International Journal of Mathematical Archive-10(7), 2019, 53-62
Available online through www.ijma.info ISSN 2229 – 5046

NEW RIDGE ESTIMATORS OF SUR MODEL WHEN THE ERRORS ARE SERIALLY CORRELATED

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(Received On: 11-07-19; Revised & Accepted On: 25-07-19)

ABSTRACT

This paper considers the seemingly unrelated regressions (SUR) model when the errors are first-order serially correlated as well as the explanatory variables are highly correlated. We proposed new ridge estimators for this model under these conditions. Moreover, the performance of the classical (Zellner's and Parks') estimators and the proposed (ridge) estimators has been examined by a Monte Carlo simulation study. The results indicated that the proposed estimators are efficient and reliable than the classical estimators.

Keywords: *Biased estimators; GLS estimation method; Multicollinearity; Parks' SUR model; Seemingly unrelated regressions model; Zellner's SUR model.*

1. INTRODUCTION

The SUR model has been proposed by Zellner (1962) under the assumption that the errors of the model are related by contemporaneous correlation. Then, Parks (1967) developed this model by assume that the errors are related by both serial and contemporaneous correlation. But the two models assume that the explanatory variables in the model are independent. But this assumption is very hard in empirical work, because most empirical datasets contain the correlated explanatory variables. This problem defined in econometrics literature with the multicollinearity problem. This problem arises in situations when the explanatory variables are highly inter-correlated. Then it becomes difficult to disentangle the separate effects of each of the explanatory variables on the dependent variable. As a result, the estimated coefficients may be statistically insignificant and/or have, unexpectedly, different signs. Thus, conducting a meaningful statistical inference would be difficult for the researcher.

To solve the multicollinearity problem in Zellner's SUR model, Srivastava and Giles (1987) proposed the general ridge estimator for this model.[1] Also several ridge estimators are proposed and compared by Alkhamisi and Shukur (2008) and Rana and Al Amin (2015). However, these papers not provided any solution of the multicollinearity problem for the SUR model under Parks' assumption. Therefore, in this paper, we develop the ridge estimators for Parks' SUR model.

This paper is organized as follows. Section 2 presents a background about Parks' SUR model. The proposed ridge estimators of this model are provided in Section 3. While in Section 4, a simulation study is conducted to investigate the performance of the proposed estimators. Finally, Section 5 offers the concluding remarks.

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[1] In general, the ridge estimator was first proposed by Hoerl and Kennard (1970a,b), and later followed by many papers, such as Saleh and Kibria (1993), Kibria (2003), and Alkhamisi et al. (2006). Also, in many empirical applications across the different fields, the ridge estimators are used to get more efficient estimation than the classical estimation, see for example, Locking et al. (2013) and Rady et al. (2018).

2. PARKS' SUR MODEL

Consider the following SUR model, with m regression equations:

$$\underset{mn\times 1}{Y} = \underset{mn\times K}{X}\ \underset{K\times 1}{B} + \underset{mn\times 1}{U} \tag{1}$$

where $Y = (y_1', \dots, y_m')'$ is the vector of endogenous variable, and $X = diag[X_i]$; with X_i (for $i = 1, \dots, m$) is the matrix of the exogenous variables of equation number i with dimension $n \times k_i$, and B is the vector of the parameters with $K = \sum_{i=1}^{m} k_i$, while $U = (u_1', \dots, u_m')'$ is the errors vector.

Assumptions:

A1: $u_i = \rho_i u_{i,-1} + \varepsilon_i$ for all $i = 1, \dots, m$, whrere $u_{i,-1}$ is the first lag vector of u_i, and ρ_i is the first-order serial correlation coefficient; where $-1 < \rho_i < 1$.

