International Journal of Mathematical Archive-10(7), 2019, 37-42
Available online through www.ijma.info ISSN 2229 – 5046

Semi generalized b-strongly b*-open sets in Topological Spaces

P. SELVAN[1*] AND K. BAGEERATHI[2]

[1,2]Department of Mathematics,
Aditanar College of Arts and Science, Tiruchendur, India.

(Received On: 21-06-19; Revised & Accepted On: 20-07-19)

ABSTRACT

In this paper a new class of open sets in topological spaces, namely semi generalized b-strongly b-open (briefly, sgbsb*-open) set is introduced. We give some basic properties and various characterizations of sgbsb*-open sets. Also we introduce sgbsb*-neighbourhood in a topological spaces and investigate some basic properties.*

Mathematics Subject Classification 2010: *54A05.*

Keywords: *sgbsb*-closed, sgbsb*-open, sgbsb*-nbhd.*

1. INTRODUCTION

In 1970, Levine[8] introduced the class of generalized closed sets. The notion of generalized closed sets has been extended and studied exclusively in recent years by many topologists. In 1996, Andrjivic [16] gave a new type of generalized closed sets in topological spaces called b-closed sets. A.Poongothai and R.Parimelazhagan [21] introduced sb*-closed sets and investigated some of their properties in 2012. Later in 2017, P.Selvan and M.J.Jeyanthi introduced generalized b-strongly b*-closed sets and investicated some of their properties.

In this paper, we introduced a new class of open sets namely semi generalized b-strongly b*-open sets sets, using the generalized b-strongly b*-interior operator instead of the interior operator in the definition of semi-open sets. The notion of semi generalized b-strongly b*-closed set and its different characterizations are given in this paper.

2. PRELIMINARIES

Throughout this paper (X, τ) represents a topological space on which no separation axiom is assumed unless otherwise mentioned. (X, τ) will be replaced by X if there is no changes of confusion. For a subset A of a topological space X, cl(A) and int(A) denote the closure of A and the interior of A respectively. We recall the following definitions and results.

Definition 2.1: Let (X, τ) be a topological space. A subset A of the space X is said to be
(i) semi-open [6] if A $\subseteq$ cl(int(A)) and semi-closed [3] if int(cl(A)) $\subseteq$ A.
(ii) α-open [7] if A $\subseteq$ int(cl(int(A))) and α-closed if cl(int(cl(A))) $\subseteq$ A.
(iii) b-open [3] if A$\subseteq$int(cl(A))$\cup$cl(int(A)) and b-closed if int(cl(A))$\cap$cl(int(A)) $\subseteq$A.
(iv) regular open[8] if int(cl(A))=A and regular closed if cl(int(A))=A.
(v) π-open [13] if A is the union of regular open sets and π-closed if A is the intersection of regular closed sets.

Definition 2.2: Let (X, τ) be a topological space and A $\subseteq$ X. The b-closure (resp.pre-closure, semi-closure, α-closure) of A, denoted by bcl(A) (resp.pcl(A),scl(A), αcl(A)) and is defined by the intersection of all b-closed (resp. pre-closed, semi-closed, α-closed) sets containing A.

Definition 2.3: Let (X, τ) be a topological space and A $\subseteq$ X. The b-interior (resp.pre- interior, semi- interior, α-interior) of A, denoted by bint(A) (resp.pint(A),sint(A), αint(A)) and is defined by the intersection of all b-open (resp. pre-open, semi-open, α-open) sets contained in A.

Corresponding Author: P.Selvan[1*],
[1,2]Department of Mathematics, Aditanar College of Arts and Science, Tiruchendur, India.

Definition 2.4: Let (X, τ) be a topological space. A subset Aof X is said to be
(i) strongly b*-closed [9](briefly sb*-closed) if cl(int(A))) $\subseteq$ U whenever A $\subseteq$ U and U is b-open in (X, τ).
(ii) Generalized b-strongly b*-closed [10](briefly gbsb*-closed) if bcl(A)⊆U, whenever A $\subseteq$ U and U is sb*-open in (X,τ).
The complements of the above mentioned closed sets are their respective open sets.

Definition 2.5: [12] A subset N of a space X, is called a neighbourhood (simply, nbhd) of A⊆X if there exists an open set U such that A⊆U⊆N.

