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MULTIPLICATIVE KULLI-BASAVA AND MULTIPLICATIVE HYPER KULLI-BASAVA INDICES OF SOME GRAPHS

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ABSTRACT

In this paper, we introduce the multiplicative Kulli-Basava indices and multiplicative hyper Kulli-Basava indices of a graph. Also we define the general multiplicative Kulli-Basava indices of a graph. We determine these indices.

Keywords: *Multiplicative Kulli-Basava indices, multiplicative hyper Kulli-Basava indices.*

Mathematics Subject Classification: *05C05, 05C07, 05C12.*

1. INTRODUCTION:

A topological index is a numerical parameter mathematically derived from the graph structure. Numerous topological indices or graph indices have been considered in Mathematical Chemistry.

Let G be a finite, simple, connected graph with vertex set $V(G)$ and edge set $E(G)$. The degree $d_G(v)$ of a vertex v is the number of edges incident to v. The degree of an edge $e = uv$ in G is defined by $d_G(e) = d_G(u)+ d_G(v) – 2$. The open neighborhood $N_G(v)$ of a vertex v is the set of all vertices adjacent to v. The edge neighborhood of a vertex v is the set of all edges incident to v and it is denoted by $N_{e(v)}$. Let $S_{e(v)}$ denote the sum of the degrees of all edges incident to a vertex v. For undefined term and notation, we refer [1].

The first and second Kulli-Basava indices were proposed in [2], defined as

$$KB_1(G) = \sum_{uv\in E(G)} \left[S_e(u)+S_e(v)\right] \qquad KB_2(G) = \sum_{uv\in E(G)} S_e(u)S_e(v).$$

In [3], Kulli introduced the first and second hyper Kulli-Basava indices, defined as

$$HKB_1(G) = \sum_{uv\in E(G)} \left[S_e(u)+S_e(v)\right]^2, \qquad HKB_2(G) = \sum_{uv\in E(G)} \left[S_e(u)S_e(v)\right]^2.$$

Recently, some variants of Kulli-Basava indices were introduced and studied such as square Kulli-Basava index [4], connectivity Kulli-Basava indices [5].

We introduce the first and second multiplicative Kulli-Basava indices, defined as

$$KB_1II(G) = \prod_{uv\in E(G)} \left[S_e(u)+S_e(v)\right], \qquad KB_2II(G) = \prod_{uv\in E(G)} S_e(u)S_e(v).$$

Also we propose the first and second multiplicative hyper Kulli-Basava indices, and they are defined as

$$HKB_1II(G) = \prod_{uv\in E(G)} \left[S_e(u)+S_e(v)\right]^2, \qquad HKB_2II(G) = \prod_{uv\in E(G)} \left[S_e(u)S_e(v)\right]^2.$$

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We now introduce the general first and second multiplicative Kulli-Basava indices of a graph G, defined as

$$KB_1^a II(G) = \prod_{uv \in E(G)} \left[S_e(u) + S_e(v)\right]^a , \tag{1}$$

$$KB_2^a II(G) = \prod_{uv \in E(G)} \left[S_e(u) S_e(v)\right]^a , \tag{2}$$

Recently, some new multiplicative indices, were studied see [6, 7, 8, 9, 10].

In this study, the first and second multiplicative Kulli-BAsava Indices, first and second multiplicative hyper. Kulli-Basava indices, general first and second multiplicative Kulli-Basava indices of regular, complete, cycle, wheel, gear and helm graphs are computed.

2. RESULTS FOR REGULAR GRAPHS

A graph G is an r-regular graph if the degree of each vertex of G is r.

Theorem 1: Let G be an r-regular graph with n vertices and m edges. Then the general first multiplicative Kulli-Basava index of G is

$$KB_1^a II(G) = \left[4r(r-1)\right]^{\frac{anr}{2}} .$$

Proof: Let G be an r-regular graph with n vertices and m edges. Then $m = \frac{nr}{2}$, $S_e(u) = 2r(r-1)$ for each vertex u of G. Thus

$$KB_1^a II(G) = \prod_{uv \in E(G)} \left[S_e(u) + S_e(v)\right]^a = \left[2r(r-1) + 2r(r-1)\right]^{am}$$

$$= \left[4r(r-1)\right]^{am} = \left[4r(r-1)\right]^{\frac{anr}{2}} .$$

Corollary 1.1: If C_n is a cycle with n vertices, then

(i) $KB_1 II(C_n) = 8^n$. (ii) $HKB_1 II(C_n) = 8^{2n}$.