A2: $E(u_1', \dots, u_m')' = E(U) = 0$, $E(\varepsilon_1', \dots, \varepsilon_m')' = E(\varepsilon) = 0$, and $Cov(U, \varepsilon) = 0$

A3: $Cov(U) = E(UU') = \begin{bmatrix} \sigma_{11}V_1 & \sigma_{12}I_n & \cdots & \sigma_{1m}I_n \\ \sigma_{21}I_n & \sigma_{22}V_2 & \cdots & \sigma_{2m}I_n \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{m1}I_n & \sigma_{m2}I_n & \cdots & \sigma_{mm}V_m \end{bmatrix} = \Omega;$

where $V_i = \frac{1}{1-\rho_i^2}\begin{pmatrix} 1 & \rho_i & \rho_i^2 & \cdots & \rho_i^{n-1} \\ \rho_i & 1 & \rho_i & \cdots & \rho_i^{n-2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho_i^{n-1} & \rho_i^{n-2} & \rho_i^{n-3} & \cdots & 1 \end{pmatrix}$.

A4: X is non-stochastic matrix, $cov(X, \varepsilon) = 0$, and $cov(X, U) = 0$.

A5: X is full column rank matrix, i.e., $rank(X) = K$.

Under these assumptions, we can apply the generalized least squares (GLS) method on equation (1) to estimate B:

$$\hat{B}_P = (X'\Omega^{-1}X)^{-1}X'\Omega^{-1}Y, \text{ and } MSE(\hat{B}_P) = (X'\Omega^{-1}X)^{-1}.$$

The model above with these assumptions is discussed by Parks (1967). But if $V_i = I_n$, we will back to Zellner's SUR model, and then Zellner's estimator is

$$\hat{B}_Z = [X'(\Sigma^{-1} \otimes I_n)X]^{-1}X'(\Sigma^{-1} \otimes I_n)Y, \text{ and } MSE(\hat{B}_Z) = [X'(\Sigma^{-1} \otimes I_n)X]^{-1}, \tag{2}$$

where $\Sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1m} \\ \sigma_{21} & \sigma_{22} & \cdots & \sigma_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{m1} & \sigma_{m2} & \cdots & \sigma_{mm} \end{pmatrix}$.

3. PROPOSED RIDGE ESTIMATORS

In this paper, we develop the ridge estimation of the Parks' model, to solve the multicollinearity problem. The ridge estimator of this model and MSE of it are given by:

$$\hat{B}_{RP} = (X'\Omega^{-1}X + R)^{-1}X'\Omega^{-1}Y;$$
$$MSE(\hat{B}_{RP}) = (X'\Omega^{-1}X + R)^{-1}(X'\Omega^{-1}X + RBB'R')(X'\Omega^{-1}X + R)^{-1}, \tag{3}$$

where R is a $K \times K$ matrix with nonnegative elements:

$$R = \begin{bmatrix} R_1 & 0 & \cdots & 0 \\ 0 & R_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & R_m \end{bmatrix}; \text{ with } R_i = diag\{r_{i1}, \dots, r_{ik_i}\}\ \forall\ i = 1, \dots, m.$$

The canonical version of model (1) is given by:

$$Y^* = Z\alpha + U^*, \tag{4}$$

where $Y^* = \Omega^{-1/2}Y$, $U^* = \Omega^{-1/2}U$, $Z = X^*\Phi$ with $X^* = \Omega^{-1/2}X$ and Φ eigenvectors of $(X^{*\prime}X^*)$, then $Z'Z = \Phi'X^{*\prime}X^*\Phi = \Lambda$, where Λ the eigenvalues of $(X^{*\prime}X^*)$. The corresponding GLS estimator of (4) is $\hat{\alpha} = (Z'Z)^{-1}Z'Y^*$. Srivastava and Giles (1987) and Firinguetti (1997) proved that the optimum elements of R are $\hat{r}_{ij} = \frac{1}{\hat{\alpha}_{ij}^2}$; $i = 1, \dots, m; j = 1, \dots, k_i$, if and only if the following conditions are holds: $\alpha'\Lambda\alpha < 1$ and $\alpha' R\, \alpha < 2$. Note that if $R = rI_K$ then these conditions are reduced to one condition: $0 < r < (2/\alpha'\alpha)$.

