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THE PERFORMANCE OF SPECKMAN ESTIMATION FOR PARTIALLY LINEAR MODEL USING KERNEL AND SPLINE SMOOTHING APPROACHES

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ABSTRACT

The Speckman method is a commonly used for estimating the partially linear model (PLM). This method used the kernel approach to estimate nonparametric part in PLM. In this paper, we suggest using the spline approach instead of the kernel approach. Then we present a comparative study of the two estimations based on two smoothing (kernel and spline) approaches. A simulation study has been conducted to evaluate the performance of these estimations. The results of our study confirmed that the spline smoothing approach was the best.

***Keywords**: Kernel smoothing; Mote Carlo simulation, Nonparametric regression; Spline smoothing; Semi-parametric regression.*

1. INTRODUCTION

The Partially linear model (PLM) is one of the most commonly used semi-parametric regression models, which offers an appealing alternative in that it allows both parametric and nonparametric specifications in the regression function. In this model, the covariates are separated into parametric components and nonparametric components, the parametric part of the model can be interpreted as a linear model, while the nonparametric part frees the model from stringent structural assumptions. A PLM is defined by:

$$y_i = x_i'\beta + g(z_i) + u_i; \;\; i = 1,2, \dots, n \tag{1}$$

where y_i denotes the response variable, u_i is the random error term, and x_i, z_i are $p \times 1$ and 1×1 of regressors, respectively. The finite dimensional parameter β is the parametric part of the model, the unknown function $g(.)$ is the non-parametric part of the model, and the random errors $(u_1, \dots, u_n)$ are independent and identically distributed and are independent of x_i, and z_i such that $E(u_i|x_i, z_i) = 0$ and $E(u_i^2|x_i, z_i) = \sigma^2$. This model has gained great popularity since it was first introduced by Engle *et al.* (1986) and has been widely applied in economics, social, and biological sciences.

The literature has several approaches to non-parametric component in the model. But the main two approaches are: (i) the spline smoothing approach, (ii) Kernel smoothing approach. The first approach used by, for example, Engle *et al.* (1986), Heckman (1986), Rice (1986), Speckman (1988), Chen and Shiau (1991), and Abonazel and Gad (2018). But the second approach used by Robinson (1988), Hamilton and Truong (1997), and Aydın (2014). Other approaches are used by Ahn and Powell (1993), Yatchew (1997, 2000, 2003), Wang *et al.* (2011), Meyer (2003), Chang and Qu (2004), and Fadili and Bullmore (2005). A review of these approaches can be found in Härdle *et al.* (2000) and Abonazel (2018a).

Speckman (1988) proposed a general estimation method for this model based on the partial residuals technique to estimate parametric and nonparametric components in PLM. However, he considered the kernel approach to estimate nonparametric component. Since, according to Speckman method, one can use any smoothing approach for non-parametric component. Therefore, in this paper, we will suggest using the spline approach instead of the kernel approach in Speckman method.

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The rest of the paper is organized as follows. In the next section, we introduce the Speckman estimation method based on the kernel and spline smoothing approaches. While in Section 3, the Monte Carlo simulation study is conducted to compare the performance of the two estimators. The concluding remarks are included in Section 4.

2. SPECKMAN ESTIMATION

Speckman (1988) suggested an estimation method to derive the PLM, in the case of identity link function and normally distributed error terms. Speckman method coincides with no updating steps are needed for the estimation of both β and g. In this section, we will present two estimators for PLM using Speckman estimation method based on kernel and spline smoothing approaches.
Beginning, we can rewrite the PLM in (1) in matrix form as:

$$y = X\beta + g(Z) + u,$$

where $y = (y_1, \dots, y_n)'$, $Z = (z_1, \dots, z_n)'$, $u = (u_1, \dots, u_n)'$, and $X' = [x_1 \quad \cdots \quad x_n]$. Taking the conditional expectation with respect to Z and differencing the two equations leads to (Härdle *et al*, 2004):

$$\tilde{y} = \tilde{X}\beta + \tilde{u}, \tag{2}$$

where $\tilde{y} = y - E(Y|Z)$, $\tilde{X} = X - E(X|Z)$, and $\tilde{u} = u - E(u|Z)$. Using the modified regression in (2), the vector of the parametric parameters (β) can be estimated separately. The modified variables $\tilde{X}$ and $\tilde{y}$ are calculated using the fact that the conditional expectation $E(Y|Z)$ can be estimated through a non-parametric regression on the explanatory variable Z. We can summarize Speckman estimation method in the following algorithm:

