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# NEIGHBORHOOD DAKSHAYANI INDICES

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***ABSTRACT:***

*Chemical Graph Theory is a branch of Mathematical Chemistry whose focus of interest is to finding graph invariants of molecular graphs. These correlate with chemical properties of the chemical compounds. In this paper, we propose new degree based topological descriptors such as first and second neighborhood Dakshayani indices, vertex neighborhood Dakshayani index, first and second hyper neighborhood Dakshayani indices of a graph and establish explicit formulas for these indices of line graphs of subdivision graphs of certain nanostructures.*

***Key words:*** *neighborhood Dakshayani indices, hyper neighborhood Dakshayani indices, nanostructures.*

***Mathematics Subject Classification:*** *05C07, 05C76, 92E10.*

## 1. INTRODUCTION

Let $G$ be a finite, simple connected graph with vertex set $V(G)$ and edge set $E(G)$. The degree $d_G(v)$ of a vertex $v$ in $G$ is the number of edges incident to $v$. The set of all vertices which adjacent to $v$ is called open neighborhood of $v$ and denoted by $N_G(v)$. The closed neighborhood set of $v$ is the set $N_G[v] = N_G(v) \cup \{v\}$. The set $N_G[v]$ is the set of closed neighborhood vertices of $v$. Let $D_G(v) = d_G(\text{v}) + \sum_{u \in N(\text{v})} d_G(u)$ is the degree sum of closed neighborhood vertices of $v$. The subdivision graph $S(G)$ is the graph obtained from $G$ by replacing each of its edge by a path of length 2. The line graph $L(G)$ of $G$ is the graph whose vertex set is $E(G)$ and two vertices of $L(G)$ are adjacent if they are adjacent in $G$. For other graph terminology and notation refer [1].

We need the following results.

**Lemma 1:** Let $G$ be a $(p, q)$ graph. Then $S(G)$ has $p+q$ vertices and $2q$ edges.

**Lemma 2:** Let $G$ be a $(p, q)$ graph. Then $L(G)$ has $q$ vertices and $\frac{1}{2}\sum_{i=1}^{p} d_G(u_i)^2 - q$ edges.

A molecular graph is a graph such that its vertices correspond to the atoms and the edges to the bonds. Chemical Graph Theory is a branch of Mathematical Chemistry which has an important effect on the development of the Chemical Sciences, Medical Sciences. A single number that can be used to characterize some property of the graph of a molecular is called a topological index for that graph. There are numerous topological indices that have found some applications in Chemistry, especially in QSPR/QSAR study, see [2, 3, 4]. Two of the best known and widely used topological indices are the first and second Zagreb indices by Gutman and Trinajstić [5]. Motivated by these definitions, we now introduce new degree based topological indices as follows:

The first neighborhood Dakshayani index is defined as

$$ND_1(G) = \sum_{uv \in E(G)} \left[D_G(u) + D_G(v)\right].$$

The second neighborhood Dakshayani index is defined as

$$ND_2(G) = \sum_{uv \in E(G)} D_G(u) D_G(v).$$

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The vertex neighborhood Dakshayani index is defined as

$$ND_v(G) = \sum_{u \in V(G)} D_G(u)^2.$$

Some results on Zagreb indices can be found in the papers [6, 7, 8].

The first hyper Zagreb index was introduced by Shirdel et al. in [9], defined as

$$HM_1(G) = \sum_{uv \in E(G)} \left[d_G(u) + d_G(v)\right]^2$$

The second hyper Zagreb index was defined as [10]

$$HM_2(G) = \sum_{uv \in E(G)} \left[d_G(u) d_G(v)\right]^2.$$

Some results on the hyper Zagreb indices can be found in the articles [11, 12].

Motivated by the above two definitions, we propose the following degree based topological indices:

