

## SOME NEW TOPOLOGICAL INDICES OF GRAPHS

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### ABSTRACT

In 2019, Basavanagoud et al. introduced new degree based topological indices called Kulli-Basava indices and studied their mathematical and chemical properties which have good response with mean isomer degeneracy. In this paper, we propose the first and second hyper Kulli-Basava indices and their polynomials of a graph and compute exact formulae for complete graphs, wheel graphs, gear graphs and helm graphs. Also we determine the Kulli-Basava indices of gear graphs and helm graphs.

**Key words:** hyper Kulli-Basava indices, polynomials, graphs.

**Mathematics Subject Classification:** 05C07, 05C76, 92E10.

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### 1. INTRODUCTION

In Chemical Sciences, topological indices have been found to be useful in chemical documentation, isomer discrimination, structure property relationships, structure activity relationships. There has been considerable interest in the general problem of determining topological indices.

Let  $G$  be a finite, simple, connected graph with vertex set  $V(G)$  and edge set  $E(G)$ . The degree  $d_G(v)$  of a vertex  $v$  is the number of edges incident to  $v$ . The edge connecting the vertices  $u$  and  $v$  will be denoted by  $uv$ . The degree of an edge  $e = uv$  in a graph  $G$  is defined by  $d_G(e) = d_G(u) + d_G(v) - 2$ . The set of all vertices adjacent to  $v$  is called the open neighborhood of  $v$  and denoted by  $N_G(v)$ . Let  $S_G(v)$  denote the sum of the degrees of all vertices adjacent to a vertex  $v$ . The set of all edges incident to  $v$  is called the edge neighborhood of  $v$  and denoted by  $N_e(v)$ . Let  $S_e(v)$  denote the sum of the degrees of all edges incident to a vertex  $v$ . We refer to [1] for undefined term and notation.

The first and second Zagreb indices are the degree based topological indices, introduced by Gutman and Trinajstić in [2]. These indices have many applications in Chemistry. Recently, Basavanagoud and Jakkannavar introduced [3] the following Kulli-Basava indices:

The first Kulli-Basava index of a graph  $G$  is defined as

$$KB_1(G) = \sum_{uv \in E(G)} [S_e(u) + S_e(v)].$$

The modified first Kulli-Basava index of a graph  $G$  is defined as

$$KB_1^*(G) = \sum_{u \in V(G)} S_e(u)^2.$$

The second Kulli-Basava index of a graph  $G$  is defined as

$$KB_2(G) = \sum_{uv \in E(G)} S_e(u)S_e(v).$$

The third Kulli-Basava index of a graph  $G$  is defined as

$$KB_3(G) = \sum_{uv \in E(G)} |S_e(u) - S_e(v)|.$$

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Considering the above Kulli-Basava indices, we define the following polynomials:

The first Kulli-Basava polynomial of a graph  $G$  is defined as

$$KB_1(G, x) = \sum_{uv \in E(G)} x^{\lfloor S_e(u) + S_e(v) \rfloor}.$$

The modified first Kulli-Basava polynomial of a graph  $G$  is defined as

$$KB_1^*(G, x) = \sum_{u \in V(G)} x^{S_e(u)^2}.$$

The second Kulli-Basava polynomial of a graph  $G$  is defined as

$$KB_2(G, x) = \sum_{uv \in E(G)} x^{S_e(u)S_e(v)}.$$

The third Kulli-Basava polynomial of a graph  $G$  is defined as

$$KB_3(G, x) = \sum_{uv \in E(G)} x^{\lfloor S_e(u) - S_e(v) \rfloor}.$$

In a recent years, the Zagreb index [4],  $F$ -index [5], sum connectivity index [6], reverse indices [7], Revan indices [8], Dakshayani index [9] were introduced and extensively studied.

