

Semi generalized b-strongly b*-open sets in Topological Spaces

P. SELVAN^{1*} AND K. BAGEERATHI²

^{1,2}Department of Mathematics,
Aditanar College of Arts and Science, Tiruchendur, India.

(Received On: 21-06-19; Revised & Accepted On: 20-07-19)

ABSTRACT

In this paper a new class of open sets in topological spaces, namely semi generalized b-strongly b*-open (briefly, sgbsb*-open) set is introduced. We give some basic properties and various characterizations of sgbsb*-open sets. Also we introduce sgbsb*-neighbourhood in a topological spaces and investigate some basic properties.

Mathematics Subject Classification 2010: 54A05.

Keywords: sgbsb*-closed, sgbsb*-open, sgbsb*-nbhd.

1. INTRODUCTION

In 1970, Levine[8] introduced the class of generalized closed sets. The notion of generalized closed sets has been extended and studied exclusively in recent years by many topologists. In 1996, Andrić [16] gave a new type of generalized closed sets in topological spaces called b-closed sets. A.Poongothai and R.Parimelazhagan [21] introduced sb*-closed sets and investigated some of their properties in 2012. Later in 2017, P.Selvan and M.J.Jeyanthi introduced generalized b-strongly b*-closed sets and investigated some of their properties.

In this paper, we introduced a new class of open sets namely semi generalized b-strongly b*-open sets sets, using the generalized b-strongly b*-interior operator instead of the interior operator in the definition of semi-open sets. The notion of semi generalized b-strongly b*-closed set and its different characterizations are given in this paper.

2. PRELIMINARIES

Throughout this paper (X, τ) represents a topological space on which no separation axiom is assumed unless otherwise mentioned. (X, τ) will be replaced by X if there is no changes of confusion. For a subset A of a topological space X , $\text{cl}(A)$ and $\text{int}(A)$ denote the closure of A and the interior of A respectively. We recall the following definitions and results.

Definition 2.1: Let (X, τ) be a topological space. A subset A of the space X is said to be

- (i) semi-open [6] if $A \subseteq \text{cl}(\text{int}(A))$ and semi-closed [3] if $\text{int}(\text{cl}(A)) \subseteq A$.
- (ii) α -open [7] if $A \subseteq \text{int}(\text{cl}(\text{int}(A)))$ and α -closed if $\text{cl}(\text{int}(\text{cl}(A))) \subseteq A$.
- (iii) b-open [3] if $A \subseteq \text{int}(\text{cl}(A)) \cup \text{cl}(\text{int}(A))$ and b-closed if $\text{int}(\text{cl}(A)) \cap \text{cl}(\text{int}(A)) \subseteq A$.
- (iv) regular open [8] if $\text{int}(\text{cl}(A)) = A$ and regular closed if $\text{cl}(\text{int}(A)) = A$.
- (v) π -open [13] if A is the union of regular open sets and π -closed if A is the intersection of regular closed sets.

Definition 2.2: Let (X, τ) be a topological space and $A \subseteq X$. The b-closure (resp.pre-closure, semi-closure, α -closure) of A , denoted by $\text{bcl}(A)$ (resp. $\text{pcl}(A)$, $\text{scl}(A)$, $\alpha\text{cl}(A)$) and is defined by the intersection of all b-closed (resp. pre-closed, semi-closed, α -closed) sets containing A .

Definition 2.3: Let (X, τ) be a topological space and $A \subseteq X$. The b-interior (resp.pre- interior, semi- interior, α -interior) of A , denoted by $\text{bint}(A)$ (resp. $\text{pint}(A)$, $\text{shint}(A)$, $\alpha\text{int}(A)$) and is defined by the intersection of all b-open (resp. pre-open, semi-open, α -open) sets contained in A .

Corresponding Author: P.Selvan^{1},*

^{1,2}Department of Mathematics, Aditanar College of Arts and Science, Tiruchendur, India.