In this paper, we propose new ridge estimators of this model basing on the following ridge parameters:

1- The median of r_{ij} that proposed by Kibria (2003) for single equation version and Alkhamisi and Shukur (2008) developed it for SUR model:

$$\hat{r}_{ij}(SK) = median\left(\frac{1}{\hat{\alpha}_{ij}^2}\right).$$

2- The max of r_{ij} that proposed by Alkhamisi and Shukur (2008):

$$\hat{r}_{ij}(AS) = max\left(\frac{1}{\hat{\alpha}_{ij}^2}\right).$$

3- Let $\bar{K} = K/m$, we suggest using the following ridge parameters:

$$\hat{r}_{ij}(new1) = \left[\hat{r}_{ij}(SK)\right]^{1/\bar{K}}\left[\hat{r}_{ij}(AS)\right]^{1/\bar{K}};$$

$$\hat{r}_{ij}(new2) = \left[sum\left(\frac{1}{\hat{\alpha}_{ij}^2}\right)\right]^{1/\bar{K}}\left[\hat{r}_{ij}(AS)\right]^{1/\bar{K}}.$$

Since $\hat{B}_R$ estimator still involves the unknown parameters, therefore it needs to estimate these parameters to make this estimator feasible. We suggest using the following consistent estimators:[2]

$$\hat{\rho}_i = \frac{\sum_{t=2}^{n} \hat{u}_{it}\hat{u}_{i,t-1}}{\sum_{t=2}^{n} \hat{u}_{i,t-1}^2}, \text{and } \hat{\sigma}_{is} = \frac{\hat{\varepsilon}_i'\hat{\varepsilon}_s}{(n-k_i)^{1/2}(n-k_s)^{1/2}} \; \forall\, i = s = 1, \dots, m,$$

where $\hat{u}_i = (\hat{u}_{i1}, \dots, \hat{u}_{in})'$ is the residuals vector obtained from applying OLS on equation number i, $\hat{\varepsilon}_i = (\hat{\varepsilon}_{i1}, \dots, \hat{\varepsilon}_{in})'$; $\hat{\varepsilon}_{i1} = \hat{u}_{i1}\sqrt{1-\hat{\rho}_i^2}$, and $\hat{\varepsilon}_{it} = \hat{u}_{it} - \hat{\rho}_i\, \hat{u}_{i,t-1}$ for $t = 2, \dots, n$.