Step-1: Given a smoother matrix S_λ, depending on smoothing parameter λ construct $\tilde{X} = (I - S_\lambda)X$ and $\tilde{y} = (I - S_\lambda)y$, respectively.
Step-2: For parametric component: the vector of the regression coefficients β can be estimated by: $\hat{\beta} = \left(\tilde{X}'\tilde{X}\right)^{-1} \tilde{X}'\,\tilde{y}$.
Step-3: For non-parametric component: $\hat{g}(Z) = S_\lambda\left(y - X\hat{\beta}\right) = S_\lambda y^*$.

2.1 Kernel Smoothing Approach

In case of a Nadaraya–Watson kernel regression (Härdle et al, 2004), we consider the smoother matrix S with elements:

$$\mathrm{S_{ij}} = \mathrm{K_h}\left(\frac{\mathrm{z_i - z_j}}{\mathrm{h}}\right) \Big/ \sum_{\mathrm{i=1}}^{\mathrm{n}} \mathrm{K_h}\left(\frac{\mathrm{z_i - z_j}}{\mathrm{h}}\right); \quad i,j = 1, \dots, n, \tag{3}$$

where $\mathrm{K}_h(.)$ denotes a Kernel function and h is the bandwidth value. In this paper, biweight function is used as a Kernel function. Using (3), the modified variables $\tilde{X}$ and $\tilde{y}$ are: $\tilde{x}_j = x_j - \sum_{i=1}^{n} S_{ij}\, x_i$, and $\tilde{y}_j = y_j - \sum_{i=1}^{n} S_{ij}\, y_i$; for $j = 1, \dots, n$. While the non-parametric component of the PLM is thus estimated by:

$$\hat{g}\,(z_j) = \sum_{i=1}^{n} S_{ij}\,(y_i - x_i'\hat{\beta}); \quad j = 1, \dots, n.$$

2.2 Spline Smoothing Approach

In case of smoothing spline approach, the smoothing estimate $\hat{g}_\lambda$ of the fitted values for $y^* = y - X\hat{\beta}$ is projected by:

$$\hat{g}_\lambda = [\hat{g}_\lambda(k_1), \dots, \hat{g}_\lambda(k_n)]' = S_\lambda(y_1^*, \dots, y_n^*)' = S_\lambda y^*,$$

where $\hat{g}_\lambda$ is a natural cubic spline with knots at $k_1, \dots, k_n$ for a fixed $\lambda > 0$, and S_λ is a well-known positive-definite smoother matrix which depends on λ and the knot points.

To gain better perspective on smoothing spline, estimation of the parameters of interest in PLM can be performed by minimizing the following sum of squares equation:

$$SS(g, \hat{\beta}) = \sum_{i=1}^{n}\left[\left(y_i - x_i'\hat{\beta}\right) - g(z_i)\right]^2 + \lambda \int_a^b [g''(z)]^2\, dz, \tag{4}$$

where $\hat{\beta}$ is the estimated parametric component that is given from step 2 in the algorithm above. To solve Equation (4), an iterative algorithm is required. In this case, the resulting estimator is called partial spline estimator; see Abonazel and Gad (2018).

3. MONTE CARLO SIMULATION STUDY

This section aims to investigate the performance of the presented estimators in section 2 above through a Monte Carlo simulation study. In fact, we make a comparison study between the Kernel and spline smoothing approaches when they are used in Speckman estimation method of PLM. R software is used to perform this study. For information about how to create Monte Carlo simulation studies using R, see Abonazel (2018b).