Definition 2.4: Let (X, τ) be a topological space. A subset A of X is said to be

- (i) strongly b^* -closed [9](briefly sb^* -closed) if $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is b -open in (X, τ) .
- (ii) Generalized b -strongly b^* -closed [10](briefly $gbsb^*$ -closed) if $bcl(A) \subseteq U$, whenever $A \subseteq U$ and U is sb^* -open in (X, τ) .

The complements of the above mentioned closed sets are their respective open sets.

Definition 2.5: [12] A subset N of a space X , is called a neighbourhood (simply, nbhd) of $A \subseteq X$ if there exists an open set U such that $A \subseteq U \subseteq N$.

Lemma 2.6: For any subset A of a topological space (X, τ) ,

- (i) $sint(A) = A \cap cl(int(A))$
- (ii) $pin(A) = A \cap int(cl(A))$
- (iii) $scl(A) = A \cup int(cl(A))$
- (iv) $pcl(A) = A \cup cl(int(A))$.

Definition 2.7: [11] Let A be a subset of a topological space (X, τ) . Then the union of all $gbsb^*$ -open sets contained in A is called the $gbsb^*$ -interior of A and it is denoted by $gbsb^*int(A)$. That is, $gbsb^*int(A) = \cup \{V : V \subseteq A \text{ and } V \in gbsb^*O(X)\}$.

Definition 2.8: [11] Let A be a subset of a topological space (X, τ) . Then the intersection of all $gbsb^*$ -closed sets in X containing A is called the $gbsb^*$ -closure of A and it is denoted by $gbsb^*cl(A)$. That is, $gbsb^*cl(A) = \cap \{F : A \subseteq F \text{ and } F \in gbsb^*C(X)\}$.

Remark 2.9: [11] For any subset A of a topological space (X, τ) ,

- (i) $X \setminus gbsb^*cl(A) = gbsb^*int(X \setminus A)$
- (ii) $X \setminus gbsb^*int(A) = gbsb^*cl(X \setminus A)$.

Definition 2.10: [11] Let A be a subset of a topological space X . A point $x \in X$ is said to be $gbsb^*$ -limit point of A if $G \cap (A \setminus \{x\}) \neq \phi$, for every $gbsb^*$ -open set G containing x .

Definition 2.11: [11] The set of all $gbsb^*$ -limit points of A is called the $gbsb^*$ -derived set of A and is denoted by $D_{gbsb^*}(A)$.

Lemma 2.12: [11] For any subset A of a topological space (X, τ) ,

- (i) $gbsb^*int(A) = A \setminus D_{gbsb^*}(X \setminus A)$
- (ii) $gbsb^*cl(A) = A \cup D_{gbsb^*}(A)$

3. Semi generalized b-strongly b^* -open set

Definition 3.1: A subset A of a topological space (X, τ) is said to be a semi-generalized b -strongly b^* -open set (briefly, semi- $gbsb^*$ -open or $sgbsb^*$ -open) if $A \subseteq cl(gbsb^*int(A))$.

Theorem 3.2: Every open set is $sgbsb^*$ -open.

Proof: Let A be an open subset of a topological space (X, τ) . Then $A = int(A) \subseteq cl(int(A)) \subseteq cl(gbsb^*int(A))$ and hence A is $sgbsb^*$ -open.

Remark 3.3: The converse of the above theorem is not true which is shown in the following example.

Example 3.4: Let $X = \{a, b, c, d\}$ with $\tau = \{\phi, \{a\}, \{a, b\}, \{a, b, c\}, X\}$. The set $\{b, c\}$ is $sgbsb^*$ -open but not an open sets.

Theorem 3.5: Every semi-open set is $sgbsb^*$ -open.

Proof: Let A be a semi-open subset of a topological space (X, τ) . Then $cl(gbsb^*int(A)) \supseteq cl(int(A)) \supseteq A$ and hence A is $sgbsb^*$ -open.