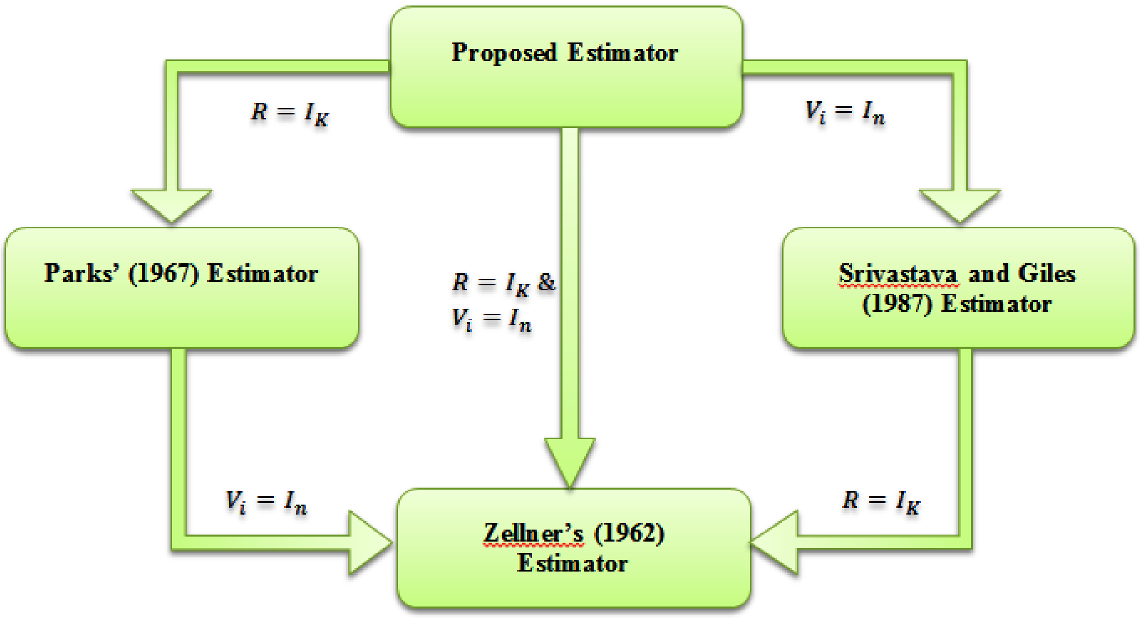

Figure-1: The transformations from the proposed estimator to other estimator

Figure 1 shows that the proposed estimator is a general estimator of three estimators (Srivastava and Giles (1987), Parks', and Zellner's estimators). In other words, we can say that these estimators are special cases of the proposed estimator.

4. MONTE CARLO SIMULATION STUDY

In this section, we conduct a comparative study between the classical (Parks' and Zellner's) estimators and the four ridge estimators (SK, AS, new1, and new2) through the Monte Carlo simulation study. In our simulation study, Monte Carlo experiments were performed based on the model in (1). To investigate the performance of these estimators in different situations, we will use different simulation factors as show in Table 1. R software is used to perform this study.[3]

[2] The properties of these estimators are studied by Parks (1967).

[3] For using R software to create the Monte Carlo simulation studies, see Abonazel (2018)

To judge the performance of six estimators, the total mean squared error (TMSE) criterion is used:

$$\text{TMSE}(\hat{\beta}) = \frac{1}{L}\sum_{l=1}^{L}(\hat{\beta}_l - \beta)'(\hat{\beta}_l - \beta),$$

where $\hat{\beta}_l$ is the vector of estimated values at l^{th} experiment of $L = 1000$ Monte Carlo experiments, while β is the vector of true coefficients.

Table-1: The simulation factors of our study

Simulation factor	Levels
The number of parameters ($\boldsymbol{B}$) in each equation (without intercept)	$k_i = 4$ or 6
The number of equations ($\boldsymbol{m}$)	m = 2, 4, or 6
The true values of $\boldsymbol{B}$ (as Månsson and Shukur, 2011 and KaÇiranlar and Dawoud, 2018)	$B'B = 1$ and $\beta_1 = \cdots = \beta_m$
The sample size in each equation ($\boldsymbol{n}$)	n = 25, 50, 75, or 100
The explanatory variables: $\boldsymbol{X} \sim \boldsymbol{MVN}(\boldsymbol{1}, \boldsymbol{\Sigma_X})$, where *diag* ($\boldsymbol{\Sigma_x}$) = 1 and *off-diag* ($\boldsymbol{\Sigma_x}$) = $\boldsymbol{\rho_x}$ (as Alkhamisi and Shukur, 2008 and Abonazel and Farghali, 2018)	ρ_x = .90, .95, or .99
The error term: $\boldsymbol{U} \sim \boldsymbol{MVN}(\boldsymbol{0}, \boldsymbol{\Omega})$, where $\boldsymbol{\sigma_{ii} = 1}$, $\boldsymbol{\sigma_{is} = .80}\ \forall\ \boldsymbol{i \neq s}$; $\boldsymbol{i = s = 1, \ldots, m}$, and $\boldsymbol{\rho_i}$ is	$\rho_i = \rho$ =.75 or .90

The results are given in Tables 2-7. Specifically, Tables 2, 4, and 6 present the TMSE values of the estimators when $k_i = 4$, while case of $k_i = 6$ is presented in Tables 3, 5, and 7. From Tables 2-7, we can summarize some effects for all estimators in the following points:

- If the value of n is increased, the values of TMSE are decreasing for all simulation situations.
- If the values of m and ρ are increased, the values of TMSE are increasing in all situations.