The first hyper neighborhood Dakshayani index of a graph *G* is defined as

$$HND_1(G) = \sum_{uv \in E(G)} \left[D_G(u) + D_G(v)\right]^2.$$

The second hyper neighborhood Dakshayani index of a graph *G* is defined as

$$HND_2(G) = \sum_{uv \in E(G)} \left[D_G(u) D_G(v)\right]^2$$

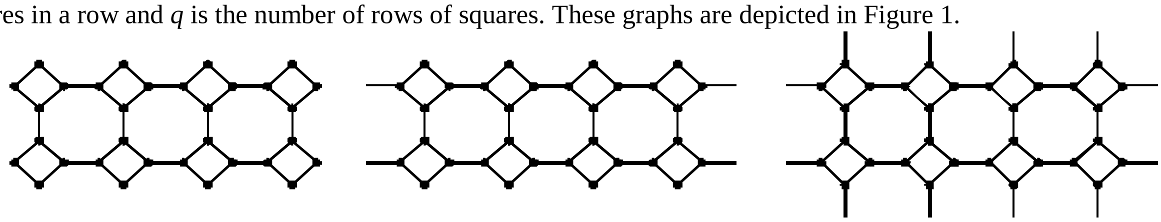
Recently, some hyper indices like hyper Revan indices [13], reverse hyper Zagreb indices [14], *K* hyper Banhatti indices [15] were introduced and studied.

Recently, the first neighborhood Zagreb index was introduced and studied by Basavanagoud et al. [16] and Mondal *et al.* [17].

We consider 2*D*-lattice, nanotube and nanotorus of $TUC_4C_8[p, q]$, see [18, 19]. In this paper, we determine the first and second neighborhood Dakshayani indices, the vertex neighborhood Dakshayani index, first and second hyper neighborhood Dakshayani indices of line graphs of subdivision graphs of 2*D*-lattice, nanotube and nanotorus of $TUC_4C_8[p, q]$.

## 2. 2-*D* LATTICE, NANOTUBE AND NANOTORUS OF $TUC_4C_8[p, q]$

In this section, we consider the graph of 2*D*-lattice, nanotube and nanotorus of $TUC_4C_8[p, q]$, where *p* is the number of squares in a row and *q* is the number of rows of squares. These graphs are depicted in Figure 1.



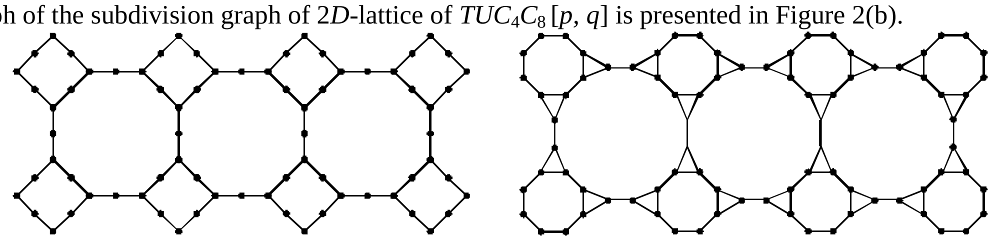
(a) (b) (c)

**Figure-1**

(a) 2D-lattice of $TUC_4C_8[4, 2]$ (b) $TUC_4C_8[4,2]$ nanotube (c) $TUC_4C_8[4, 2]$ nanotorus

## 3. RESULTS FOR 2*D*-LATTICE OF $TUC_4C_8[p, q]$

The line graph of the subdivision graph of 2*D*-lattice of $TUC_4C_8[p, q]$ is presented in Figure 2(b).



(a) subdivision graph of 2D-lattice of $TUC_4C_8[4,2]$

(b) line graph of the subdivision graph of $TUC_4C_8[4, 2]$

**Figure-2**

**Theorem 1:** Let $G$ be the line graph of the subdivision graph of $2D$-lattice of $TUC_4C_8$ $[p, q]$. Then the first neighborhood Dakshayani index of $G$ is given by

$$ND_1(G) = 432pq - 628(p+q) + 8, \quad \text{if } p > 1, q > 1,$$
$$= 260p - 164, \quad \text{if } p > 1, q = 1.$$

**Proof:** The $2D$-lattice of $TUC_4C_8[p, q]$ is a graph with $4pq$ vertices and $6pq - p - q$ edges. By Lemma 1, the subdivision graph of $2D$ lattice of $TUC_4C_8[p, q]$ is a graph with $10pq - p - q$ vertices and $2(6pq - p - q)$ edges. Hence by Lemma 2, $G$ has $2(6pq - p - q)$ vertices and $18pq - 5p - 5q$ edges. Clearly, the vertices of $G$ are either of degree 2 or 3, see Figure 2(b). Thus the edge partition of $G$ is given in Table 1 and Table 2.

| $D_G(u), D_G(v) \setminus uv \in E(G)$ | (6, 6) | (6,7) | (7, 7) | (7, 11) | (11, 12) | (12, 12) |
|---|---|---|---|---|---|---|
| Number of edges | 4 | 8 | $2(p+q-4)$ | $4(p+q-2)$ | $8(p+q-2)$ | $2(9pq+10) - 19(p+q)$ |