Remark 3.6: The converse of the above theorem is not true which is shown in the following example.

Example 3.7: Let $X = \{a, b, c, d\}$ with $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}, \{b, c, d\}, X\}$. The set $\{b\}$ is $sgbsb^*$ -open but not a semi-open set.

Theorem 3.8: Every α -open set is sgbsb*-open.

Proof: Let A be a α -open subset of a topological space (X, τ) . Then $A \subseteq \text{int}(\text{cl}(\text{int}(A))) \subseteq \text{cl}(\text{int}(A)) \subseteq \text{cl}(\text{gbsb}^*\text{int}(A))$ and hence A is sgbsb*-open.

Remark 3.9: The converse of the above theorem is not true which is shown in the following example.

Example 3.10: Let $X = \{a, b, c, d\}$ with $\tau = \{\emptyset, \{a\}, \{a,b\}, \{a,b,c\}, X\}$. The set $\{a,c\}$ is sgbsb*-open set but not a α -open set.

Theorem 3.11: Every regular open set is sgbsb*-open.

Proof: Let A be a regular open subset of a topological space (X, τ) . Since every regular open set is open and by Theorem 3.2, A is sgbsb*-open.

Remark 3.12: The converse of the above theorem is not true which is shown in the following example.

Example 3.13: Let $X = \{a, b, c, d\}$ with $\tau = \{\emptyset, \{a\}, \{b\}, \{a,b\}, \{b,c\}, \{a,b,c\}, \{b,c,d\}, X\}$. The set $\{a,b,c\}$ is sgbsb*-open but not a regular-open set.

Theorem 3.14: Every π -open set is sgbsb*-open.

Proof: Let A be a π -open subset of a topological space (X, τ) . Since every π -open set is open and by Theorem 3.2, A is sgbsb*-open.

Remark 3.15: The converse of the above theorem is not true which is shown in the following example.

Example 3.16: Let $X = \{a, b, c, d\}$ with $\tau = \{\emptyset, \{a\}, \{a,b\}, \{a,b,c\}, X\}$. The sets $\{a,c\}$ and $\{a,d\}$ are sgbsb*-open sets but not a π -open sets.

Theorem 3.17: Every gbsb*-open set is sgbsb*-open.

Proof: Let A be a gbsb*-open subset of a topological space (X, τ) . Then $A \subseteq \text{cl}(A) = \text{cl}(\text{gbsb}^*\text{int}(A))$ and hence A is sgbsb*-open.

Remark 3.18: The converse of the above theorem is not true which is shown in the following example.

Example 3.19: Let $X = \{a, b, c, d\}$ with $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}, \{b, c, d\}, X\}$. The set $\{a, c, d\}$ is sgbsb*-open but not a gbsb*-open set.

Theorem 3.20: Every b-open set is sgbsb*-open.

Proof: Let A be a b-open subset of a topological space (X, τ) . Then and hence $A \subseteq \text{cl}(A) = \text{cl}(\text{bint}(A)) \subseteq \text{cl}(\text{gbsb}^*\text{int}(A))$ is sgbsb*-open.

Remark 3.21: The converse of the above theorem is not true which is shown in the following example.

Example 3.22: Let $X = \{a, b, c, d\}$ with $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}, \{b, c, d\}, X\}$. The set $\{a, c, d\}$ is sgbsb*-open but not a b-open set.

Theorem 3.23: A subset A of X is sgbsb*-open if and only if there exists a gbsb*-open set U such that $U \subseteq A \subseteq \text{cl}(U)$.

Proof: Necessity. If A is sgbsb*-open, then $A \subseteq \text{cl}(\text{gbsb}^*\text{int}(A))$. Take $U = \text{gbsb}^*\text{int}(A)$. Then U is an gbsb*-open set in X such that $U \subseteq A \subseteq \text{cl}(U)$.

Sufficiency. Assume that there is an gbsb*-open set U such that $U \subseteq A \subseteq \text{cl}(U)$.