Corollary 1.2: If K_n is a complete graph with n vertices, then

(i) $KB_1 II(K_n) = \left[4(n-1)(n-2)\right]^{\frac{n(n-1)}{2}}$.

(ii) $HKB_1 II(K_n) = \left[4(n-1)(n-2)\right]^{n(n-1)}$.

Corollary 1.3: If G is an r-regular graph with n vertices, then

(i) $KB_1 II(G) = \left[4r(r-1)\right]^{\frac{nr}{2}}$. (ii) $HKB_1 II(G) = \left[4r(r-1)\right]^{nr}$.

Theorem 2: The general second multiplicative Kulli-Basava index of an r-regular graph G is

$$KB_2^a II(G) = \left[2r(r-1)\right]^{anr} .$$

Proof: Let G be an r-regular graph with n vertices.

Then $|E(G)| = \frac{nr}{2}$ and $S_e(u) = 2r(r-1)$ for any vertex u in G. Thus

$$KB_2^a II(G) = \prod_{uv \in E(G)} \left[S_e(u) + S_e(v)\right]^a$$

$$= \left[\left(2r(r-1) \times 2r(r-1)\right)^a\right]^m$$

$$= \left[4r^2(r-1)^2\right]^{am} = \left[2r(r-1)\right]^{anr} .$$

Corollary 2.1: If G is an r-regular graph with n vertices, then

(i) $KB_2II(G) = [2r(r-1)]^{nr}$ (ii) $HKB_2II(G) = [2r(r-1)]^{2nr}$

Corollary 2.2: If C_n is a cycle n vertices, then

(i) $KB_2II(C_n) = 4^{2n}$. (ii) $HKB_2II(C_n) = 4^{4n}$.

Corollary 2.3: Let K_n be a complete graph with n vertices. Then

(i) $KB_2II(K_n) = [2(n-1)(n-2)]^{n(n-1)}$

(ii) $HKB_2II(K_n) = [2(n-1)(n-2)]^{2n(n-1)}$.

3. RESULTS FOR WHEEL GRAPHS

A wheel W_n is the join of C_n and K_1. We see that W_n has n+1 vertices and $2n$ edges. The vertices of C_n are called rim vertices and the vertex K_1 is called apex. A graph W_n is shown in Figure 1.

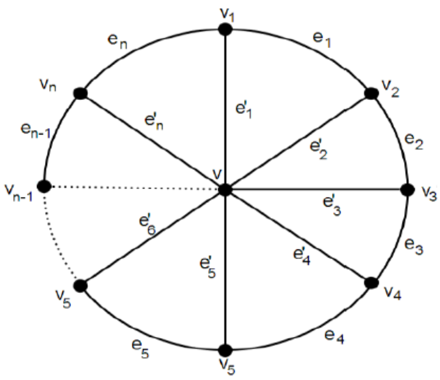

Figure-1: Wheel W_n

Lemma 3: If W_n is a wheel with n+1 vertices and $2n$ edges, then W_n has two types of edges as given below:

$E_1 = \{uv \in E(W_n) \mid S_e(u) = n(n+1), S_e(v) = n+9\}$, $|E_1| = n$.

$E_2 = \{uv \in E(W_n) \mid S_e(u) = S_e(v) = n+9\}$, $|E_2| = n$.

Theorem 4: Let W_n be a wheel with n+1 vertices and $2n$ edges. The general first multiplicative Kulli-Basava index of W_n is

$$KB_1^aII(W_n) = (n^2+2n+9)^{an} \times (2n+18)^{an}.$$

Proof: By using equation (1) and Lemma 3, we deduce

$$KB_1^aII(W_n) = \prod_{uv\in E(W_n)} [S_e(u)+S_e(v)]^a$$

$$= \left[(n(n+1)+(n+9))^a\right]^n \times \left[((n+9)+(n+9))^a\right]^n$$

$$= [n^2+2n+9]^{an} \times [2n+18]^{an}.$$

Corollary 4.1: The first multiplicative Kulli-Basava index of W_n is

$$KB_1II(W_n) = (n^2+2n+9)^n \times (2n+18)^n.$$

Proof: Put a = 1 in equation (3), we get the desired result.

Corollary 4.2: The first multiplicative hyper Kulli-Basava index of W_n is

$$HKB_1II(W_n) = (n^2+2n+9)^{2n} \times (2n+18)^{2n}.$$

Proof: Put a = 2, in equation (3), we obtain the desired result.