The hyper Zagreb index of a graph  $G$  was introduced by Shirdel et al. in [10] and it is defined as

$$HM_1(G) = \sum_{uv \in E(G)} [d_G(u) + d_G(v)]^2.$$

In [11], Gao et al. proposed the second hyper Zagreb index, defined as

$$HM_2(G) = \sum_{uv \in E(G)} [d_G(u) d_G(v)]^2.$$

We now introduce the first and second hyper Kulli-Basava indices of a graph  $G$ , defined as

$$HKB_1(G) = \sum_{uv \in E(G)} [S_e(u) + S_e(v)]^2,$$

$$HKB_2(G) = \sum_{uv \in E(G)} [S_e(u) S_e(v)]^2.$$

Considering the first and second hyper Kulli-Basava indices, we propose the first and second hyper Kulli-Basava polynomials of a graph, defined as

$$HKB_1(G, x) = \sum_{uv \in E(G)} x^{\lfloor S_e(u) + S_e(v) \rfloor^2},$$

$$HKB_2(G, x) = \sum_{uv \in E(G)} x^{\lfloor S_e(u) S_e(v) \rfloor^2}.$$

## 2. COMPLETE GRAPHS

Let  $K_n$  be a complete graph with  $n$  vertices. Then the degree of each vertex of  $K_n$  is  $n - 1$  and the number of edges in  $K_n$  is  $\frac{n(n-1)}{2}$ .

**Lemma 1:** Let  $K_n$  be a complete graph with  $n$  vertices,  $n \geq 2$ . Then

- (i)  $V_1 = \{u \in V(K_n) \mid S_e(u) = 2(n-1)(n-2)\}$ ,  $|V_1| = n$ .
- (ii)  $E_1 = \{uv \in E(K_n) \mid S_e(u) = S_e(v) = 2(n-1)(n-2)\}$ ,  $|E_1| = \frac{n(n-1)}{2}$ .

**Theorem 2:** If  $K_n$  is a complete graph with  $n \geq 2$  vertices, then

$$KB_1(K_n, x) = \frac{n(n-1)}{2} x^{4(n-1)(n-2)}$$

$$KB_1^*(K_n, x) = nx^{4(n-1)^2(n-2)^2}$$

$$KB_2(K_n, x) = \frac{n(n-1)}{2} x^{4(n-1)^2(n-2)^2}$$

$$KB_3(K_n, x) = \frac{n(n-1)}{2} x^0.$$

**Proof:** By using definitions and Lemma 1, we obtain

$$KB_1(K_n, x) = \sum_{uv \in E(K_n)} x^{S_e(u)+S_e(v)} = |E_1| x^{4(n-1)(n-2)} = \frac{n(n-1)}{2} x^{4(n-1)(n-2)}$$

$$KB_1^*(K_n, x) = \sum_{u \in V(K_n)} x^{S_e(u)^2} = |V_1| x^{4(n-1)^2(n-2)^2} = nx^{4(n-1)^2(n-2)^2}$$

$$KB_2(K_n, x) = \sum_{u \in E(K_n)} x^{S_e(u)S_e(v)} = |E_1| x^{4(n-1)^2(n-2)^2} = \frac{n(n-1)}{2} x^{4(n-1)^2(n-2)^2}$$

$$KB_3(K_n, x) = \sum_{uv \in E(K_n)} x^{|S_e(u)-S_e(v)|} = |E_1| x^0 = \frac{n(n-1)}{2} x^0.$$

**Theorem 3:** The first hyper Kulli-Basava index and its polynomial of a complete graph are given by

(i)  $HKB_1(K_n) = 8n(n-1)^3(n-2)^2$

(ii)  $HKB_1(K_n, x) = \frac{n(n-1)}{2} x^{16(n-1)^2(n-2)^2}.$

**Proof:**

(i) Let  $K_n$  be a complete graph with  $n \geq 2$  vertices. By using definition and Lemma 1, we derive

$$HKB_1(K_n) = \sum_{uv \in E(K_n)} [S_e(u) + S_e(v)]^2 = |E_1| [4(n-1)(n-2)]^2$$