Now $U \subseteq A \Rightarrow U = \text{gbsb}^*\text{int}(U) \subseteq \text{gbsb}^*\text{int}(A) \Rightarrow A \subseteq \text{cl}(U) \subseteq \text{cl}(\text{gbsb}^*\text{int}(A))$. Therefore A is sgbsb*-open.

Theorem 3.24: The union of two sgbsb*-open sets is also a sgbsb*-open set.

Proof: Let A and B be two sgbsb*-open sets in a topological space (X, τ) . Then $A \subseteq \text{cl}(\text{gbsb}^*\text{int}(A))$ and $B \subseteq \text{cl}(\text{gbsb}^*\text{int}(B))$. Now, $A \cup B \subseteq \text{cl}(\text{gbsb}^*\text{int}(A)) \cup \text{cl}(\text{gbsb}^*\text{int}(B)) \subseteq \text{cl}(\text{gbsb}^*\text{int}(A) \cup \text{gbsb}^*\text{int}(B)) \subseteq \text{cl}(\text{gbsb}^*\text{int}(A \cup B))$. Therefore, $A \cup B$ is sgbsb*-open.

Remark 3.25: Arbitrary union of sgbsb*-open sets of a topological space is also a sgbsb*-open set.

Remark 3.26: The finite intersection of sgbsb*-open sets need not be a sgbsb*-open, which is shown in the following example.

Example 3.27: Let $X = \{a, b, c, d\}$ with $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}, \{b, c, d\}, X\}$. The sets $\{b, c, d\}$ and $\{a, c, d\}$ are sgbsb*-open set but their intersection $\{c, d\}$ is not a sgbsb*-open set.

Theorem 3.28: If a topological space (X, τ) , let $\tau_{sgbsb^*} = \{U \in \text{sgbsb}^*\text{-O}(X, \tau) / U \cap A \in \text{sgbsb}^*\text{-O}(X, \tau) \text{ for all } A \in \text{sgbsb}^*\text{-O}(X, \tau)\}$. Then τ_{sgbsb^*} is a topology on X.

Proof: Clearly $\phi, X \in \tau_{sgbsb^*}$. Let $U_\beta \in \tau_{sgbsb^*}$ and $U = \cup U_\beta$. Since each $U_\beta \in \tau_{sgbsb^*}$, then by Remark 3.23, $U \in \text{sgbsb}^*\text{-O}(X, \tau)$. Let $A \in \text{sgbsb}^*\text{-O}(X, \tau)$. Then $U_\beta \cap A \in \text{sgbsb}^*\text{-O}(X, \tau)$ for each β . Hence $U \cap A = (\cup U_\beta) \cap A = \cup (U_\beta \cap A) \in \text{sgbsb}^*\text{-O}(X, \tau)$. Therefore $U \in \tau_{sgbsb^*}$. Let $U_1, U_2 \in \tau_{sgbsb^*}$. Then $U_1, U_2 \in \text{sgbsb}^*\text{-O}(X, \tau)$ and from definition of τ_{sgbsb^*} , $U_1 \cap U_2 \in \text{sgbsb}^*\text{-O}(X, \tau)$. If $A \in \text{sgbsb}^*\text{-O}(X, \tau)$, and from definition of τ_{sgbsb^*} , $U_1 \cap U_2 \cap A \in \text{sgbsb}^*\text{-O}(X, \tau)$. Hence $U_1 \cap U_2 \in \tau_{sgbsb^*}$. This shows that τ_{sgbsb^*} is closed under finite intersection. Hence τ_{sgbsb^*} is a topology on X.

Theorem 3.29: A subset A is sgbsb*-open iff $\text{cl}(A) = \text{cl}(\text{gbsb}^*\text{int}(A))$.