**Table-1:** Edge partition of $G$ with $p > 1$, $q > 1$

| $D_G(u), D_G(v) \setminus uv \in E(G)$ | (6, 6) | (6,7) | (7, 7) | (7, 11) | (11, 11) | (11, 12) | (12, 12) |
|---|---|---|---|---|---|---|---|
| Number of edges | 6 | 4 | $2(p-2)$ | $4(p-1)$ | $2(p-1)$ | $4(p-1)$ | $(p-1)$ |

**Table-2:** Edge partition of $G$ with $p > 1$ and $q = 1$

**Case-1:** Let $p > 1$ and $q > 1$.

The edge partition based on the degree sum of closed neighborhood vertices of each vertex is obtained, as given in Table 1. To compute $ND_1(G)$, we see that

$$ND_1(G) = \sum_{uv \in E(G)} \left[D_G(u) + D_G(v)\right]$$
$$= 4(6+6) + 8(6+7) + 2(p+q-4)(7+7) + 4(p+q-2)(7+11)$$
$$+ 8(p+q-2)(11+12) + \left[2(9pq+10) - 19(p+q)\right](12+12)$$
$$= 432pq - 628(p+q) + 8.$$

**Case-2:** Let $p > 1$ and $q = 1$.

The edge partition based on the degree sum of closed neighborhood vertices of each vertex is obtained, as given in Table 2. To compute $ND_1(G)$, we see that

$$ND_1(G) = \sum_{uv \in E(G)} \left[D_G(u) + D_G(v)\right]$$
$$= 6(6+6) + 4(6+7) + 2(p-2)(7+7) + 4(p-1)(7+11)$$
$$+ 2(p-1)(11+11) + 4(p-1)(11+12) + (p-1)(12+12)$$
$$= 260p - 164.$$

**Theorem 2:** Let $G$ be the line graph of the subdivision graph of $2D$-lattice of $TUC_4C_8[p, q]$. Then the second neighborhood Dakshayani index of $G$ is given by

$$ND_2(G) = 2592pq - 4010(p+q) + 436, \quad \text{if } p > 1, q > 1,$$
$$= 1586pq - 1202, \quad \text{if } p > 1, q = 1.$$

**Proof:**
**Case-1:** Let $p > 1$ and $q > 1$.

The edge partition based on the degree sum of closed neighborhood vertices of each vertex is obtained, as given in table 1. To compute $ND_2(G)$, we see that

$$ND_2(G) = \sum_{uv \in E(G)} \left[D_G(u) D_G(v)\right]$$
$$= 4(6\times6) + 8(6\times7) + 2(p+q-4)(7\times7) + 4(p+q-2)(7\times11)$$
$$+ 8(p+q-2)(11\times12) + \left[2(9pq+10) - 19(p+q)\right](12\times12)$$
$$= 2592pq - 4010(p+q) + 436.$$

**Case-2:** Let $p > 1$ and $q = 1$.
The edge partition based on the degree sum of closed neighborhood vertices of each vertex is obtained, as given in Table 2.

To compute $ND_2(G)$, we see that

$$ND_2(G) = \sum_{uv \in E(G)} D_G(u) D_G(v)$$
$$= 6(6\times 6) + 4(6\times 7) + 2(p-2)(7\times 7) + 4(p-1)(7\times 11)$$
$$+2(p-1)(11\times 11) + 4(p-1)(11\times 12) + (p-1)(12\times 12)$$
$$= 1586p - 1202.$$

**Theorem-3:** Let $G$ be the line graph of the subdivision graph of 2D-lattice of $TUC_4C_8$ $[p, q]$. Then the first hyper neighborhood Dakshayani index of $G$ is given by

$HND_1(G) = 10368pq - 5024(p+q) + 1608$, if $p > 1$, $q > 1$,
$= 5348p - 4200$, if $p > 1$, $q = 1$.

**Proof:**
**Case-1:** Let $p > 1$ and $q > 1$.

The edge partition based on the degree sum of closed neighborhood vertices of each vertex is obtained, as given in Table 1.

