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MATTER, SPACE, MASS, INERTIAL MASS, GRAVITATIONAL MASS AND THEIR RELATIONS

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ABSTRACT

Some fundamental concepts required to understand, analyze and solve problems in any area are defined through their properties understandable at a given time and space. Properties of fundamental concepts known at a given time and space are not in general invariant at all times, consequently a definition of a fundamental concept will change. Further a fundamental concept is associated with many properties known at a given time, it may have properties unknown at that time. A fundamental concept may give rise to many properties but a given set of properties cannot uniquely define the fundamental concepts. This is the drawback in the definition of a fundamental concept through its properties. For example the definition of death, we define death by the failure of certain organs, no doubt for a dead person many organs may fail but failure of these organs may not lead to death. Another fundamental concept in philosophy is God. God is defined through many properties, God may have all these properties but if any entity also has all these properties it does not mean that the entity is the God.

With this background we now come to the definition of the fundamental concepts mentioned in the title.

Definition 1: Matter
Matter is defined by the property that it can occupy space.

In this definition of matter we used another concept called "space" but space is not defined yet. To define space we need a third concept, but instead of looking for a different concept we may use matter to define the space(thus reducing the number of concepts).

Definition 2: Space
Space is defined by the property that it can be occupied by matter.

The two definitions of space and matter show that neither of them is defined independent of the other. In other words, matter defines and is defined by space. The question as to which is first cannot be answered at this stage (may never be answered at all).

Once we defined matter and space, it is necessary to measure them for practical use.

Mass represents a measure of matter and is defined by :

Definition 3: Mass
Mass of an object is defined as the "quantity of matter contained in that object".
But quantity is a non-zero positive real number, hence the above definition can be modified to ,

Definition 4:
Mass of an object is a non-zero positive real number associated with the given object.
Mathematically association is called a mapping, hence a mathematical definition of mass is,

Definition 5:
Let S be the set of all physical objects and R^+ be the set of all non-zero positive real numbers and let $f: S \to R^+$ be a mapping.

If $A \in S$ then the value $F(A)$ is called the mass of A.

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For practical applications, we associate an arbitrary number with the given standard object. If P is a standard object we associate an arbitrary but fixed number m_0 $i.e.,$ $F(P) = m_0$.

If A is any other object then $F(A) = \lambda m_0$; $\lambda =$ positive scalar.

Although the above definition is practical and is in current use, it is not scientific.

We can also associate a number with the given object scientifically by using Newton's laws.

Scientifically we can associate a number with a given object in two ways leading to inertial mass and gravitational mass and for this we require the definition of force and inertia. Force and inertia are defined through Newton's first law of motion which states that "Every object remains in its state of rest or in uniform motion along a straight line unless disturbed by an external force".

Immediately we deduce the definition of force and inertia.

Definition 6: Force
Force is a concept which changes or tends to change the state of rest or uniform motion of an object.

Definition 7: Inertia
Inertia is a property of matter which allows it to remain in its state of rest or of uniform motion along a straight line unless matter is disturbed by an external agent (force).

From the above definition, it is clear that inertia of an object is proportional to the quantity of matter in the object i.e., it is proportional to the mass of the object, but inertia is defined through force. These aspects of mass, inertia and force led Einstein to declare that he does not understand the concept of "inertia".

From Newton's second law of motion, we know that a force 'F' acting on an object produces acceleration 'a' proportional to 'F' which results in the equation $F = \mu a$, where μ is a constant.

We conclude that a force 'F' associates a number 'μ' with the given object, since the force has to overcome the inertia of the object and inertia depends on mass we see that 'μ' is some kind of mass of the object. 'μ' is called the inertial mass 'm_i' of the object,

Therefore $F = m_i a$

The force can also associate a number with a given object as explained below.
Consider an object 'A' above the Earth Surface. By Newton's Universal law Earth exerts a force ω on the object and ω depends on the quantity of matter in A. This quantity of matter associated with ω.

Therefore ω is proportional to the quantity of matter. We denote the quantity of matter in this case by m_g and is called the gravitational mass of A. Therefore $m = m_g \mu$ where μ is a constant. To interpret this concept physically we note the force ω produces an acceleration in A given by $\omega = m_i a$

Therefore $m_i a = m_g \mu$ then $\mu = a$ acceleration of A produced by the gravitational force ω due to earth hence μ is the acceleration due to gravity. Therefore $\omega = m_g g$.
We have taken $m_i = m_{g.}$

Note:
If $m_i \neq m_g$then $\omega = m_i a$ and $\omega = m_g g$ imply that the same force ω has two kinds of effects on the object. It is not physically reasonable therefore $m_i = m_g$.

It has been shown by experiments with a high degree of accuracy that $m_i = m_g$.
We now define some of the above concepts purely mathematically using the basics mathematical concept "mappings".

Definition 8:
As before let $f: S \to R^+$ be the mapping then
1) $f(A)$ is called the mass of $A \in S$.
2) If $f(A) = m_i a$ where a is some variable then m_i is called the inertial mass of A.
3) If $f(A) = m_g g$ where g is a constant then m_g depends on A, m_g is called the gravitational mass of A.

Since f is a mapping we have $m_i a = m_g g$. Again $a = g$ if $m_i = m_g$. We would like to continue our work on these lines.

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