Proof: Necessity. Since A is sgbsb*-open, $A \subseteq \text{cl}(\text{gbsb}^*\text{int}(A))$. Hence $\text{cl}(A) \subseteq \text{cl}(\text{gbsb}^*\text{int}(A))$. Also we have, $\text{cl}(\text{gbsb}^*\text{int}(A)) \subseteq \text{cl}(A)$. Hence $\text{cl}(A) = \text{cl}(\text{gbsb}^*\text{int}(A))$.

Sufficiency. Take $U = \text{gbsb}^*\text{int}(A)$. Then U is a gbsb*-open set in X such that $U \subseteq A \subseteq \text{cl}(A) = \text{cl}(\text{gbsb}^*\text{int}(A)) = \text{cl}(U)$. Therefore by Theorem 3.23, A is sgbsb*-open.

Theorem 3.30: Let A be sgbsb*-open and $B \subseteq X$ such that $A \subseteq B \subseteq \text{cl}(A)$. Then B is sgbsb*-open.

Proof: Since A is sgbsb*-open, $A \subseteq \text{cl}(\text{gbsb}^*\text{int}(A))$. Since $\text{gbsb}^*\text{int}(A) \subseteq \text{gbsb}^*\text{int}(B)$, $\text{cl}(\text{gbsb}^*\text{int}(A)) \subseteq \text{cl}(\text{gbsb}^*\text{int}(B))$. Therefore by the above theorem, $B \subseteq \text{cl}(A) = \text{cl}(\text{gbsb}^*\text{int}(A)) \subseteq \text{cl}(\text{gbsb}^*\text{int}(B))$. Hence B is sgbsb*-open.

Theorem 3.31: For a subset A of a topological space (X, τ) the following statements are equivalent:

- (i) A is sgbsb*-open.
- (ii) $A \subseteq \text{cl}(\text{gbsb}^*\text{int}(A))$.
- (iii) $\text{cl}(\text{gbsb}^*\text{int}(A)) = \text{cl}(A)$.
- (iv) $\text{cl}(A \setminus \text{D}_{\text{gbsb}^*}(X \setminus A)) = \text{cl}(A)$.

4. Semi-generalized b-strongly b*-closed set.

Definition 4.1: A subset A of a space (X, τ) is called a semi-generalized b-strongly b*-closed set (briefly, semi-gbsb*-closed or sgbsb*-closed) if $X \setminus A$ is sgbsb*-open. The set of all sgbsb*-open sets in (X, τ) is denoted by $\text{sgbsb}^*\text{-O}(X, \tau)$.

Theorem 4.2: For a topological space (X, τ) ,

- (i) Every closed set is sgbsb*-closed.
- (ii) Every semi-closed set is sgbsb*-closed.
- (iii) Every α -closed set is sgbsb*-closed.
- (iv) Every regular closed set is sgbsb*-closed.
- (v) Every π -closed set is sgbsb*-closed.
- (vi) Every gbsb*-closed set is sgbsb*-closed.
- (vii) Every b-closed set is sgbsb*-closed.

Theorem 4.3: A subset A of a space (X, τ) is sgbsb*-closed if and only if there is a gbsb*-closed set F in (X, τ) such that $\text{int}(F) \subseteq A \subseteq F$.

Proof: Necessity. Suppose A is sgbsb*-closed. Then $X \setminus A$ is sgbsb*-open. Then there exists a gbsb*-open set U in X such that $U \subseteq X \setminus A \subseteq \text{cl}(U)$ which implies $X \setminus U \supseteq A \supseteq X \setminus \text{cl}(U)$. That implies, $X \setminus U \supseteq A \supseteq \text{int}(X \setminus U)$ where $X \setminus U$ is gbsb*-closed in X.

Sufficiency. Suppose there is a gbsb*-closed set F in (X, τ) such that $\text{int}(F) \subseteq A \subseteq F$ which implies $X \setminus \text{int}(F) \supseteq X \setminus A \supseteq X \setminus F$. Since $X \setminus \text{int}(F) = \text{cl}(X \setminus F)$, we have $\text{cl}(X \setminus F) \supseteq X \setminus A \supseteq X \setminus F$ where $X \setminus F$ is a gbsb*-open set. Hence $X \setminus A$ is sgbsb*-open. Therefore A is sgbsb*-closed.