However, if the values of k_i, ρ_x, and ρ are increased, the TMSE values of Parks' and Zellner's estimators are increased more than ridge estimators. In other words, the ridge estimators are efficient than Parks' and Zellner's estimators. Specifically, the new2 estimator is the best estimator because it has minimum TMSE in all simulation situations.

5. CONCLUSION

In this paper, we developed new ridge estimators for SUR model when there are high inter-correlations between the explanatory variables as well as the errors are serially correlated. A Monte Carlo simulation study was conducted to evaluate the performance of the classical estimators and different ridge estimators. The simulation results indicated that the ridge estimators are efficient than classical estimators in all situations. Moreover, the ridge estimators which based on the proposed parameters $\hat{r}_{ij}(new1)$ and $\hat{r}_{ij}(new2)$ are more efficient than other ridge estimators. And the new2 estimator is the best ridge estimator for this model.

Table-2: TMSE values for the different estimators when $m = 2$ and $k_i = 4$

$\boldsymbol{n}$	**GLS estimators**		**Ridge estimators**			
	Zellner	**Parks**	**SK**	**AS**	**new1**	**new2**
			$\boldsymbol{\rho = .75, \rho_x = .90}$			
25	3.3093	2.8132	0.5662	0.4290	0.4107	0.2434
50	1.9638	0.8717	0.1898	0.4322	0.1995	0.1365
75	1.4756	0.8921	0.2873	0.4425	0.2978	0.2187
100	0.7433	0.5846	0.1850	0.4023	0.2111	0.1444
			$\boldsymbol{\rho = .75, \rho_x = .95}$			
25	10.5349	6.8406	0.6441	0.3898	0.3819	0.2005
50	3.2643	2.4891	0.4964	0.3292	0.3774	0.1977
75	1.9457	1.5986	0.3309	0.3485	0.2808	0.1617
100	1.7389	1.0083	0.2293	0.3833	0.2359	0.1537
			$\boldsymbol{\rho = .75, \rho_x = .99}$			
25	64.5727	29.0269	1.1529	0.2699	0.3252	0.1682
50	14.9609	15.3605	1.5979	0.2907	0.4549	0.1938
75	10.6514	6.8152	0.7066	0.2744	0.3761	0.2069
100	5.4922	4.0121	0.5068	0.2736	0.2491	0.1095
			$\boldsymbol{\rho = .90, \rho_x = .90}$			
25	6.9896	8.7574	1.2625	0.4675	0.6063	0.3352
50	7.3234	2.2401	0.4410	0.4503	0.3603	0.2265
75	5.2763	1.8146	0.4578	0.4246	0.4092	0.2603

100	1.6578	1.0216	0.2294	0.4804	0.2302	0.1695
			$\rho=.90, \rho_x=.95$			
25	17.3703	13.4405	1.3889	0.4474	0.5614	0.3325
50	7.2449	3.5487	0.4274	0.4183	0.2965	0.1898
75	4.7687	2.1814	0.3775	0.4086	0.2872	0.1822
100	6.4977	1.7847	0.3213	0.3787	0.2833	0.1820
			$\rho=.90, \rho_x=.99$			
25	76.2155	40.0421	2.2786	0.3703	0.5024	0.2536
50	35.1001	15.7828	0.8277	0.2770	0.3008	0.1646
75	24.9208	20.3456	2.1830	0.3659	0.6939	0.3422
100	37.7388	7.9543	0.6389	0.2339	0.2798	0.1299