In our simulation study, Monte Carlo experiments were carried out based on Equation (1). The simulated model is generated as follows:

1. The number of parametric coefficients are p =2, 8**;** with β_a=1; for $a = 1, \dots, p$.
2. Four functions have been used for the nonparametric component in the model:
   $g_1 = 1.5\, sin(\pi z)$; $g_2 = 1.5\, sin(\pi z^2)$; $g_3 = 3\, sin(\pi z^3)$, $g_4 = z + z^2 + z^4$
3. The explanatory variables x_a(for $a = 1, \dots, p$) are generated from uniform from zero to 1. While the variable z is generated from uniform from -1 to 1.

4. The errors are generated from normal distribution with mean zero and standard deviation $\sigma = 0.5$, 1.
5. The different sample sizes have been used as: N =100, 150, 200, 300, and 500.
6. All Monte Carlo experiments involved 1000 replications and all the results of all separate experiments are obtained by precisely the same series of random numbers.

The goodness of fit of $\hat{g}$ and $\hat{\beta}$ can be quantified by computing the average and the median of mean squared error values (MSEs) for $\hat{g}$ and $\hat{\beta}$ at each iteration run $l = 1, \dots, 1000$. The MSEs of $\hat{g}$ and $\hat{\beta}$ are calculated as:

$$MSE_l(\hat{g}) = \frac{1}{n}\sum_{i=1}^{n}[\hat{g}(z_i) - g(z_i)]^2\,; MSE_l(\hat{\beta}) = \frac{1}{p}\sum_{a=1}^{p}[\hat{\beta}_a - \beta_a]^2,$$

where $\hat{g}(z_i)$ and $\hat{\beta}_a$ are the estimated values of $g(z_i)$ and β_a, respectively.

The results of simulation are recorded in Tables 1–8. These tables present the average of MSE (AMSE) and the median of MSE (MMSE) for $\hat{g}$ and $\hat{\beta}$ using error terms with different standard deviations, different sample sizes, different shapes of the nonparametric component, and different number of explanatory variables.

From Tables 1–8, we can summarize these effects for the kernel and spline estimators in the following points:

- As n increases, the AMSEs and MMSEs decrease.
- As σ increases, the AMSEs and MMSEs increase.
- As p increases, the AMSEs and MMSEs increase.

In general, we can conclude that the AMSEs and MMSEs of spline estimator are smaller than the AMSEs and MMSEs of kernel estimator, for all simulation situations. But we note that, for the parametric component, the AMSEs and MMSEs of the kernel and spline estimators are relatively close.

Graphically, we illustrate the degree of goodness of fit of the kernel and spline estimators for nonparametric functions ($g_1, \dots, g_4$) via different simulated PLMs. These models are generated based on different factors (n, p, and σ). The fitted curves of the estimators based on four nonparametric functions are shown in Figures 1-4, respectively. From Figure 1, we find that the fitted curve based on spline estimator is closer to the true curve than kernel estimator. The same results can be concluded from other figures. This means that spline estimator performs better regardless of the form of nonparametric function

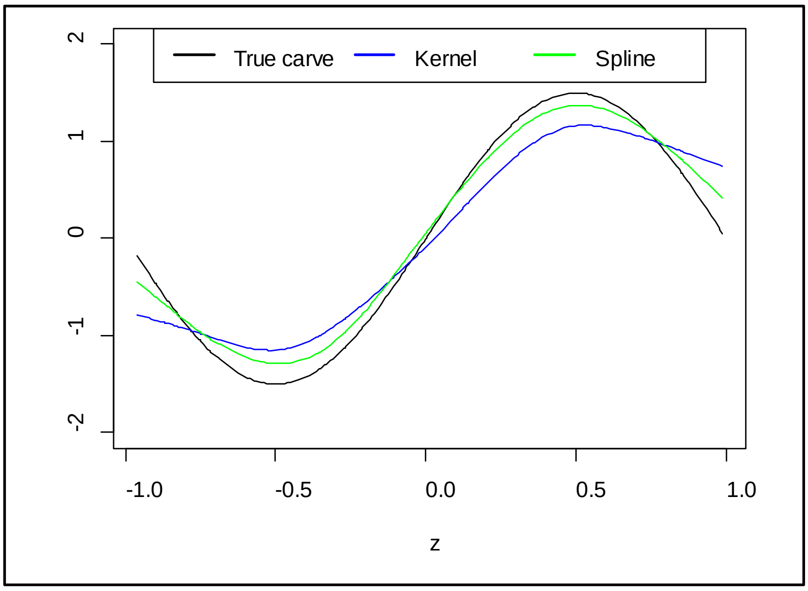

Figure-1: Fitted values for the estimators of g_1 when $n = 200,\ p = 12$, and $\sigma = .5$