Lemma 2.6: For any subset A of a topological space (X, τ),
(i) sint(A)=A∩cl(int(A))
(ii) pin(A)=A∩int(cl(A))
(iii) scl(A)=A∪int(cl(A))
(iv) pcl(A)=A∪cl(int(A)).

Definition 2.7: [11] Let A be a subset of a topological space (X, τ). Then the union of all gbsb*-open sets contained in A is called the gbsb*-interior of A and it is denoted by gbsb*int(A). That is, gbsb*int(A)=∪{V:V$\subseteq$A and V∈gbsb*-O(X)}.

Definition 2.8: [11] Let A be a subset of a topological space (X, τ). Then the intersection of all gbsb*-closed sets in X containing A is called the gbsb*-closure of A and it is denoted by gbsb*cl(A). That is, gbsb*cl(A)=∩{F: A$\subseteq$F and F∈gbsb*-[illegible](X)}.

Remark 2.9: [11] For any subset A of a topological space (X, τ),
(i) X\gbsb*cl(A)=gbsb*int(X\A)
(ii) X\gbsb*int(A)=gbsb*cl(X\A).

Definition 2.10: [11] Let A be a subset of a topological space X. A point x∈X is said to be gbsb*-limit point of A if G∩(A\{x})≠ ϕ, for every gbsb*-open set G containing x.

Definition 2.11: [11] The set of all gbsb*-limit points of A is called the gbsb*-derived set of A and is denoted by D_{gbsb*}(A).

Lemma 2.12: [11] For any subset A of a topological space (X, τ),
(i) gbsb*int(A)= A\D_{gbsb*}(X\A)
(ii) gbsb*cl(A)= A∪D_{gbsb*}(A)

3. Semi generalized b-strongly b*-open set

Definition 3.1: A subset A of a topological space (X,τ) is said to be a semi-generalized b-strongly b*-open set(briefly, semi-gbsb*-open or sgbsb*-open) if A⊆cl(gbsb*int(A)).

Theorem 3.2: Every open set is sgbsb*-open.

Proof: Let A be an open subset of a topological space (X,τ). Then A=int(A)⊆cl(int(A)) ⊆cl(gbsb*int(A)) and hence A is sgbsb*-open.

Remark 3.3: The converse of the above theorem is not true which is shown in the following example.

Example 3.4: Let X = {a, b, c, d} with τ= {ϕ, {a}, {a,b}, {a,b,c}, X}. The set {b,c} is sgbsb*-open but not an open sets.

Theorem 3.5: Every semi-open set is sgbsb*-open.

Proof: Let A be a semi-open subset of a topological space (X, τ). Then cl(gbsb*int(A))⊇cl(int(A))⊇A and hence A is sgbsb*-open.

Remark 3.6: The converse of the above theorem is not true which is shown in the following example.

Example 3.7: Let X = {a, b, c, d} with τ= {ϕ, {a}, {b}, {a,b}, {b,c}, {a,b,c}, {b,c,d}, X}. The set {b} is sgbsb*-open but not a semi-open set.

Theorem 3.8: Every α-open set is sgbsb*-open.

Proof: Let A be a α-open subset of a topological space (X, τ). Then A⊆int(cl(int(A))) ⊆cl(int(A)) ⊆ cl(gbsb*int(A)) and hence A is sgbsb*-open.

Remark 3.9: The converse of the above theorem is not true which is shown in the following example.

Example 3.10: Let X = {a, b, c, d} with τ= {ϕ, {a}, {a,b}, {a,b,c}, X}. The set {a,c} is sgbsb*-open set but not a α-open set.

Theorem 3.11: Every regular open set is sgbsb*-open.

Proof: Let A be a regular open subset of a topological space (X, τ). Since every regular open set is open and by Theorem 3.2, A is sgbsb*-open.

Remark 3.12: The converse of the above theorem is not true which is shown in the following example.

Example 3.13: Let X = {a, b, c, d} with τ={ϕ,{a},{b},{a,b},{b,c},{a,b,c},{b,c,d}, X}. The set {a,b,c} is sgbsb*-open but not a regular-open set.

Theorem 3.14: Every π-open set is sgbsb*-open.

Proof: Let A be a π-open subset of a topological space (X, τ). Since every π-open set is open and by Theorem 3.2, A is sgbsb*-open.