Theorem 4.4: $A \subseteq X$ is $sgbsb^*$ -closed if and only if $\text{int}(gbsb^*cl(A)) \subseteq A$.

Proof: Necessity. Suppose A is $sgbsb^*$ -closed. Then $X \setminus A$ is $sgbsb^*$ -open. Therefore $X \setminus A \subseteq \text{cl}(gbsb^*int(X \setminus A))$ and hence $\text{int}(gbsb^*cl(A)) \subseteq A$.

Sufficiency. Assume that $\text{int}(gbsb^*cl(A)) \subseteq A$. Take $F = gbsb^*cl(A)$. Then F is a $gbsb^*$ -closed set in X such that $\text{int}(F) \subseteq A \subseteq F$ and hence A is $sgbsb^*$ -closed.

Theorem 4.5: If A is $sgbsb^*$ -closed in X and $B \subseteq X$ is such that $\text{int}(A) \subseteq B \subseteq A$. Then B is $sgbsb^*$ -closed in X .

Theorem 4.6: The intersection of two $sgbsb^*$ -closed sets is also $sgbsb^*$ -closed.

Proof: Let A and B be two $sgbsb^*$ -closed sets in a topological space (X, τ) . Then $\text{int}(gbsb^*cl(A)) \subseteq A$ and $\text{int}(gbsb^*cl(B)) \subseteq B$. Now, $\text{int}(gbsb^*cl(A \cap B)) \subseteq \text{int}(gbsb^*cl(A) \cap gbsb^*cl(B)) = \text{int}(gbsb^*cl(A)) \cap \text{int}(gbsb^*cl(B)) \subseteq A \cap B$. Therefore, $A \cap B$ is $sgbsb^*$ -closed.

Remark 4.7: Arbitrary intersection of $sgbsb^*$ -closed sets of a topological space is also a $sgbsb^*$ -closed set.

Remark 4.8: The union of $sgbsb^*$ -closed sets need not be a $sgbsb^*$ -closed set.

Theorem 4.9: If A is $sgbsb^*$ -closed and U is $sgbsb^*$ -open in X , then $A \setminus U$ is $sgbsb^*$ -closed in X .

Proof: Since $A \setminus U = A \cap (X \setminus U)$, A and $X \setminus U$ are $sgbsb^*$ -closed sets, by Theorem 4.6, $A \setminus U$ is $sgbsb^*$ -closed in X .

Theorem 4.10: A subset A is $sgbsb^*$ -closed iff $\text{int}(A) = \text{int}(gbsb^*cl(A))$.

Proof: Necessity. Since A is $sgbsb^*$ -closed, $\text{int}(gbsb^*cl(A)) \subseteq A$. Hence $\text{int}(gbsb^*cl(A)) \subseteq \text{int}(A)$. Also we have, $\text{int}(A) \subseteq \text{int}(gbsb^*cl(A))$. Hence $\text{int}(A) = \text{int}(gbsb^*cl(A))$.

Sufficiency. Take $U = gbsb^*cl(A)$. Then U is a $gbsb^*$ -closed set in X such that $\text{int}(U) \subseteq A \subseteq U$. Therefore by Theorem 4.3, A is $sgbsb^*$ -closed.

Theorem 4.11: For a subset A of a topological space (X, τ) the following statements are equivalent:

- (i) A is $sgbsb^*$ -closed.
- (ii) $\text{int}(gbsb^*cl(A)) \subseteq A$.
- (iii) $\text{int}(gbsb^*cl(A)) = \text{int}(A)$.
- (iv) $cl(A \cup D_{gbsb^*}(A)) = cl(A)$.

