw^3$

Taking I.E.T. of the above equation we obtain

$y(x) = 2 + cx.$ This is solution of equation (1.3.2).

Thus we have solved equation (1.3.1) by using Elzaki transform method and obtained solution which is in real form.

1.4 CONCLUDING REMARKS

In this paper we have solved ordinary differential equation (1.3.1) by applying Elzaki transform and its inverse and we obtained its solution in real form. Thus we conclude that if Laplace transform exists then Elzaki transform exists but not conversely.

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