Remark 3.15: The converse of the above theorem is not true which is shown in the following example.

Example 3.16: Let X = {a, b, c, d} with τ= {ϕ, {a}, {a,b}, {a,b,c}, X}. The sets {a,c} and {a,d} are sgbsb*-open sets but not a π-open sets.

Theorem 3.17: Every gbsb*-open set is sgbsb*-open.

Proof: Let A be a gbsb*-open subset of a topological space (X, τ). Then A⊆cl(A)=cl(gbsb*int(A)) and hence A is sgbsb*-open.

Remark 3.18: The converse of the above theorem is not true which is shown in the following example.

Example 3.19: Let X = {a, b, c, d} with τ= {ϕ, {a}, {b}, {a, b}, {b, c}, {a, b, c}, {b, c, d}, X}. The set {a, c, d} is sgbsb*-open but not a gbsb*-open set.

Theorem 3.20: Every b-open set is sgbsb*-open.

Proof: Let A be a b-open subset of a topological space (X, τ). Then and hence A⊆cl(A)= cl(bint(A))⊆cl(gbsb*int(A)) is sgbsb*-open.

Remark 3.21: The converse of the above theorem is not true which is shown in the following example.

Example 3.22: Let X = {a, b, c, d} with τ= {ϕ, {a}, {b}, {a, b}, {b, c}, {a, b, c}, {b, c, d}, X}. The set {a, c, d} is sgbsb*-open but not a b-open set.

Theorem 3.23: A subset A of X is sgbsb*-open if and only if there exists a gbsb*-open set U such that U⊆A⊆cl(U).

Proof: Necessity. If A is sgbsb*-open, then A⊆cl(gbsb*int(A)). Take U=gbsb*int(A). Then U is an gbsb*-open set in X such that U⊆A⊆cl(U).

Sufficiency. Assume that there is an gbsb*-open set U such that U⊆A⊆cl(U).
Now U⊆A ⟹ U=gbsb*int(U)⊆gbsb*int(A) ⟹ A⊆cl(U)⊆cl(gbsb*int(A)). Therefore A is sgbsb*-open.

Theorem 3.24: The union of two sgbsb*-open sets is also a sgbsb*-open set.

Proof: Let A and B be two sgbsb*-open sets in a topological space (X, τ). Then A⊆cl(gbsb*int(A)) and B⊆cl(gbsb*int(B)). Now, A∪B⊆ cl(gbsb*int(A))∪cl(gbsb*int(B)) ⊆cl(gbsb*int(A) ∪gbsb*int(B))⊆ cl(gbsb*int(A∪B)). Therefore, A∪B is sgbsb*-open.

Remark 3.25: Arbitrary union of sgbsb*-open sets of a topological space is also a sgbsb*-open set.

Remark 3.26: The finite intersection of sgbsb*-open sets need not be a sgbsb*-open, which is shown in the following example.

Example 3.27: Let X = {a, b, c, d} with τ= {ϕ, {a}, {b}, {a, b}, {b, c}, {a, b, c}, {b, c, d}, X}. The sets {b, c, d} and {a, c, d} are sgbsb*-open set but their intersection {c, d} is not a sgbsb*-open set.

Theorem 3.28: If a topological space (X, τ), let τ_{sgbsb^*}={U∈sgbsb*-O(X, τ)/ U∩A∈sgbsb*-O(X, τ) for all A∈sgbsb*-O(X, τ)}. Then τ_{sgbsb^*} is a topology on X.

Proof: Clearly ϕ, X∈ τ_{sgbsb^*}. Let $U_\beta \in \tau_{sgbsb^*}$ and U=∪ U_β. Since each $U_\beta \in \tau_{sgbsb^*}$, then by Remark 3.23, U∈sgbsb*-O(X, τ). Let A∈sgbsb*-O(X, τ). Then U_β ∩A∈sgbsb*-O(X, τ) for each β. Hence U∩A=(∪ U_β) ∩A=∪ (U_β ∩A) ∈sgbsb*-O(X, τ). Therefore U∈ τ_{sgbsb^*}. Let U_1,U_2∈ τ_{sgbsb^*}. Then U_1,U_2∈sgbsb*-O(X, τ) and from definition of τ_{sgbsb^*}, U_1∩U_2∈sgbsb*-O(X, τ). If A∈sgbsb*-O(X, τ), and from definition of τ_{sgbsb^*}, U_1∩U_2∩A ∈sgbsb*-O(X, t). Hence U_1∩U_2∈ τ_{sgbsb^*}. This shows that τ_{sgbsb^*} is closed under finite intersection. Hence τ_{sgbsb^*} is a topology on X.