Table-3: TMSE values for the different estimators when $m = 2$ and $k_i = 6$

n	GLS estimators		Ridge estimators			
	Zellner	**Parks**	**SK**	**AS**	**new1**	**new2**
			$\rho=.75, \rho_x=.90$			
25	6.0869	4.9353	0.4805	0.4744	0.5549	0.2602
50	2.1677	1.6487	0.2374	0.4843	0.4070	0.2136
75	2.1376	1.6900	0.3605	0.4338	0.5612	0.3078
100	1.1635	0.8936	0.1945	0.4622	0.3883	0.2320
			$\rho=.75, \rho_x=.95$			
25	18.8283	10.7105	1.4818	0.3611	1.0309	0.4630
50	8.0270	5.0371	0.6325	0.3790	0.6987	0.3335
75	3.9324	2.9416	0.5416	0.3878	0.6590	0.3314
100	2.8857	1.5957	0.2380	0.4277	0.4339	0.2247
			$\rho=.75, \rho_x=.99$			
25	80.0961	50.8054	4.8144	0.3225	1.0006	0.4121
50	23.0518	21.7883	1.9871	0.2899	0.7741	0.3087
75	33.2683	12.1523	1.0189	0.2484	0.6152	0.2440
100	16.0482	7.0348	0.6724	0.2693	0.5094	0.1950
			$\rho=.90, \rho_x=.90$			
25	22.7738	17.0336	2.4325	0.5397	1.3022	0.6262
50	10.0682	6.0696	0.7376	0.5305	0.7956	0.3895
75	6.2711	3.0931	0.5817	0.5007	0.7453	0.4126
100	3.2779	1.3125	0.1958	0.5475	0.3745	0.2122
			$\rho=.90, \rho_x=.95$			
25	26.8653	22.1972	1.2221	0.4851	0.6291	0.2887
50	12.8952	6.6694	0.6216	0.4551	0.5674	0.2573
75	9.9596	9.5709	1.6845	0.3875	1.2132	0.5635
100	6.1629	3.0764	0.3267	0.4544	0.4918	0.2307
			$\rho=.90, \rho_x=.99$			
25	136.6920	92.4660	6.9209	0.4154	0.9572	0.4599
50	52.7980	28.2602	1.2997	0.2877	0.4473	0.1843
75	43.6389	24.2276	1.7502	0.2691	0.6378	0.2645
100	26.2401	16.3303	1.2590	0.2946	0.5539	0.2267

Table-4: TMSE values for the different estimators when $m = 4$ and $k_i = 4$

n	GLS estimators		Ridge estimators			
	Zellner	**Parks**	**SK**	**AS**	**new1**	**new2**
			$\rho = .75, \rho_x = .90$			
25	7.0932	5.4229	0.6376	0.6180	0.4221	0.2427
50	2.7313	1.8013	0.2767	0.6812	0.2699	0.1900
75	2.4684	1.6725	0.3218	0.6406	0.3099	0.1996
100	1.3302	0.6680	0.0903	0.7315	0.1246	0.1242
			$\rho = .75, \rho_x = .95$			
25	20.7810	15.8471	1.3541	0.5581	0.5878	0.2783
50	5.5935	3.6247	0.4217	0.5929	0.3089	0.1707
75	6.6071	4.7564	0.6663	0.5255	0.4559	0.2136
100	2.1297	1.2059	0.1279	0.6167	0.1536	0.1091
			$\rho = .75, \rho_x = .99$			
25	113.6974	95.0786	7.8941	0.4621	0.8608	0.3342
50	25.9463	16.9079	0.7986	0.4420	0.2752	0.1318
75	27.6831	16.4686	0.9874	0.3851	0.3463	0.1339
100	9.3053	5.5652	0.3359	0.5057	0.1669	0.0846
			$\rho = .90, \rho_x = .90$			
25	15.8385	11.3784	1.3109	0.6949	0.7106	0.4109
50	6.2572	2.9468	0.3763	0.7426	0.3202	0.2751
75	7.1595	2.7141	0.4380	0.6666	0.3467	0.2228
100	3.4778	1.2267	0.1622	0.7427	0.1786	0.1834
			$\rho = .90, \rho_x = .95$			
25	42.7804	23.5079	1.8264	0.6190	0.6827	0.3665
50	11.3087	5.6150	0.5294	0.6180	0.3570	0.2307
75	11.3110	5.6140	0.7155	0.6171	0.4577	0.2646
100	5.2760	2.1381	0.2182	0.6520	0.1989	0.1549
			$\rho = .90, \rho_x = .99$			
25	173.9707	105.3340	4.8775	0.4448	0.5924	0.2755
50	51.3406	23.0601	0.9171	0.5306	0.3147	0.2129
75	54.7160	24.7351	1.4043	0.4557	0.3935	0.2078
100	19.0941	9.1932	0.4137	0.5390	0.2216	0.1566