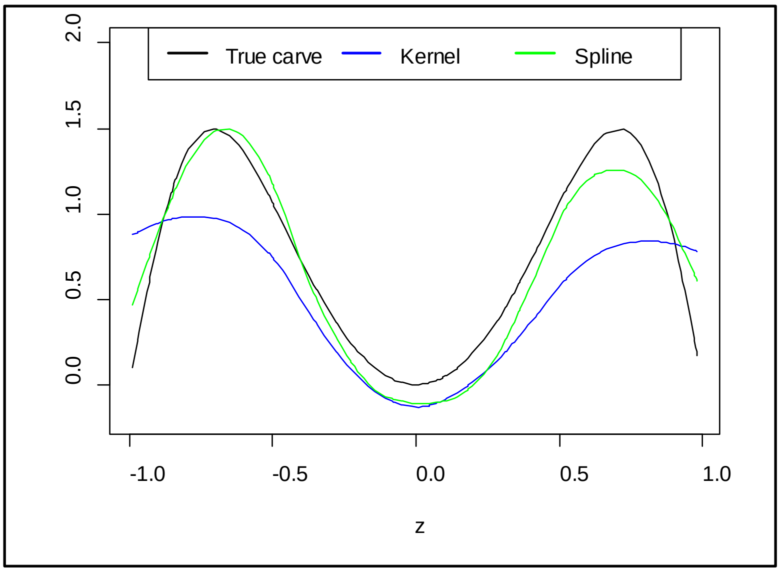

Figure-2: Fitted values for the estimators of g_2 when $n = 150$, $p = 5$, and $\sigma = 1$

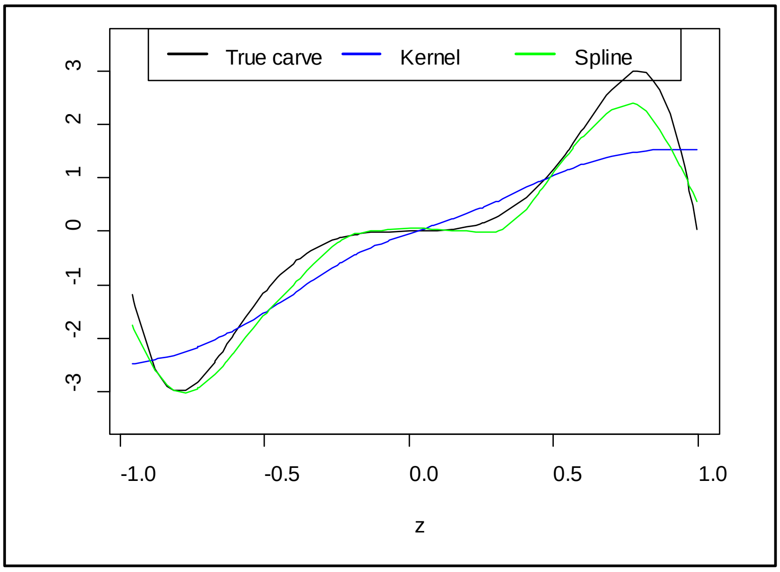

Figure-3: Fitted values for the estimators of g_3 when $n = 100$, $p = 2$, and $\sigma = .5$