$$= 8n(n-1)^3(n-2)^2.$$

(ii) From definition and by using Lemma 1, we obtain

$$HKB_1(K_n, x) = \sum_{uv \in E(K_n)} x^{[S_e(u)+S_e(v)]^2} = |E_1| x^{[4(n-1)(n-2)]^2}$$

$$= \frac{n(n-1)}{2} x^{16(n-1)^2(n-2)^2}.$$

**Theorem 4:** The second hyper Kulli-Basava index and its polynomial of a complete graph are given by

(i)  $HKB_2(K_n) = 8n(n-1)^5(n-2)^4.$

(ii)  $HKB_2(K_n, x) = \frac{n(n-1)}{2} x^{16(n-1)^4(n-2)^4}.$

**Proof:**

(i) Let  $K_n$  be a complete graph with  $n \geq 2$  vertices. Then by using definition and Lemma 1, we deduce

$$HKB_2(K_n) = \sum_{uv \in E(K_n)} [S_e(u)S_e(v)]^2 = |E_1| [4(n-1)^2(n-2)^2]^2$$

$$= 8n(n-1)^5(n-2)^4.$$

(ii) By using definition and Lemma 1, we obtain

$$HKB_2(K_n, x) = \sum_{uv \in E(K_n)} x^{[S_e(u)S_e(v)]^2} = |E_1| x^{[4(n-1)^2(n-2)^2]^2}$$

$$= \frac{n(n-1)}{2} x^{16(n-1)^4(n-2)^4}.$$

### 3. WHEEL GRAPHS

A wheel  $W_n$  is the join of  $K_1$  and  $C_n$ . Clearly  $W_n$  has  $n+1$  vertices and  $2n$  edges. A graph  $W_n$  is depicted in Figure 1. The vertices of  $C_n$  are called rim vertices and the vertex of  $K_1$  is called apex.

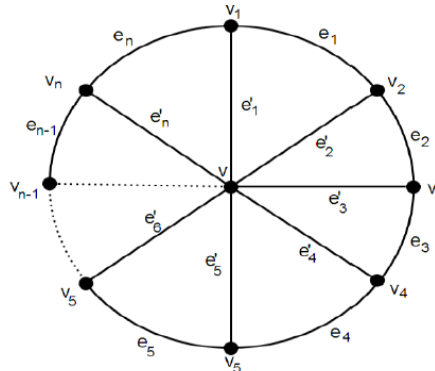


Figure-1: Wheel  $W_n$

**Lemma 5:** Let  $W_n$  be a wheel with  $n+1$  vertices. Then  $W_n$  has two types of vertices as given below:

$$V_1 = \{u \in V(W_n) \mid S_e(u) = n(n+1)\}, \quad |V_1| = 1.$$

$$V_2 = \{u \in V(W_n) \mid S_e(u) = n+9\}, \quad |V_2| = n.$$

**Lemma 6:** Let  $W_n$  be a wheel with  $2n$  edges. Then  $W_n$  has two types of edges as given below:

$$E_1 = \{uv \in E(W_n) \mid S_e(u) = n+9, S_e(v) = n(n+1)\}, \quad |E_1| = n.$$

$$E_2 = \{uv \in E(W_n) \mid S_e(u) = n+9, S_e(v) = n+9\}, \quad |E_2| = n.$$

**Theorem 7:** Let  $W_n$  be a wheel with  $n+1$  vertices and  $2n$  edges,  $n \geq 3$ . Then

$$KB_1(W_n, x) = nx^{n^2+2n+9} + nx^{2n+18}$$

$$KB_1^*(W_n, x) = x^{n^2(n+1)^2} + nx^{(n+9)^2}$$

$$KB_2(W_n, x) = nx^{n(n+1)(n+9)} + nx^{(n+9)^2}$$

$$KB_3(W_n, x) = nx^{|n^2-9|} + nx^0$$

**Proof:** By using definitions and Lemmas 5 and 6, we deduce

$$KB_1(W_n, x) = \sum_{uv \in E(W_n)} x^{S_e(u)+S_e(v)} = |E_1| x^{n+9+n(n+1)} + |E_2| x^{n+9+n+9}$$