Theorem 3.29: A subset A is sgbsb*-open iff cl(A)=cl(gbsb*int(A)).

Proof: Necessity. Since A is sgbsb*-open, A⊆cl(gbsb*int(A)). Hence cl(A)⊆cl(gbsb*int(A)). Also we have, cl(gbsb*int(A))⊆cl(A). Hence cl(A)=cl(gbsb*int(A)).

Sufficiency. Take U=gbsb*int(A). Then U is a gbsb*-open set in X such that U⊆A⊆cl(A)=cl(gbsb*int(A))=cl(U). Therefore by Theorem 3.23, A is sgbsb*-open.

Theorem 3.30: Let A be sgbsb*-open and B⊆X such that A⊆B⊆cl(A). Then B is sgbsb*-open.

Proof: Since A is sgbsb*-open, A⊆cl(gbsb*int(A)). Since gbsb*int(A)⊆gbsb*int(B), cl(gbsb*int(A))⊆cl(gbsb*int(B)). Therefore by the above theorem, B⊆cl(A)=cl(gbsb*int(A))⊆cl(gbsb*int(B)). Hence B is sgbsb*-open.

Theorem 3.31: For a subset A of a topological space (X, τ) the following statements are equivalent:
- (i) A is sgbsb*-open.
- (ii) A⊆cl(gbsb*int(A)).
- (iii) cl(gbsb*int(A))=cl(A).
- (iv) cl(A\\D_{gbsb*}(X\\A))=cl(A).

4. Semi-generalized b-strongly b*-closed set.

Definition 4.1: A subset A of a space (X, τ) is called a semi-generalized b-strongly b*-closed set(briefly, semi-gbsb*-closed or sgbsb*-closed) if X\A is sgbsb*-open. The set of all sgbsb*-open sets in (X, τ) is denoted by sgbsb*-O(X, τ).

Theorem 4.2: For a topological space (X, τ),
- (i) Every closed set is sgbsb*-closed.
- (ii) Every semi-closed set is sgbsb*-closed.
- (iii) Every α-closed set is sgbsb*-closed.
- (iv) Every regular closed set is sgbsb*-closed.
- (v) Every π-closed set is sgbsb*-closed.
- (vi) Every gbsb*-closed set is sgbsb*-closed.
- (vii) Every b-closed set is sgbsb*-closed.

Theorem 4.3: A subset A of a space (X, τ) is sgbsb*-closed if and only if there is a gbsb*-closed set F in (X, τ) such that int(F)⊆A⊆F.

Proof: Necessity. Suppose A is sgbsb*-closed. Then X\A is sgbsb*-open. Then there exists a gbsb*-open set U in X such that U⊆X\A⊆cl(U) which implies X\U⊇A⊇X\cl(U). That implies, X\U⊇A⊇int(X\U) where X\U is gbsb*-closed in X.

Sufficiency. Suppose there is a gbsb*-closed set F in (X, τ) such that int(F)⊆A⊆F which implies X\int(F)⊇X\A⊇ X\F. Since X\int(F)=cl(X\F), we have cl(X\F)⊇X\A⊇X\F where X\F is a gbsb*-open set. Hence X\A is sgbsb*-open. Therefore A is sgbsb*-closed.

Theorem 4.4: A⊆X is sgbsb*-closed if and only if int(gbsb*cl(A))⊆A.

Proof: Necessity. Suppose A is sgbsb*-closed. Then X\A is sgbsb*-open. Therefore X\A⊆cl(gbsb*int(X\A)) and hence int(gbsb*cl(A))⊆A.

Sufficiency. Assume that int(gbsb*cl(A))⊆A. Take F=gbsb*cl(A). Then F is a gbsb*-closed set in X such that int(F)⊆A⊆F and hence A is sgbsb*-closed.

Theorem 4.5: If A is sgbsb*-closed in X and B⊆X is such that int(A)⊆B⊆A. Then B is sgbsb*-closed in X.