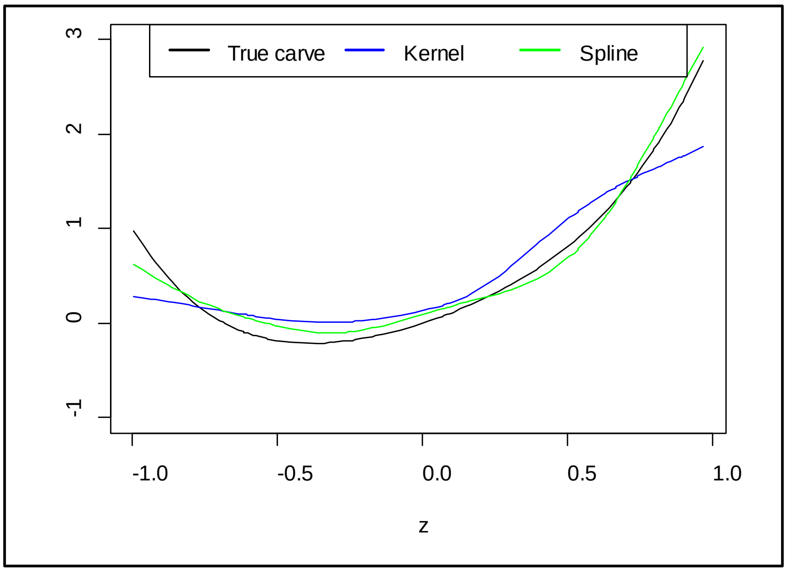

Figure-4: Fitted values for the estimators of g_4 when $n = 100$, $p = 3$, and $\sigma = .5$

Table-1: Average and median of MSEs of the estimators when $p = 2$ and using g_1

Sample	Parametric Component				Nonparametric Component			
	AMSE		MMSE		AMSE		MMSE	
	kernel	Spline	kernel	Spline	kernel	Spline	kernel	Spline
Sigma = .5								
100	0.0340	0.0311	0.0234	0.0208	0.1152	0.0345	0.1086	0.0250
150	0.0236	0.0213	0.0168	0.0161	0.0906	0.0229	0.0866	0.0172
200	0.0169	0.0159	0.0122	0.0109	0.0761	0.0177	0.0726	0.0131
300	0.0112	0.0105	0.0074	0.0071	0.0603	0.0113	0.0586	0.0087
500	0.0064	0.0062	0.0042	0.0041	0.0445	0.0071	0.0433	0.0054
Sigma = 1								
100	0.1330	0.1327	0.0908	0.0905	0.1812	0.1282	0.1468	0.0900
150	0.0848	0.0845	0.0619	0.0614	0.1360	0.0850	0.1164	0.0627
200	0.0661	0.0650	0.0460	0.0456	0.1126	0.0654	0.0976	0.0498
300	0.0398	0.0395	0.0277	0.0272	0.0834	0.0427	0.0748	0.0328
500	0.0247	0.0243	0.0167	0.0164	0.0583	0.0261	0.0533	0.0192

Table-2: Average and median of MSEs of the estimators when $p = 2$ and using g_2

Sample	Parametric Component				Nonparametric Component			
	AMSE		MMSE		AMSE		MMSE	
	kernel	Spline	kernel	Spline	kernel	Spline	kernel	Spline
Sigma = .5								
100	0.0389	0.0321	0.0263	0.0219	0.1240	0.0420	0.1161	0.0328
150	0.0259	0.0215	0.0181	0.0164	0.1041	0.0284	0.1006	0.0233
200	0.0192	0.0161	0.0129	0.0112	0.0923	0.0219	0.0883	0.0177
300	0.0120	0.0106	0.0086	0.0071	0.0775	0.0142	0.0763	0.0116
500	0.0072	0.0063	0.0048	0.0042	0.0637	0.0089	0.0624	0.0073
Sigma = 1								
100	0.1377	0.1346	0.0957	0.0944	0.1907	0.1488	0.1533	0.1105
150	0.0905	0.0857	0.0649	0.0614	0.1489	0.0988	0.1279	0.0768
200	0.0674	0.0656	0.0465	0.0457	0.1281	0.0765	0.1110	0.0611
300	0.0412	0.0397	0.0289	0.0273	0.1009	0.0503	0.0923	0.0410
500	0.0251	0.0244	0.0168	0.0163	0.0783	0.0317	0.0728	0.0257