$$= nx^{n^2+2n+9} + nx^{2n+18}$$

$$KB_1^*(W_n, x) = \sum_{u \in V(W_n)} x^{S_e(u)^2} = |V_1| x^{n^2(n+1)^2} + |V_2| x^{(n+9)^2}$$

$$= x^{n^2(n+1)^2} + nx^{(n+9)^2}$$

$$KB_2(W_n, x) = \sum_{uv \in E(W_n)} x^{S_e(u)S_e(v)} = |E_1| x^{(n+9)n(n+1)} + |E_2| x^{(n+9)(n+9)}$$

$$= nx^{n(n+1)(n+9)} + nx^{(n+9)^2}$$

$$KB_3(W_n, x) = \sum_{uv \in E(W_n)} x^{|S_e(u)-S_e(v)|} = |E_1| x^{|n+9-n(n+1)|} + |E_2| x^{|n+9-n-9|}$$

$$= nx^{|n^2-9|} + nx^0$$

**Theorem 8:** The first hyper Kulli-Basava index and its polynomial of a wheel  $W_n$  are given by

- (i)  $HKB_1(W_n) = n(n^2 + 2n + 9)^2 + n(2n + 18)^2$ .
- (ii)  $HKB_1(W_n, x) = nx^{(n^2+2n+9)^2} + nx^{(2n+18)^2}$ .

**Proof:**

(i) Let  $W_n$  be a wheel with  $n \geq 3$  vertices. Then by using definition and Lemma 6, we deduce

$$\begin{aligned}
 HKB_1(W_n) &= \sum_{uv \in E(W_n)} [S_e(u) + S_e(v)]^2 = |E_1| [n + 9 + n(n+1)]^2 + |E_2| (n + 9 + n + 9)^2 \\
 &= n(n^2 + 2n + 9)^2 + n(2n + 18)^2.
 \end{aligned}$$

(ii) 
$$\begin{aligned}
 HKB_1(W_n, x) &= \sum_{uv \in E(W_n)} x^{[S_e(u) + S_e(v)]^2} \\
 &= nx^{(n^2 + 2n + 9)^2} + nx^{(2n + 18)^2}.
 \end{aligned}$$

**Theorem 9:** The second hyper Kulli-Basava index and its polynomial of a wheel  $W_n$  are given by

(i) 
$$HKB_2(W_n) = n(n + 9)^2 [n^2(n + 1)^2 + (n + 9)^2].$$

(ii) 
$$HKB_2(W_n, x) = nx^{n^2(n+1)^2(n+9)^2} + nx^{(n+9)^4}.$$

**Proof:**

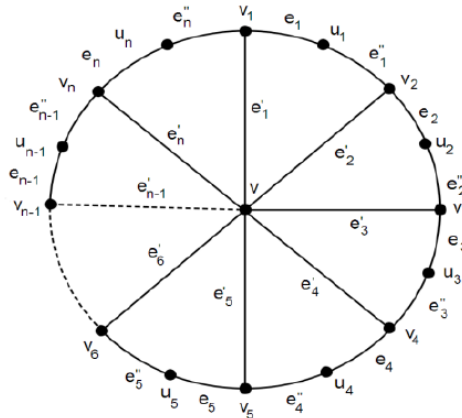
(i) By using definition and Lemma 6, we derive

$$\begin{aligned}
 HKB_2(W_n) &= \sum_{uv \in E(W_n)} [S_e(u) S_e(v)]^2 = |E_1| [(n + 9)n(n + 1)]^2 + |E_2| [(n + 9)(n + 9)]^2 \\
 &= n(n + 9)^2 [n^2(n + 1)^2 + (n + 9)^2].
 \end{aligned}$$

(ii) 
$$\begin{aligned}
 HKB_2(W_n, x) &= \sum_{uv \in E(W_n)} x^{[S_e(u) S_e(v)]^2} \\
 &= nx^{n^2(n+1)^2(n+9)^2} + nx^{(n+9)^4}.
 \end{aligned}$$

**4. GEAR GRAPHS**

A graph is a gear graph obtained from  $W_n$  by adding a vertex between each pair of adjacent rim vertices and it is denoted by  $G_n$ . Clearly  $G_n$  has  $2n+1$  vertices and  $3n$  edges. A graph  $G_n$  is shown in Figure 2.