Theorem 5: Let W_n be a wheel with n+1 vertices and 2n edges. The general second multiplicative Kulli-Basava index of W_n is

$$KB_1^a II\left(W_n\right)=\left[n(n+1)\right]^{an}\times(n+9)^{3an}. \tag{4}$$

Proof: From equation (2) and Lemma 3, we derive

$$KB_2^a II\left(W_n\right)=\prod_{uv\in E(W_n)}\left[S_e(u)S_e(v)\right]^a$$
$$=\left[\left(n(n+1)(n+9)\right)^a\right]^n\times\left[\left((n+9)(n+9)\right)^a\right]^n$$
$$=\left[n(n+1)\right]^{an}\times\left[n+9\right]^{3an}.$$

Corollary 5.1: The second multiplicative Kulli-Basava index of W_n is

$$KB_1 II\left(W_n\right)=\left[n(n+1)\right]^{n}\times(n+9)^{3n}.$$

Proof: Put a = 1 is equation (4), we get the desired result.

Corollary 5.2: The second multiplicative hyper Kulli-Basava index of W_n is

$$HKB_2 II\left(W_n\right)=\left[n(n+1)\right]^{2n}\times(n+9)^{6n}.$$

Proof: Put a = 2 in equation (4), we obtain the desired result.

4. RESULTS FOR GEAR GRAPHS

A graph is a gear graph obtained from W_n by adding a vertex between each pair of adjacent rim vertices and it is denoted by G_n. Clearly G_n has 2n+1 vertices and 3n edges. A gear graph G_n is presented in Figure 2.

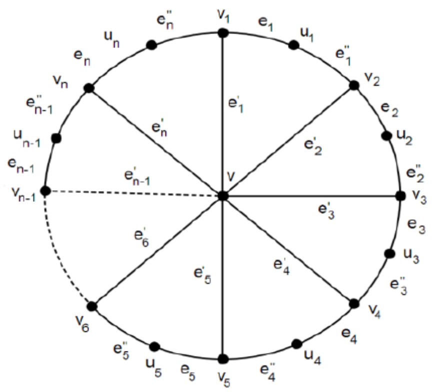

Figure-2: Gear graph G_n

Lemma 6: Let G_n be a gear graph with 2n +1 vertices and 3n edges. Then G_n has two types of edges as follows:

$E_1 = \{uv \in E(G_n) \mid S_e(u) = n(n+1), S_e(v) = n+7\}$, $|E_1| = n$.
$E_2 = \{uv \in E(G_n) \mid S_e(u) = n+7, S_e(v) = 6\}$, $|E_2| = 2n$.

Theorem 7: Let G_n be a gear graph with 2n+1 vertices and 3n edges. The general first multiplicative Kulli-Basava index of G_n is given by

$$KB_1^a II\left(G_n\right)=\left(n^2+2n+7\right)^{an}\times(n+13)^{2an}. \tag{5}$$

Proof: By using equation (1) and Lemma 6, we obtain

$$KB_2^a II\left(G_n\right)=\prod_{uv\in E(G_n)}\left[S_e(u)+S_e(v)\right]^a$$
$$=\left[n(n+1)+n+7\right]^n\times(n+7+6)^{a2n}$$
$$=\left(n^2+2n+7\right)^{an}\times(n+13)^{2an}.$$

Corollary 7.1: The first multiplicative Kulli-Basava index of G_n is

$$KB_1II(G_n) = (n^2+2n+7)^n \times (n+13)^{2n}.$$

Proof: Put $a = 1$ is equation (5), we get the desired result.

Corollary 7.2: The first multiplicative hyper Kulli-Basava index of G_n is

$$HKB_1II(G_n) = (n^2+2n+7)^{2n} \times (n+13)^{4n}.$$

Proof: Put $a = 2$ in equation (5), we obtain the desired result.

Theorem 8: Let G_n be a gear graph with $2n$+1 vertices and $3n$ edges. The general second multiplicative Kulli-Basava index of G_n is given by

$$KB_2^aII(G_n) = 6^{2an}\left[n(n+1)\right]^{an} \times (n+7)^{3an}. \tag{6}$$

Proof: From equation (2) and by using Lemma 6, we have

$$KB_2^aII(G_n) = \prod_{uv\in E(G_n)} \left[S_e(u)S_e(v)\right]^a$$

$$= \left[n(n+1)+n+7\right]^{an} \times \left[6(n+7)\right]^{2an}$$

$$= 6^{2an}\left[n(n+1)\right]^{an} \times (n+7)^{3an}.$$

Corollary 8.1: The second multiplicative Kulli-Basava index of G_n is

$$KB_2II(G_n) = 6^{2n}\left[n(n+1)\right]^{n} \times (n+7)^{3n}.$$

Proof: Put $a = 1$ in equation (6), we get the desired result.