To compute $HND_1(G)$, we see that

$$HND_1(G) = \sum_{uv \in E(G)} \left[D_G(u) + D_G(v)\right]^2$$
$$= 4(6+6)^2 + 8(6+7)^2 + 2(p+q-4)(7+7)^2 + 4(p+q-2)(7+11)^2$$
$$+8(p+q-2)(11+12)^2 + \left[2(9pq+10) - 19(p+q)\right](12+12)^2$$
$$= 10368pq - 5024(p+q) + 1608.$$

**Case-2:** Let $p>1$ and $q = 1$.

The edge partition based on the degree sum of closed neighborhood vertices of each vertex is obtained, as given in Table 2. To compute $HND_1(G)$, we see that

$$HND_1(G) = \sum_{uv \in E(G)} \left[D_G(u) + D_G(v)\right]^2$$
$$= 6(6+6)^2 + 4(6+7)^2 + 2(p-2)(7+7)^2 + 4(p-1)(7+11)^2$$
$$+2(p-1)(11+11)^2 + 4(p-1)(11+12)^2 + 2(p-1)(12+12)^2$$
$$= 5348p - 4200.$$

**Theorem 4:** Let $G$ be the line graph of the subdivision graph of 2$D$-lattice of $TUC_4C_8$ $[p, q]$. Then the second hyper neighborhood Dakshayani index of $G$ is given by

$HND_2(G) = 374248pq - 226074(p+q) + 106116$, if $p > 1$, $q > 1$,
$= 148232p - 138202$, if $p > 1$, $q = 1$.

**Proof:**
**Case-1:** Let $p >1$ and $q >1$.

The edge partition based on the degree sum of closed neighborhood vertices of each vertex is obtained as given in Table 1.

To compute $HND_2(G)$, we see that

$$HND_2(G) = \sum_{uv \in E(G)} \left[D_G(u) D_G(v)\right]^2$$
$$= 4(6\times 6)^2 + 8(6\times 7)^2 + 2(p+q-4)(7\times 7)^2 + 4(p+q-2)(7\times 11)^2$$
$$+8(p+q-2)(11\times 12)^2 + \left[2(9pq+10) - 19(p+q)\right](12\times 12)^2$$
$$= 374248p - 226074(p+q) + 106116.$$

**Case-2:** Let $p >1$ and $q = 1$.

The edge partition based on the degree sum of closed neighborhood vertices of each vertex is obtained, as given in Table 2.

To compute $HND_2(G)$, we see that

$$HND_2(G) = \sum_{uv \in E(G)} \left[D_G(u) D_G(v)\right]^2$$
$$= 6(6\times 6)^2 + 4(6\times 7)^2 + 2(p-2)(7\times 7)^2 + 4(p-1)(7\times 11)^2$$
$$+2(p-1)(11\times 11)^2 + 4(p-1)(11\times 12)^2 + (p-1)(12\times 12)^2$$
$$= 148232p - 138202.$$

**Theorem-5:** Let $G$ be the line graph of the subdivision graph of 2$D$-lattice of $TUC_4C_8$ [$p$, $q$]. Then the vertex neighborhood Dakshayani index of $G$ is

$ND_v(G) = 1728pq - 760(p+q) + 80$, if $p > 1$, $q > 1$,
$= 968p - 680$, if $p > 1$, $q = 1$.

**Proof:** In Theorem 1, it is known that $G$ has $2(6pq - p - q)$ vertices. The vertex partition based on the degree sum of closed neighborhood vertices of each vertex is obtained, as given in Table 3 and Table 4.

| $D_G(u) \setminus u \in V(G)$ | 6 | 7 | 11 | 12 |
|---|---|---|---|---|
| Number of vertices | 8 | $4(p+q-2)$ | $4(p+q-2)$ | $2(6pq-5p-5q+4)$ |

**Table-3:** Vertex partition of $G$ with p>1 and $q > 1$

| $D_G(u) \setminus u \in V(G)$ | 6 | 7 | 11 | 12 |
|---|---|---|---|---|
| Number of vertices | 8 | $4(p-1)$ | $4(p-1)$ | $2(p-1)$ |

**Table-4:** Vertex partition of $G$ with $p > 1$ and $q = 1$

**Case-1:** Let $p > 1$ and $q > 1$.

By using the definition and Table 3, we deduce

$$ND_v(G) = \sum_{u \in V(G)} D_G(u)^2$$
$$= 8\times 6^2 + 4(p+q-2)7^2 + 4(p+q-2)11^2 + 2(6pq-5p-5q+4)12^2$$
$$= 1728pq - 760(p+q) + 80.$$

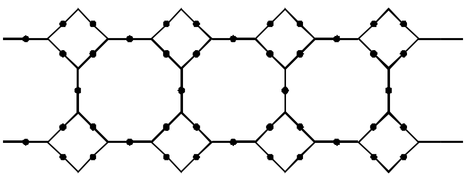
**Case-2:** Let $p > 1$ and $q = 1$.