**Figure-2:** Gear graph  $G_n$

**Lemma 10:** Let  $G_n$  be a gear graph with  $2n+1$  vertices. Then  $G_n$  has three types of vertices as given below.

$$\begin{aligned}
 V_1 &= \{u \in V(G_n) \mid S_e(u) = n(n+1)\}, & |V_1| &= 1. \\
 V_2 &= \{u \in V(G_n) \mid S_e(u) = n+7\}, & |V_2| &= n. \\
 V_3 &= \{u \in V(G_n) \mid S_e(u) = 6\}, & |V_3| &= n.
 \end{aligned}$$

**Lemma 11:** Let  $G_n$  be a gear graph with  $3n$  edges. Then  $G_n$  has two types of edges as shown below.

$$\begin{aligned}
 E_1 &= \{uv \in E(G_n) \mid S_e(u) = n(n+1), S_e(v) = n+7\}, & |E_1| &= n. \\
 E_2 &= \{uv \in E(G_n) \mid S_e(u) = n+7, S_e(v) = 6\}, & |E_2| &= 2n.
 \end{aligned}$$

**Theorem 12:** Let  $G_n$  be a gear graph with  $2n+1$  vertices and  $3n$  edges. Then

$$\begin{aligned}
 KB_1(G_n) &= n^3 + 4n^2 + 33n. \\
 KB_1^*(G_n) &= n^4 + 3n^3 + 15n^2 + 85n.
 \end{aligned}$$

$$KB_2(G_n) = n(n+7)(n^2+n+12).$$

$$KB_3(G_n) = n|n^2-7|+2n(n+1).$$

**Proof:** By using definitions and Lemmas 10 and 11, we deduce

$$\begin{aligned} KB_1(G_n) &= \sum_{uv \in E(G_n)} [S_e(u) + S_e(v)] \\ &= n[n(n+1) + n+7] + 2n(n+7+6) \\ &= n^3 + 4n^2 + 33n. \end{aligned}$$

$$\begin{aligned} KB_1^*(G_n) &= \sum_{u \in V(G_n)} S_e(u)^2 \\ &= [n(n+1)]^2 + n(n+7)^2 + n6^2 \\ &= n^4 + 3n^3 + 15n^2 + 85n. \end{aligned}$$

$$\begin{aligned} KB_2(G_n) &= \sum_{uv \in E(G_n)} S_e(u)S_e(v) \\ &= n[n(n+1)(n+7)] + 2n[(n+7)6] \\ &= n(n+7)(n^2+n+12). \end{aligned}$$

$$\begin{aligned} KB_3(G_n) &= \sum_{uv \in E(G_n)} |S_e(u) - S_e(v)| \\ &= n|n(n+1) - (n+7)| + 2n|n+7-6| \\ &= n|n^2-7| + 2n(n+1). \end{aligned}$$

**Theorem 13:** Let  $G_n$  be a gear graph with  $2n+1$  vertices and  $3n$  edges. Then

$$KB_1(G_n, x) = nx^{n^2+2n+7} + 2nx^{n+13}.$$

$$KB_1^*(G_n, x) = x^{n^2(n+1)^2} + nx^{(n+7)^2} + nx^{36}.$$

$$KB_2(G_n, x) = nx^{n(n+1)(n+7)} + 2nx^{6(n+7)}.$$

$$KB_3(G_n, x) = nx^{|n^2-7|} + 2nx^{n+1}.$$

**Proof:** By using definitions and Lemmas 10 and 11, we derive

$$\begin{aligned} KB_1(G_n, x) &= \sum_{uv \in E(G_n)} x^{S_e(u)+S_e(v)} = |E_1|x^{n(n+1)+n+7} + |E_2|x^{n+7+6} \\ &= nx^{n^2+2n+7} + 2nx^{n+13} \end{aligned}$$