Table-5: TMSE values for the different estimators when $m = 4$ and $k_i = 6$

n	GLS estimators		Ridge estimators			
	Zellner	**Parks**	**SK**	**AS**	**new1**	**new2**
			$\rho = .75, \rho_x = .90$			
25	16.8465	13.1986	1.0562	0.6676	0.9840	0.3417
50	4.6038	3.4177	0.3246	0.7259	0.6209	0.2279
75	3.7594	3.2273	0.4822	0.7142	0.7731	0.3087
100	1.7489	1.1432	0.1022	0.7779	0.3624	0.1579
			$\rho = .75, \rho_x = .95$			
25	36.5553	25.6178	2.2447	0.5874	1.2485	0.3990
50	10.8707	8.8997	0.8008	0.6222	0.8868	0.3196
75	7.4498	5.8000	0.6238	0.6173	0.8033	0.2815
100	3.5236	1.8671	0.1404	0.7208	0.4102	0.1541
			$\rho = .75, \rho_x = .99$			
25	169.7513	150.4132	9.8967	0.4188	1.1344	0.2985
50	55.0262	38.1009	1.5649	0.4687	0.6665	0.1833
75	38.6596	31.7640	2.5263	0.4106	1.0364	0.2811
100	17.5981	10.3540	0.5447	0.5344	0.5054	0.1225
			$\rho = .90, \rho_x = .90$			
25	41.9803	22.4930	1.9984	0.7424	1.2546	0.4909
50	11.2528	4.8031	0.4819	0.8075	0.6931	0.3232
75	6.7004	4.5999	0.5688	0.7442	0.7907	0.3618
100	4.1897	1.6760	0.1359	0.8086	0.3526	0.1524
			$\rho = .90, \rho_x = .95$			
25	67.5143	72.4064	5.5347	0.6184	1.6124	0.5655
50	19.8076	10.2837	0.6387	0.7189	0.6434	0.2393
75	15.8237	10.0435	1.0654	0.6136	0.9662	0.3661
100	8.2728	2.7418	0.1579	0.7359	0.3676	0.1346
			$\rho = .90, \rho_x = .99$			
25	492.4736	270.4675	18.6617	0.4949	1.0279	0.3705
50	130.6941	46.6131	1.5592	0.5094	0.5434	0.1714
75	99.9280	35.1452	1.3777	0.4972	0.5260	0.1627
100	42.6739	20.6565	0.8736	0.5207	0.5462	0.1520