By using the definition and Table 4, we derive

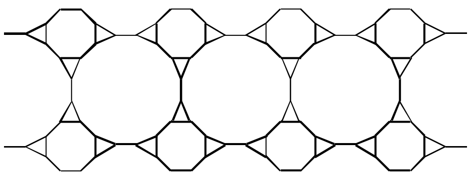
$$ND_v(G) = \sum_{u \in V(G)} D_G(u)^2$$
$$= 8\times 6^2 + 4(p-1)7^2 + 4(p-1)11^2 + 2(p-1)12^2$$
$$= 968p - 680.$$

## 4. RESULTS FOR $TUC_4C_8$ [$p$, $q$] NANOTUBE

The line graph of the subdivision graph of $TUC_4C_8$ [$p$, $q$] nanotube is presented in Figure 3 (b).



(a)
Subdivision graph
of $TUC_4C_8$[4, 2] nanotube



(b)
Line graph of subdivision
graph of $TUC_4C_8$[4, 2]

**Figure-3**

**Theorem 6:** Let $H$ be the line graph of the subdivision graph of $TUC_4C_8[p, q]$ nanotube. Then the first neighborhood Dakshayani index of $H$ is

$ND_1(H) = 432pq - 172p$, if $p > 1$, $q > 1$,
$= 260p$, if $p > 1$, $q = 1$.

**Proof:** The $TUC_4C_8[p, q]$ nanotube is a graph with $4pq$ vertices and $6pq - p$ edges. By Lemma 1, the subdivision graph of $TUC_4C_8[p, q]$ nanotube is a graph with $10pq - p$ vertices and $12pq - 2p$ edges. Therefore by Lemma 2, $H$ has $12pq - 2p$ vertices and $18pq - 5p$ edges. Clearly the vertices of $H$ are either of degree 2 or 3, see Figure 3 (b). Thus the partition of the edge set of $H$ is as shown in Table 5 and Table 6.

| $D_H(u), D_H(v) \setminus uv \in E(H)$ | (7,7) | (7, 11) | (11, 12) | (12, 12) |
|---|---|---|---|---|
| Number of edges | $2p$ | $4p$ | $8p$ | $18pq - 19p$ |

**Table-5:** Edge partition of $H$ when $p > 1$ and $q > 1$

| $D_H(u), D_H(v) \setminus uv \in E(H)$ | (7,7) | (7, 11) | (11, 11) | (11, 12) | (12, 12) |
|---|---|---|---|---|---|
| Number of edges | $2p$ | $4p$ | $2p$ | $4p$ | $p$ |

**Table-6:** Edge partition of $H$ when $p > 1$ and $q = 1$

**Case-1:** Let $p > 1$ and $q > 1$.

The edge partition based on the degree sum of closed neighborhood vertices of each vertex is given in Table 5.

To compute $ND_1(H)$, we see that

$$ND_1(H) = \sum_{uv \in E(H)} \left[D_H(u) + D_H(v)\right]$$
$$= 2p(7+7) + 4p(7+11) + 8p(11+12) + (18pq - 19p)(12+12)$$
$$= 432pq - 172p.$$

**Case-2:** Let $p > 1$ and $q = 1$.

The edge partition based on the degree sum of closed neighborhood vertices of each vertex is obtained, as given in Table 6.

By using definition and Table 6, we deduce

$$ND_1(H) = \sum_{uv \in E(H)} \left[D_H(u) + D_H(v)\right]$$
$$= 2p(7+7) + 4p(7+11) + 2p(11+11) + 4p(11+12) + p(12+12)$$
$$= 260p.$$

**Theorem 7:** Let $H$ be the line graph of the subdivision graph of $TUC_4C_8[p, q]$ nanotube. Then the second neighborhood Dakshayani index of $H$ is

$ND_2(H) = 2592pq - 1274p$, if $p > 1$, $q > 1$,
$= 1320p$, if $p > 1$, $q = 1$.