Table-3: Average and median of MSEs of the estimators when $p = 2$ and using g_3

Sample	Parametric Component				Nonparametric Component			
	AMSE		MMSE		AMSE		MMSE	
	kernel	Spline	kernel	Spline	kernel	Spline	kernel	Spline
Sigma = .5								
100	0.0643	0.0340	0.0436	0.0232	0.3730	0.0545	0.3639	0.0454
150	0.0421	0.0221	0.0289	0.0166	0.3354	0.0361	0.3316	0.0311
200	0.0305	0.0165	0.0200	0.0113	0.3092	0.0284	0.3059	0.0247
300	0.0190	0.0109	0.0129	0.0073	0.2794	0.0186	0.2774	0.0163
500	0.0103	0.0064	0.0072	0.0042	0.2434	0.0119	0.2426	0.0104
Sigma = 1								
100	0.1730	0.1423	0.1236	0.0981	0.4452	0.1860	0.4143	0.1476
150	0.1040	0.0877	0.0711	0.0609	0.3798	0.1231	0.3641	0.1039
200	0.0796	0.0672	0.0533	0.0473	0.3480	0.0971	0.3341	0.0821
300	0.0469	0.0403	0.0333	0.0280	0.2993	0.0648	0.2930	0.0560
500	0.0290	0.0247	0.0197	0.0173	0.2582	0.0410	0.2556	0.0349

Table-4: Average and median of MSEs of the estimators when $p = 2$ and using g_4

Sample	Parametric Component				Nonparametric Component			
	AMSE		MMSE		AMSE		MMSE	
	kernel	Spline	kernel	Spline	kernel	Spline	kernel	Spline
Sigma = .5								
100	0.0334	0.0307	0.0229	0.0208	0.0840	0.0339	0.0771	0.0245
150	0.0225	0.0211	0.0165	0.0163	0.0659	0.0231	0.0613	0.0175
200	0.0165	0.0158	0.0112	0.0108	0.0560	0.0180	0.0531	0.0137
300	0.0110	0.0105	0.0074	0.0070	0.0450	0.0117	0.0424	0.0091
500	0.0064	0.0062	0.0043	0.0042	0.0336	0.0074	0.0325	0.0057
Sigma = 1								
100	0.1344	0.1310	0.0936	0.0894	0.1535	0.1215	0.1184	0.0843
150	0.0842	0.0834	0.0599	0.0595	0.1104	0.0816	0.0901	0.0599
200	0.0659	0.0648	0.0447	0.0456	0.0909	0.0629	0.0753	0.0463
300	0.0398	0.0394	0.0272	0.0265	0.0676	0.0418	0.0581	0.0319
500	0.0244	0.0243	0.0166	0.0167	0.0475	0.0260	0.0415	0.0188

Table-5: Average and median of MSEs of the estimators when $p = 8$ and using g_1

Sample	Parametric Component				Nonparametric Component			
	AMSE		MMSE		AMSE		MMSE	
	kernel	Spline	kernel	Spline	kernel	Spline	kernel	Spline
Sigma = .5								
100	0.0388	0.0346	0.0352	0.0316	0.1792	0.0907	0.1426	0.0543
150	0.0240	0.0219	0.0219	0.0197	0.1262	0.0552	0.1040	0.0336
200	0.0170	0.0158	0.0154	0.0146	0.1018	0.0423	0.0866	0.0240
300	0.0111	0.0105	0.0103	0.0097	0.0801	0.0308	0.0687	0.0180
500	0.0063	0.0062	0.0057	0.0055	0.0538	0.0167	0.0487	0.0102
Sigma = 1								
100	0.1443	0.1431	0.1298	0.1283	0.4120	0.3732	0.2392	0.1890
150	0.0879	0.0870	0.0794	0.0796	0.2727	0.2235	0.1808	0.1294
200	0.0637	0.0633	0.0588	0.0582	0.2022	0.1583	0.1373	0.0876
300	0.0415	0.0413	0.0384	0.0377	0.1490	0.1084	0.1024	0.0624
500	0.0249	0.0248	0.0228	0.0225	0.0961	0.0632	0.0706	0.0363