Table-6: TMSE values for the different estimators when $m = 6$ and $k_i = 4$

n	GLS estimators		Ridge estimators			
	Zellner	**Parks**	**SK**	**AS**	**new1**	**new2**
			$\boldsymbol{\rho = .75, \rho_x = .90}$			
25	13.1908	9.3313	0.9860	0.7433	0.5123	0.2598
50	4.1172	2.3364	0.2908	0.8153	0.2747	0.2332
75	2.4507	1.1278	0.1460	0.8166	0.1754	0.1759
100	1.9554	1.0619	0.1622	0.7737	0.1971	0.1535
			$\boldsymbol{\rho = .75, \rho_x = .95}$			
25	41.2118	30.5880	3.2372	0.6794	0.8124	0.3095
50	8.1734	4.3377	0.3959	0.7029	0.2801	0.1620
75	4.8312	2.4020	0.2178	0.7478	0.2061	0.1556
100	4.1129	2.2568	0.2517	0.7152	0.2349	0.1464
			$\boldsymbol{\rho = .75, \rho_x = .99}$			
25	127.5892	98.9370	5.3107	0.5734	0.6111	0.2635
50	41.3215	25.0368	0.9554	0.5450	0.2801	0.1258
75	22.1921	12.1928	0.5790	0.5915	0.2004	0.0887
100	16.7432	9.3874	0.5003	0.5712	0.2275	0.1115
			$\boldsymbol{\rho = .90, \rho_x = .90}$			
25	37.9816	19.5995	1.5264	0.7903	0.7251	0.4153
50	10.5094	4.3923	0.4768	0.8169	0.3684	0.3001
75	6.0622	1.7671	0.1880	0.8502	0.1949	0.2428
100	5.3421	1.7021	0.2544	0.8174	0.2505	0.2270
			$\boldsymbol{\rho = .90, \rho_x = .95}$			
25	54.0946	31.5410	2.3574	0.7735	0.8462	0.4899
50	17.1570	6.6839	0.4138	0.7620	0.2668	0.2371
75	15.9178	4.6703	0.3030	0.7567	0.2340	0.1910
100	11.4185	3.2598	0.3106	0.7554	0.2568	0.2030
			$\boldsymbol{\rho = .90, \rho_x = .99}$			
25	447.2925	245.7928	12.6826	0.6118	0.7526	0.3805
50	76.3002	32.6852	0.8465	0.6340	0.3093	0.2282
75	53.2714	15.2228	0.4093	0.6560	0.1846	0.1642
100	49.6410	14.2505	0.4808	0.5930	0.2342	0.1640

Table-7: TMSE values for the different estimators when $m = 6$ and $k_i = 6$

n	GLS estimators		Ridge estimators			
	Zellner	Parks	SK	AS	new1	new2
			$\rho = .75, \rho_x = .90$			
25	23.5063	17.8416	1.2763	0.7880	1.2509	0.3610
50	8.2999	4.7767	0.4063	0.8438	0.7827	0.2695
75	4.4318	2.2069	0.1833	0.8652	0.5262	0.1872
100	3.9531	2.0619	0.2135	0.8251	0.5929	0.2216
			$\rho = .75, \rho_x = .95$			
25	64.7327	52.2059	4.1619	0.7294	1.8123	0.4940
50	17.4715	9.9942	0.7164	0.7382	0.9191	0.2613
75	7.4082	4.1413	0.2701	0.8049	0.6118	0.1781
100	7.8235	4.0963	0.3471	0.7642	0.7084	0.2102
			$\rho = .75, \rho_x = .99$			
25	330.3648	245.4258	11.0351	0.6163	1.1191	0.2919
50	80.5724	48.9693	2.2113	0.6169	0.8576	0.1929
75	40.1462	20.8563	0.8985	0.6257	0.6670	0.1408
100	38.7441	19.6973	1.1073	0.5804	0.8298	0.1753
			$\rho = .90, \rho_x = .90$			
25	56.9650	36.1794	3.3353	0.8242	1.9120	0.6542
50	21.8188	10.5090	0.7616	0.8405	0.9688	0.3395
75	10.9613	3.9171	0.2821	0.8793	0.5785	0.2293
100	9.2270	2.9419	0.2515	0.8748	0.5663	0.2257
			$\rho = .90, \rho_x = .95$			
25	117.9301	70.2598	3.6470	0.7898	1.3298	0.4357
50	39.7162	17.3205	0.7861	0.7818	0.7812	0.2409
75	17.7821	5.6742	0.2749	0.8510	0.5013	0.1763
100	15.9975	5.0113	0.2991	0.7953	0.5590	0.1758
			$\rho = .90, \rho_x = .99$			
25	535.6719	277.0767	14.1436	0.5895	1.1646	0.3146
50	271.4577	92.6541	2.5172	0.6084	0.6868	0.1785
75	93.6418	31.1750	0.8587	0.6557	0.5084	0.1284
100	68.2881	25.6952	0.9064	0.6249	0.6197	0.1820

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Source of support: Nil, Conflict of interest: None Declared.