**Proof:** Case 1. Let $p > 1$ and $q > 1$.

The edge partition based on the degree sum of closed neighborhood vertices of each vertex is given in Table 5.

By using definition and Table 5, we obtain

$$ND_2(H) = \sum_{uv \in E(\mathrm{H})} D_H(u) D_H(v)$$
$$= 2p(7\times7) + 4p(7\times11) + 8p(11\times12) + (18pq - 19p)(12\times12)$$
$$= 1592pq - 1274p.$$

**Case-2:** Let $p > 1$ and $q = 1$.

The edge partition based on the degree sum of closed neighborhood vertices of each vertex is obtained as given in Table 6.

By using definition and Table 6, we derive

$$ND_2(H) = \sum_{uv \in E(H)} D_H(u) D_H(v)$$

$$= 2p(7\times7) + 4p(7\times11) + 2p(11\times11) + 4p(11\times12) + p(12\times12)$$

$$= 1320p.$$

**Theorem 8:** Let $H$ be the line graph of the subdivision graph of $TUC_4C_8$ [$p$, $q$] nanotube. Then the first hyper neighborhood Dakshayani indices of $H$ is

$HND_1(H) = 10368pq - 4424p$, if $p > 1$, $q > 1$,
$= 5348p$, if $p > 1$, $q = 1$.

**Proof: Case 1.** Let $p > 1$, $q > 1$.

The edge partition based on the degree sum of closed neighborhood vertices of each vertex is given in Table 5.

By using definition and Table 5, we obtain

$$HND_1(H) = \sum_{uv \in E(H)} \left[D_H(u) + D_H(v)\right]^2$$

$$= 2p(7+7)^2 + 4p(7+11)^2 + 8p(11+12)^2 + (18pq - 19p)(12+12)^2$$

$$= 10368pq - 4424p.$$

**Case-2:** Let $p > 1$ and $q = 1$.

The edge partition based on the degree sum of closed neighborhood vertices of each vertex is obtained, as given in Table 6.

To compute $HND_1(H)$, we see that

$$HND_1(H) = \sum_{uv \in E[H]} \left[D_H(u) + D_H(v)\right]^2$$

$$= 2p(7+7)^2 + 4p(7+11)^2 + 2p(11+11)^2 + 4p(11+12)^2 + p(12+12)^2$$

$$= 5348p.$$

**Theorem 9:** Let $H$ be the line graph of the subdivision graph of $TUC_4C_8$ [$p$, $q$] nanotube. Then the second hyper neighborhood Dakshayani index of $H$ is

$HND_2(H) = 373248pq - 226074p$, if $p > 1$, $q > 1$,
$= 148232p$, if $p > 1$, $q = 1$.

**Proof: Case 1.** Suppose $p > 1$, $q > 1$.

The edge partition based on the degree sum of closed neighborhood vertex of each vertex is given in Table 5.

By using definition and Table 5, we have

$$HND_2(H) = \sum_{uv \in E(H)} \left[D_H(u) D_H(v)\right]^2$$

$$= 2p(7\times7)^2 + 4p(7\times11)^2 + 8p(11\times12)^2 + (18pq - 19p)(12\times12)^2$$

$$= 373248pq - 226074p.$$

**Case-2:** Suppose $p > 1$, $q = 1$.

The edge partition based on the degree sum of closed neighborhood vertex of each vertex is obtained as given in Table 6.

By using definition and Table 6, we obtain

$$HND_2(H) = \sum_{uv \in E(H)} \left[D_H(u) D_H(v)\right]^2$$

$$= 2p(7\times7)^2 + 4p(7\times11)^2 + 2p(11\times11)^2 + 4p(11\times12)^2 + p(12\times12)^2$$

$$= 148232p.$$

**Theorem 10:** Let $H$ be the line graph of the subdivision graph of $TUC_4C_8$ $[p, q]$ nanotube. Then the vertex neighborhood Dakshayani index of $H$ is

$ND_v(H) = 1728pq – 760p$, if $p > 1$, $q > 1$,
$= 968p$, if $p > 1$, $q = 1$.