Theorem 4.6: The intersection of two sgbsb*-closed sets is also sgbsb*-closed.

Proof: Let A and B be two sgbsb*-closed sets in a topological space (X, τ). Then int(gbsb*cl(A))⊆A and int(gbsb*cl(B))⊆B. Now, int(gbsb*cl(A∩B))⊆ int(gbsb*cl(A)∩gbsb*cl(B))= int(gbsb*cl(A))∩int(gbsb*cl(B))⊆A∩B. Therefore, A∩B is sgbsb*-closed.

Remark 4.7: Arbitrary intersection of sgbsb*-closed sets of a topological space is also a sgbsb*-closed set.

Remark 4.8: The union of sgbsb*-closed sets need not be a sgbsb*-closed set.

Theorem 4.9: If A is sgbsb*-closed and U is sgbsb*-open in X, then A\U is sgbsb*-closed in X.

Proof: Since A\U=A∩(X\U), A and X\U are sgbsb*-closed sets, by Theorem 4.6, A\U is sgbsb*-closed in X.

Theorem 4.10: A subset A is sgbsb*-closed iff int(A)=int(gbsb*cl(A)).

Proof: Necessity. Since A is sgbsb*-closed, int(gbsb*cl(A))⊆A. Hence int(gbsb*cl(A))⊆int(A). Also we have, int(A))⊆int(gsb*cl(A)). Hence int(A)=int(gbsb*cl(A)).

Sufficiency. Take U=gbsb*cl(A). Then U is a gbsb*-closed set in X such that int(U)⊆A⊆U. Therefore by Theorem 4.3, A is sgbsb*-closed.

Theorem 4.11: For a subset A of a topological space (X, τ) the following statements are equivalent:
- (i) A is sgbsb*-closed.
- (ii) int(gbsb*cl(A))⊆A.
- (iii) int(gbsb*cl(A))=int(A).
- (iv) cl(A∪D_{gbsb*}(A))=cl(A).

Theorem 4.12: Let A be a sgbsb*-closed in X. Then
- (i) sint(A) is sgbsb*-closed.
- (ii) If A is regular open, then pint(A) and scl(A) are also sgbsb*-closed.
- (iii) If A is regular closed, then pcl(A) is also sgbsb*-closed.

Proof: Let A be a sgbsb*-closed set of X.
- (i) Since cl(int(A)) is closed, then by Theorem 4.2, cl(int(A)) is sgbsb*-closed. By Lemma 2.6, sint(A) is sgbsb ∗-closed.
- (ii) Suppose A is regular open, then int(cl(A))=A. By Lemma 2.6, scl(A)=A. Since A is sgbsb*-closed, then scl(A) is sgbsb*-closed. Similarly pint(A) is sgbsb*-closed.
- (iii) Suppose A is regular closed, cl(int(A))=A. Then by Lemma 2.6, pcl(A)=A, and hence sgbsb*-closed.

5. sgbsb*-neighbourhood

Definition 5.1: Let X be a topological space and let x∈X. A subset N of X is said to be a sgbsb*-neighbourhood (shortly, sgbsb*-nbhd) of x if there exists a sgbsb*-open set U such that x∈U⊆N.

Definition 5.2: A subset N of a space X, is called a sgbsb*-nbhd of A⊆X if there exists an sgbsb*-open set U such that A⊆U⊆N.

Theorem 5.3: Every nbhd N of x∈X is a sgbsb*-nbhd of x.

Proof: Let N be anbhd of point x∈ X. Then there exists an open set U such that x∈U⊆N. Since every open set is sgbsb*-open, U is a sgbsb*-open set such that x∈U⊆N. This implies, N is a sgbsb*-nbhd of x.

Remark 5.4: The converse of the above theorem need not be true which is shown in the following example.

Example 5.5: Let X = {a, b, c, d} with topology τ = {ϕ, {a},{b}, {a,b}, {b,c}, {a,b,c}, {b,c,d}, X}. In this topological space (X, τ), sgbsb*-O(X) = { ϕ, {a}, {b}, {a,b},{a,d}, {b,c}, {a, b, c}, {a,b, d}, {a, c, d}, {b, c d}, X} The set {a,d} is the sgbsb*-nbhd of d, since {a,d} is sgbsb*-open set such that d∈{a,d}⊆ {a, d}. However, the set {b, d} is not a nbhd of the point d.