Theorem 4.12: Let A be a $sgbsb^*$ -closed in X . Then

- (i) $\text{sint}(A)$ is $sgbsb^*$ -closed.
- (ii) If A is regular open, then $\text{pint}(A)$ and $\text{scl}(A)$ are also $sgbsb^*$ -closed.
- (iii) If A is regular closed, then $\text{pcl}(A)$ is also $sgbsb^*$ -closed.

Proof: Let A be a $sgbsb^*$ -closed set of X .

- (i) Since $cl(\text{int}(A))$ is closed, then by Theorem 4.2, $cl(\text{int}(A))$ is $sgbsb^*$ -closed. By Lemma 2.6, $\text{sint}(A)$ is $sgbsb^*$ -closed.
- (ii) Suppose A is regular open, then $\text{int}(cl(A)) = A$. By Lemma 2.6, $\text{scl}(A) = A$. Since A is $sgbsb^*$ -closed, then $\text{scl}(A)$ is $sgbsb^*$ -closed. Similarly $\text{pint}(A)$ is $sgbsb^*$ -closed.
- (iii) Suppose A is regular closed, $cl(\text{int}(A)) = A$. Then by Lemma 2.6, $\text{pcl}(A) = A$, and hence $sgbsb^*$ -closed.

5. $sgbsb^*$ -neighbourhood

Definition 5.1: Let X be a topological space and let $x \in X$. A subset N of X is said to be a $sgbsb^*$ -neighbourhood (shortly, $sgbsb^*$ -nbhd) of x if there exists a $sgbsb^*$ -open set U such that $x \in U \subseteq N$.

Definition 5.2: A subset N of a space X , is called a $sgbsb^*$ -nbhd of $A \subseteq X$ if there exists an $sgbsb^*$ -open set U such that $A \subseteq U \subseteq N$.

Theorem 5.3: Every nbhd N of $x \in X$ is a $sgbsb^*$ -nbhd of x .

Proof: Let N be nbhd of point $x \in X$. Then there exists an open set U such that $x \in U \subseteq N$. Since every open set is $sgbsb^*$ -open, U is a $sgbsb^*$ -open set such that $x \in U \subseteq N$. This implies, N is a $sgbsb^*$ -nbhd of x .

Remark 5.4: The converse of the above theorem need not be true which is shown in the following example.

Example 5.5: Let $X = \{a, b, c, d\}$ with topology $\tau = \{\phi, \{a\}, \{b\}, \{a,b\}, \{b,c\}, \{a,b,c\}, \{b,c,d\}, X\}$. In this topological space (X, τ) , $sgbsb^*-O(X) = \{\phi, \{a\}, \{b\}, \{a,b\}, \{a,d\}, \{b,c\}, \{a, b, c\}, \{a,b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$. The set $\{a,d\}$ is the $sgbsb^*$ -nbhd of d , since $\{a,d\}$ is $sgbsb^*$ -open set such that $d \in \{a,d\} \subseteq \{a, d\}$. However, the set $\{b, d\}$ is not a nbhd of the point d .

Remark 5.6: Every $sgbsb^*$ -open set is a $sgbsb^*$ -nbhd of each of its points.

Theorem 5.7: If F is a $sgbsb^*$ -closed subset of X and $x \in X \setminus F$, then there exists a $sgbsb^*$ -nbhd N of x such that $N \cap F = \phi$.

Proof: Let F be $sgbsb^*$ -closed subset of X and $x \in X \setminus F$. Then $X \setminus F$ is $sgbsb^*$ -open set of X . By Theorem 4.6, $X \setminus F$ contains a $sgbsb^*$ -nbhd of each of its points. Hence there exists a $sgbsb^*$ -nbhd N of x such that $N \subseteq X \setminus F$. Hence $N \cap F = \phi$.

Definition 5.8: The collection of all $sgbsb^*$ -neighborhoods of $x \in X$ is called the $sgbsb^*$ -neighborhood system of x and is denoted by $sgbsb^*-N(x)$.