$$\begin{aligned} KB_1^*(G_n, x) &= \sum_{u \in V(G_n)} x^{S_e(u)^2} = |V_1|x^{n^2(n+1)^2} + |V_2|x^{(n+7)^2} + |V_3|x^{6^2} \\ &= x^{n^2(n+1)^2} + nx^{(n+7)^2} + nx^{36}. \end{aligned}$$

$$\begin{aligned} KB_2(G_n, x) &= \sum_{uv \in E(G_n)} x^{S_e(u)S_e(v)} = |E_1|x^{n(n+1)(n+7)} + |E_2|x^{6(n+7)} \\ &= nx^{n(n+1)(n+7)} + 2nx^{6(n+7)} \end{aligned}$$

$$\begin{aligned} KB_3(G_n, x) &= \sum_{uv \in E(G_n)} x^{|S_e(u)-S_e(v)|} = |E_1|x^{|n(n+1)-(n+7)|} + |E_2|x^{|n+7-6|} \\ &= nx^{|n^2-7|} + 2nx^{n+1}. \end{aligned}$$

**Theorem 14:** The first hyper Kulli-Basava index and its polynomial of a gear graph  $G_n$  are given by

(i)  $HKB_1(G_n) = n(n^2 + 2n + 7)^2 + 2n(n + 13)^2.$

(ii)  $HKB_1(G_n, x) = nx^{(n^2+2n+7)^2} + 2nx^{(n+13)^2}.$

**Proof:** Let  $G_n$  be a gear graph with  $2n+1$  vertices and  $3n$  edges.

(i) By using definition and Lemma 11, we obtain

$$\begin{aligned} HKB_1(G_n) &= \sum_{uv \in E(G_n)} [S_e(u) + S_e(v)]^2 = |E_1| [n(n+1) + n + 7]^2 + |E_2| [(n+7) + 6]^2 \\ &= n(n^2 + 2n + 7)^2 + 2n(n+13)^2. \end{aligned}$$

(ii) 
$$HKB_1(G_n, x) = \sum_{uv \in E(G_n)} x^{[S_e(u) + S_e(v)]^2} = nx^{(n^2 + 2n + 7)^2} + 2nx^{(n+13)^2}.$$

**Theorem 15:** The second hyper Kulli-Basava index and its polynomial of a gear graph  $G_n$  are given by

(i) 
$$HKB_2(G_n) = n(n+7)^2 [n^2(n+1)^2 + 72].$$

(ii) 
$$HKB_2(G_n, x) = nx^{n^2(n+1)^2(n+7)^2} + 2nx^{36(n+7)^2}.$$

**Proof:**

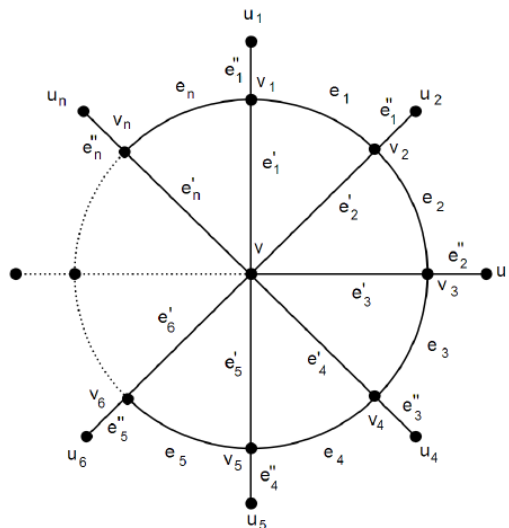
(i) By using definition and Lemma 11, we have

$$\begin{aligned} HKB_2(G_n) &= \sum_{uv \in E(G_n)} [S_e(u) S_e(v)]^2 = |E_1| [n(n+1)(n+7)]^2 + |E_2| [(n+7)6]^2 \\ &= n(n+7)^2 [n^2(n+1)^2 + 72]. \end{aligned}$$

(ii) 
$$HKB_2(G_n, x) = \sum_{uv \in E(G_n)} x^{[S_e(u) S_e(v)]^2} = nx^{n^2(n+1)^2(n+7)^2} + 2nx^{36(n+7)^2}.$$

**5. HELM GRAPHS**

A helm graph  $H_n$  is a graph obtained from  $W_n$  by attaching an end edge to each rim vertex. Clearly  $H_n$  has  $2n+1$  vertices and  $3n$  edges, see Figure 3.