Remark 5.6: Every sgbsb*-open set is a sgbsb*-nbhd of each of its points.

Theorem 5.7: If F is a sgbsb*-closed subset of X and x∈X\F, then there exists a sgbsb*-nbhd N of x such that N∩F=ϕ.

Proof: Let F be sgbsb*-closed subset of X and x∈ X\F. Then X\F is sgbsb*-open set of X. By Theorem 4.6, X\F contains a sgbsb*-nbhd of each of its points. Hence there exists a sgbsb*-nbhd N of x such that N⊆ X\F. Hence N∩F=ϕ.

Definition 5.8: The collection of all sgbsb*-neighborhoods of x∈X is called the sgbsb*-neighborhood system of x and is denoted by sgbsb*-N(x).

Theorem 5.9: Let (X, t) be a topological space and x ∈ X. Then
- (i) sgbsb*-N(x)≠ φ and x∈each member of sgbsb*-N(x)
- (ii) If N ∈ sgbsb*-N(x) and N ⊆ M, then M ∈ sgbsb*-N(x).
- (iii) Each member N ∈ sgbsb ∗-N(x) is a superset of a member G∈ sgbsb ∗-N(x) where G is a sgbsb*-open set.

Proof:
- (i) Since X is sgbsb*-open set containing x, it is a sgbsb*-nbhd of every x∈X. Thus for each x∈X, there exists atleast one sgbsb*-nbhd, namely X. Therefore, sgbsb*-N(x)≠ φ. Let N∈sgbsb*-N(x). Then N is a sgbsb*-nbhd of x. Hence there exists a sgbsb*-open set G such that x∈G ⊆N, so x ∈ N. Therefore x∈every member N of sgbsb*-N(x).
- (ii) If N sgbsb*-N(x), then there is a sgbsb*-open set G such that x∈G⊆N. Since N⊆M, M is sgbsb*-nbhd of x. Hence M∈sgbsb*-N(x).
- (iii) Let N∈sgbsb*-N(x). Then there is a sgbsb*-open set G, such that x ∈ G⊆N. Since G is sgbsb*-open and x∈G, G is sgbsb*-nbhd of x. Therefore G∈sgbsb*-N(x) and also G⊆N.

REFERENCES

1. Adiya.K.Hussein, New type of generalized closed sets, Mathematical Theory and Modeling, Vol.3 (4), 2013.
2. Ahmad Al-Omari and Mohd.Salmi Md. Noorani, On Generalized b-closed sets. Bull. Malays.math. Sci. Soc (2) 32(1) (2009),19-30.
3. D.Andrijevic, On b-open sets, Mat.Vesnik 48(1996), 59-64.
4. M.Anitha and P.Thangavelu, Generalized closed sets in submaximal spaces, Antarctica J. Math., 4(1) (2007), 99-108.
5. N. Levine, Generalized closed sets in topology, Rand. Circ. Mat. Palermo, 19(2) (1970), 89-96.
6. N. Levine, Semi-open sets and semi-continuity in Topological Spaces, Amer. Mat. Monthly 70(1)(1963), 36- 41.
7. O. Njastad, Some classes of nearly open sets, Pacific J. Math., 15(1965), 961- 970.
8. N.Palaniappan and K.C. Rao, Regular generalized closed sets, Kyungpook Math.J., 33(2) (1993), 211-219.
9. A.Poongothai and R.Parimelazhagan, sb*-closed sets in a topological spaces, Int. Journal of Math. Analysis 6(47), (2012), 2325-2333.
10. P. Selvan and M.J.Jeyanthi, Generalized b-strongly b*-closed sets in topological spaces, Int. Journal of Math. Archive-8(5), (2017), 27-34.
11. P. Selvan and M.J.Jeyanthi, More on gbsb*-closed and sets in topological spaces, Scholars Journal of Physics, Mathematics and Statistics, 4(13), 2017, 113-120.
12. Willard S., General Topology, Addison Wesley, 1970.
13. V.Zaitsav, On certain classes of topological spaces and their bicompactifications, Dokl. Akad.Nauk SSSR, 178 (1968), 778-779.

Source of support: Nil, Conflict of interest: None Declared.