Theorem 5.9: Let (X, τ) be a topological space and $x \in X$. Then

- (i) $sgbsb^*-N(x) \neq \phi$ and $x \in$ each member of $sgbsb^*-N(x)$
- (ii) If $N \in sgbsb^*-N(x)$ and $N \subseteq M$, then $M \in sgbsb^*-N(x)$.
- (iii) Each member $N \in sgbsb^*-N(x)$ is a superset of a member $G \in sgbsb^*-N(x)$ where G is a $sgbsb^*$ -open set.

Proof:

- (i) Since X is $sgbsb^*$ -open set containing x , it is a $sgbsb^*$ -nbhd of every $x \in X$. Thus for each $x \in X$, there exists at least one $sgbsb^*$ -nbhd, namely X . Therefore, $sgbsb^*-N(x) \neq \phi$. Let $N \in sgbsb^*-N(x)$. Then N is a $sgbsb^*$ -nbhd of x . Hence there exists a $sgbsb^*$ -open set G such that $x \in G \subseteq N$, so $x \in N$. Therefore $x \in$ every member N of $sgbsb^*-N(x)$.
- (ii) If $N \in sgbsb^*-N(x)$, then there is a $sgbsb^*$ -open set G such that $x \in G \subseteq N$. Since $N \subseteq M$, M is $sgbsb^*$ -nbhd of x . Hence $M \in sgbsb^*-N(x)$.
- (iii) Let $N \in sgbsb^*-N(x)$. Then there is a $sgbsb^*$ -open set G , such that $x \in G \subseteq N$. Since G is $sgbsb^*$ -open and $x \in G$, G is $sgbsb^*$ -nbhd of x . Therefore $G \in sgbsb^*-N(x)$ and also $G \subseteq N$.

REFERENCES

1. Adiya.K.Hussein, New type of generalized closed sets, Mathematical Theory and Modeling, Vol.3 (4), 2013.
2. Ahmad Al-Omari and Mohd.Salmi Md. Noorani, On Generalized b-closed sets. Bull. Malays.math. Sci. Soc (2) 32(1) (2009),19-30.
3. D.Andrijevic, On b-open sets, Mat.Vesnik 48(1996), 59-64.
4. M.Anitha and P.Thangavelu, Generalized closed sets in submaximal spaces, Antarctica J. Math., 4(1) (2007), 99-108.
5. N. Levine, Generalized closed sets in topology, Rand. Circ. Mat. Palermo, 19(2) (1970), 89-96.
6. N. Levine, Semi-open sets and semi-continuity in Topological Spaces, Amer. Mat. Monthly 70(1)(1963), 36- 41.
7. O. Njastad, Some classes of nearly open sets, Pacific J. Math., 15(1965), 961- 970.
8. N.Palaniappan and K.C. Rao, Regular generalized closed sets, Kyungpook Math.J., 33(2) (1993), 211-219.
9. A.Poongothai and R.Parimelazhagan, sb*-closed sets in a topological spaces, Int. Journal of Math. Analysis 6(47), (2012), 2325-2333.
10. P. Selvan and M.J.Jeyanthi, Generalized b-strongly b*-closed sets in topological spaces, Int. Journal of Math. Archive-8(5), (2017), 27-34.
11. P. Selvan and M.J.Jeyanthi, More on gbsb*-closed and sets in topological spaces, Scholars Journal of Physics, Mathematics and Statistics, 4(13), 2017, 113-120.
12. Willard S., General Topology, Addison Wesley, 1970.
13. V.Zaitsav, On certain classes of topological spaces and their bicompatifications, Dokl. Akad.Nauk SSSR, 178 (1968), 778-779.

Source of support: Nil, Conflict of interest: None Declared.

[Copy right © 2019. This is an Open Access article distributed under the terms of the International Journal of Mathematical Archive (IJMA), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.]