**Proof:** In Theorem 6, it is known that $H$ has $12pq – 2p$ vertices. The vertex partition based on the degree sum of closed neighborhood vertices of each vertex is given in Table 7 and Table 8.

| $D_H(u) \setminus u \in V(H)$ | 7 | 11 | 12 |
|---|---|---|---|
| Number of vertices | $4p$ | $4p$ | $12pq – 10p$ |

**Table-7:** Vertex partition of $H$ when $p > 1$, $q > 1$

| $D_H(u) \setminus u \in V(H)$ | 7 | 11 | 12 |
|---|---|---|---|
| Number of vertices | $4p$ | $4p$ | $2p$ |

**Table-8:** Vertex partition of $H$ when $p > 1$, $q = 1$

**Case-1:**. Let $p > 1$ and $q > 1$.

By using the definition and Table 7, we derive

$$ND_v(H) = \sum_{u \in V(H)} D_H(u)^2$$
$$= 4p \times 7^2 + 4p \times 11^2 + (12pq - 10p)12^2$$
$$= 1728pq - 760p.$$

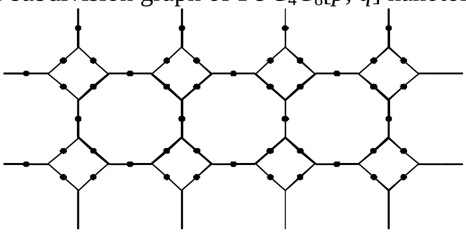
**Case-2:** Let $p > 1$ and $q = 1$.

From the definition and by using Table 8, we deduce

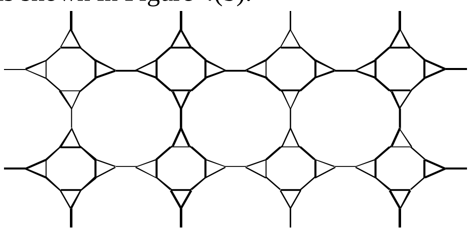
$$ND_v(H) = \sum_{u \in V(H)} D_H(u)^2$$
$$= 4p \times 7^2 + 4p \times 11^2 + 2p \times 12^2$$
$$= 968p.$$

## 5. RESULTS FOR $TUC_4C_8$ [p, q] NANOTORUS

The line of the subdivision graph of $TUC_4C_8[p, q]$ nanotorus is shown in Figure 4(b).



(a)
Subdivision graph of
$TUC_4C_8[p, q]$ nanotorus



(b)
Line graph of subdivision
graph of $TUC_4C_8[p, q]$ nanotorus

**Figure-4**

**Theorem 11:** Let $K$ be the line graph of the subdivision graph of $TUC_4C_8$ $[p, q]$ nanotorus. Then

$ND_1(K) = 432pq$.
$ND_2(K) = 2592pq$.
$HND_1(K) = 10368pq$.
$HND_2(K) = 373248pq$.
$N_1^*(K) = 1728pq$.

**Proof:** A graph of $TUC_4C_8[p, q]$ nanotorus has $4pq$ vertices and $6pq$ edges. By Lemma 1, the subdivision graph of $TUC_4C_8$ $[p, q]$ nanotorus is a graph with $10pq$ vertices and $12pq$ edges. Hence by Lemma 2, $K$ has $12pq$ vertices and $18pq$ edges. Clearly the degree of each vertex is 3. The edge partition based on the degree sum of closed neighborhood vertices of each vertex is as given in Table 9.

| $D_K(u), D_K(v) \setminus uv \in E(K)$ | (12, 12) |
|---|---|
| Number of edges | $18pq$ |

**Table-9:** Edge partition of K

By using definitions and Table 9, we have

$$ND_1(K) = \sum_{uv \in E(K)} \left[D_K(u) + D_K(v)\right] = 18pq(12+12) = 432pq.$$

$$ND_2(K) = \sum_{uv \in E(K)} D_K(u) D_K(v) = 18pq(12 \times 12) = 2592pq.$$

$$HND_1(K) = \sum_{uv \in E(K)} \left[D_K(u) + D_K(v)\right]^2 = 18pq(12+12)^2 = 10368pq.$$

$$HND_2(K) = \sum_{uv \in E(K)} \left[D_K(u) D_K(v)\right]^2 = 18pq(12 \times 12)^2 = 373248pq.$$

The vertex partition based on the degree sum of closed neighborhood vertices of each vertex is as given in Table 10.

| $D_K(u)/\ u \in V(K)$ | 12 |
|---|---|
| Number of vertices | $12pq$ |

**Table-10:** Vertex partition of K

From definition and using Table 10, we obtain

$$ND_v(K) = \sum_{u \in V(K)} D_K(u)^2 = 12pq \times 12^2 = 1728pq.$$

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