**Figure-3:** Helm graph  $H_n$

**Lemma 16:** Let  $H_n$  be a helm graph with  $2n+1$  vertices.

Then  $H_n$  has three types of vertices as follows:

- $V_1 = \{u \in V(H_n) \mid S_e(u) = n(n+2)\}, \quad |V_1| = 1.$
- $V_2 = \{u \in V(H_n) \mid S_e(u) = n+17\}, \quad |V_2| = n.$
- $V_3 = \{u \in V(H_n) \mid S_e(u) = 3\}, \quad |V_3| = n.$

**Lemma 17:** Let  $H_n$  be a helm graph with  $3n$  edges. Then  $H_n$  has three types of edges as follows.

- $E_1 = \{uv \in E(H_n) \mid S_e(u) = n(n+2), S_e(v) = n+17\}, \quad |E_1| = n.$
- $E_2 = \{uv \in E(H_n) \mid S_e(u) = S_e(v) = n+17\}, \quad |E_2| = n.$
- $E_3 = \{uv \in E(H_n) \mid S_e(u) = n+17, S_e(v) = 3\}, \quad |E_3| = n.$

**Theorem 18:** Let  $H_n$  be a helm graph with  $2n + 1$  vertices and  $3n$  edges. Then

$$KB_1(H_n) = 3n^3 + 4n^2 + 71n.$$

$$KB_1^*(H_n) = n^4 + 5n^3 + 38n^2 + 298n.$$

$$KB_2(H_n) = n(n + 7)(n^2 + 3n + 20).$$

$$KB_3(H_n) = n|n^2 + n - 17| + n(n + 14).$$

**Proof:** By using definitions and Lemmas 16 and 17, we obtain

$$KB_1(H_n) = \sum_{uv \in E(H_n)} [S_e(u) + S_e(v)]$$

$$= n[n(n + 2) + n + 17] + n[n + 17 + n + 17] + n(n + 17 + 3)$$

$$= 3n^3 + 4n^2 + 71n.$$

$$KB_1^*(H_n) = \sum_{u \in V(H_n)} S_e(u)^2$$

$$= [n(n + 2)]^2 + n(n + 17)^2 + n \times 3^2$$

$$= n^4 + 5n^3 + 38n^2 + 298n.$$

$$KB_2(H_n) = \sum_{uv \in E(H_n)} S_e(u)S_e(v)$$

$$= n[n(n + 2)(n + 17)] + n[(n + 17)(n + 17)] + n[(n + 17)3]$$

$$= n(n + 17)(n^2 + 3n + 20).$$

$$KB_3(H_n) = \sum_{uv \in E(H_n)} |S_e(u) - S_e(v)|$$

$$= n|n(n + 2) - (n + 17)| + n|(n + 17) - (n + 17)| + n|n + 17 - 3|$$

$$= n|n^2 + n - 17| + n(n + 14).$$

**Theorem 19:** Let  $H_n$  be a helm graph with  $2n+1$  vertices and  $3n$  edges. Then

$$KB_1(H_n, x) = nx^{n^2+3n+17} + nx^{2(n+17)} + nx^{n+20}.$$

$$KB_1^*(H_n, x) = x^{[n(n+2)]^2} + nx^{(n+17)^2} + nx^9.$$

$$KB_2(H_n, x) = nx^{n(n+2)(n+17)} + nx^{(n+17)^2} + nx^{3(n+17)}.$$

$$KB_3(H_n, x) = nx^{|n^2+n-17|} + nx^0 + nx^{n+14}.$$

**Proof:** By using definitions and Lemmas 16 and 17, we deduce

$$KB_1(H_n, x) = \sum_{uv \in E(H_n)} x^{S_e(u)+S_e(v)}$$