Table-6: Average and median of MSEs of the estimators when $p = 8$ and using g_2

Sample	Parametric Component				Nonparametric Component			
	AMSE		MMSE		AMSE		MMSE	
	kernel	Spline	kernel	Spline	kernel	Spline	kernel	Spline
Sigma = .5								
100	0.0433	0.0359	0.0390	0.0327	0.1908	0.1051	0.1498	0.0667
150	0.0272	0.0223	0.0252	0.0204	0.1428	0.0628	0.1181	0.0419
200	0.0187	0.0160	0.0174	0.0147	0.1207	0.0487	0.1030	0.0312
300	0.0121	0.0106	0.0112	0.0098	0.0993	0.0350	0.0862	0.0224
500	0.0069	0.0062	0.0064	0.0056	0.0742	0.0190	0.0678	0.0125
Sigma = 1								
100	0.1478	0.1450	0.1346	0.1307	0.4342	0.4033	0.2648	0.2305
150	0.0910	0.0882	0.0826	0.0808	0.2943	0.2429	0.1898	0.1430
200	0.0662	0.0638	0.0609	0.0583	0.2215	0.1711	0.1491	0.1002
300	0.0428	0.0416	0.0390	0.0379	0.1688	0.1174	0.1200	0.0688
500	0.0257	0.0249	0.0235	0.0225	0.1169	0.0694	0.0885	0.0428

Table-7: Average and median of MSEs of the estimators when $p = 8$ and using g_3

Sample	Parametric Component				Nonparametric Component			
	AMSE		MMSE		AMSE		MMSE	
	kernel	Spline	kernel	Spline	kernel	Spline	kernel	Spline
Sigma = .5								
100	0.0725	0.0372	0.0667	0.0340	0.4935	0.1198	0.4302	0.0844
150	0.0443	0.0231	0.0399	0.0210	0.4029	0.0749	0.3681	0.0525
200	0.0316	0.0166	0.0287	0.0154	0.3530	0.0572	0.3331	0.0410
300	0.0193	0.0108	0.0176	0.0098	0.3107	0.0401	0.2966	0.0288
500	0.0103	0.0063	0.0094	0.0056	0.2603	0.0226	0.2551	0.0165
Sigma = 1								
100	0.1732	0.1508	0.1569	0.1319	0.7154	0.4476	0.5255	0.2696
150	0.1090	0.0911	0.1011	0.0831	0.5447	0.2749	0.4337	0.1729
200	0.0762	0.0652	0.0693	0.0590	0.4631	0.1966	0.3858	0.1233
300	0.0485	0.0421	0.0442	0.0390	0.3783	0.1326	0.3307	0.0837
500	0.0290	0.0252	0.0263	0.0227	0.3018	0.0791	0.2777	0.0537

Table-8: Average and median of MSEs of the estimators when $p = 8$ and using g_4

Sample	Parametric Component				Nonparametric Component			
	AMSE		MMSE		AMSE		MMSE	
	kernel	Spline	kernel	Spline	kernel	Spline	kernel	Spline
Sigma = .5								
100	0.0375	0.0346	0.0342	0.0313	0.1424	0.0916	0.1057	0.0543
150	0.0232	0.0218	0.0209	0.0197	0.1015	0.0560	0.0799	0.0347
200	0.0169	0.0158	0.0155	0.0146	0.0813	0.0442	0.0656	0.0255
300	0.0111	0.0105	0.0102	0.0096	0.0651	0.0323	0.0530	0.0197
500	0.0064	0.0062	0.0058	0.0056	0.0436	0.0176	0.0377	0.0110
Sigma = 1								
100	0.1438	0.1419	0.1278	0.1266	0.3787	0.3651	0.2081	0.1874
150	0.0884	0.0865	0.0805	0.0789	0.2498	0.2181	0.1483	0.1195
200	0.0642	0.0631	0.0581	0.0574	0.1881	0.1578	0.1149	0.0861
300	0.0417	0.0412	0.0389	0.0378	0.1365	0.1080	0.0878	0.0608
500	0.0250	0.0248	0.0228	0.0226	0.0865	0.0636	0.0608	0.0369

4. CONCLUSION

In this paper, the performance of Speckman estimation of PLM based on kernel and spline smoothing approaches is investigated in different shapes of the nonparametric component. The Monte Carlo simulation study is conducted to evaluate and compare the performance of these estimators (kernel and spline estimators) under different situations (such as; different shapes of the nonparametric component, different number of parametric variables, different sample sizes, and different standard deviations of error term). The simulation results confirm that the uses of spline smoothing approach in Speckman estimation of PLM better than the use of kernel smoothing approach, because it have smallest MSE values.

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