Corollary 8.2: The second multiplicative hyper Kulli-Basava index of G_n is

$$HKB_2II(G_n) = 6^{4n}\left[n(n+1)\right]^{2n} \times (n+7)^{6n}.$$

Proof: Put $a = 2$ in equation (6), we obtain the required result.

5. RESULTS FOR HELM GRAPHS

A helm graph is a graph obtained from W_n by attaching an end edge to each rim vertex and it is denoted by H_n. Clearly H_n has $2n$+1 vertices and $3n$ edges. A graph H_n is shown in Figure 3.

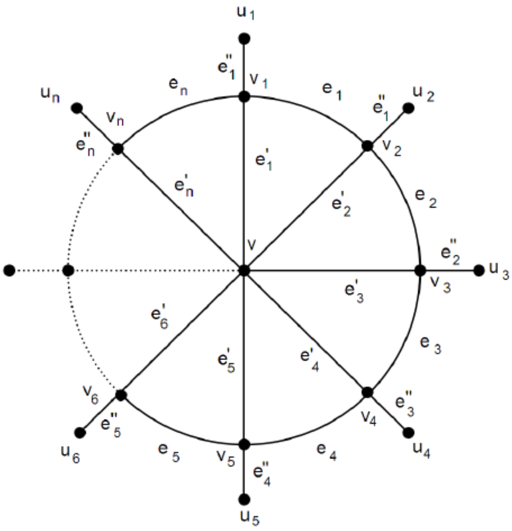

Figure-3: Helm graph H_n

Lemma 9: Let H_n be a helm graph with $2n$+1 vertices and $3n$ edges. Then H_n has three types of edges as follows:

$E_1 = \{uv \in E(H_n) \mid S_e(u) = n(n+2), S_e(v) = n+17\}$, $|E_1| = n$.
$E_2 = \{uv \in E(H_n) \mid S_e(u) = S_e(v) = n+17\}$, $|E_2| = n$.
$E_3 = \{uv \in E(H_n) \mid S_e(u) = n+17, S_e(v) = 3\}$, $|E_3| = n$.

Theorem 10: Let H_n be a helm graph with $2n+1$ vertices and $3n$ edges. The general first multiplicative Kulli-Basava index of H_n is

$$KB_2^a II(H_n) = (n^2+3n+17)^{an}(2n+34)^{an}(n+20)^{an}. \tag{7}$$

Proof: By using equation (1) and Lemma 9, we deduce

$$KB_1^a II(H_n) = \prod_{uv\in E(H_n)} \left[S_e(u)+S_e(v)\right]^a$$
$$= [n(n+2)+n+17]^{an} \times [n+17+n+17]^{an} \times [n+17+3]^{an}$$
$$= (n^2+3n+17)^{an} \times (2n+34)^{n} \times (n+20)^{an}.$$

Corollary 10.1: The first multiplicative Kulli-Basava index of H_n is

$$KB_1 II(H_n) = (n^2+3n+17)^{n} \times (2n+34)^{n} \times (n+20)^{n}.$$

Proof: Put a = 1 in equation (7), we obtain the desired result.

Corollary 10.2: The first multiplicative hyper Kulli-Basava index of H_n is

$$HKB_1 II(H_n) = (n^2+3n+7)^{2n} \times (2n+34)^{2n} \times (n+20)^{2n}.$$

Proof: Put a = 2 in equation (7) we get the desired result.

Theorem 11: Let H_n be a helm graph with $2n+1$ vertices and $3n$ edges. The general second multiplicative Kulli-Basava index of H_n is

$$KB_2^a II(H_n) = [3n(n+2)]^{an}(n+17)^{4an}. \tag{8}$$

Proof: From equation (2) and by using Lemma 9, we derive

$$KB_2^a II(H_n) = \prod_{uv\in E(H_n)} \left[S_e(u)S_e(v)\right]^a$$
$$= [n(n+2)(n+17)]^{an} \times [(n+17)(n+17)]^{an} \times [3(n+17)]^{an}$$
$$= [3n(n+2)]^{an} \times (n+17)^{4an}.$$

Corollary 11.1: The second multiplicative Kulli-Basava index of H_n is

$$KB_2 II(H_n) = [3n(n+2)]^{n}(n+17)^{4n}.$$

Proof: Put a = 1 in equation (8), we get the desired result.

Corollary 11.2: The second multiplicative hyper Kulli-Basava index of H_n is

$$KB_2 II(H_n) = [3n(n+2)]^{2n}(n+17)^{8n}.$$

Proof: Put a = 2 in equation (8), we obtain the required result.

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