$$= nx^{n(n+2)+n+17} + nx^{n+17+n+17} + nx^{n+17+3}.$$

$$= nx^{n^2+3n+17} + nx^{2(n+17)} + nx^{n+20}.$$

$$KB_1^*(H_n, x) = \sum_{u \in V(H_n)} x^{S_e(u)^2}$$

$$= x^{[n(n+2)]^2} + nx^{(n+17)^2} + nx^9.$$

$$KB_2(H_n, x) = \sum_{uv \in E(H_n)} x^{S_e(u)S_e(v)}$$

$$= nx^{n(n+2)(n+17)} + nx^{(n+17)^2} + nx^{3(n+17)}.$$

$$KB_3(H_n, x) = \sum_{uv \in E(H_n)} x^{|S_e(u)-S_e(v)|}$$

$$= nx^{|n(n+2)-(n+17)|} + nx^{|(n+17)-(n+17)|} + nx^{|n+17-3|}.$$

$$= nx^{|n^2+n-17|} + nx^0 + nx^{n+14}.$$



**Theorem 20:** The first hyper Kulli-Basava index and its polynomial of a helm graph  $H_n$  are

$$(i) \quad HKB_1(H_n) = n(n^2 + 3n + 17)^2 + 4n(n + 17)^2 + n(n + 20)^2.$$

$$(ii) \quad HKB_1(H_n, x) = nx^{(n^2+3n+17)^2} + nx^{(2n+34)^2} + nx^{(n+20)^2}.$$

**Proof:** Let  $H_n$  be a helm graph with  $2n + 1$  vertices and  $3n$  edges.

(i) By using definition and Lemma 17, we deduce

$$\begin{aligned} HKB_1(H_n) &= \sum_{uv \in E(H_n)} [S_e(u) + S_e(v)]^2 = n[n(n + 2) + n + 17]^2 + n[n + 17 + n + 17]^2 + n(n + 17 + 3)^2 \\ &= n(n^2 + 3n + 17)^2 + 4n(n + 17)^2 + n(n + 20)^2. \end{aligned}$$

$$\begin{aligned} (ii) \quad HKB_1(H_n, x) &= \sum_{uv \in E(H_n)} x^{[S_e(u)+S_e(v)]^2} \\ &= nx^{(n^2+3n+17)^2} + nx^{(2n+34)^2} + nx^{(n+20)^2}. \end{aligned}$$

**Theorem 21:** The second hyper Kulli-Basava index or its polynomial of a helm graph  $H_n$  are

$$(i) \quad HKB_2(H_n) = n(n + 17)^2 [n^2(n + 2)^2 + (n + 17)^2 + 9].$$

$$(ii) \quad HKB_2(H_n, x) = nx^{n^2(n+2)^2(n+17)^2} + nx^{(n+17)^4} + 2nx^{9(n+17)^2}.$$

**Proof:** Let  $H_n$  be a helm graph with  $2n+1$  vertices and  $3n$  edges.

$$\begin{aligned} (i) \quad HKB_2(H_n) &= \sum_{uv \in E(H_n)} [S_e(u) S_e(v)]^2 = n[n(n + 2)(n + 17)]^2 + n[(n + 17)(n + 17)]^2 + n[(n + 17)3]^2 \\ &= n(n + 17)^2 [n^2(n + 2)^2 + (n + 17)^2 + 9]. \end{aligned}$$

$$\begin{aligned} (ii) \quad HKB_2(H_n, x) &= \sum_{uv \in E(H_n)} x^{[S_e(u)S_e(v)]^2} \\ &= nx^{[n(n+2)(n+17)]^2} + nx^{(n+17)^4} + nx^{9(n+17)^2}. \end{aligned}$$

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