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NON-EXISTENCE OF COSMIC STRINGS IN BIANCHI TYPE-V COSMOLOGY

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ABSTRACT

***B**ianchi type-V string cosmological model is investigated in the presence and absence of bulk viscous fluid in general relativity. It has been shown that cosmic strings do not exist in Bianchi type-V cosmology. The physical and geometrical aspects of the model are also discussed.*

***Keywords**: Bianchi type-V model, Bulk viscous fluid, cosmic strings.*

1. INTRODUCTION

The study of Bianchi type-V cosmological models plays an important role in the study of Universe and the study is more interesting as these models contain isotropic special cases and permit arbitrary small anisotropy levels at some point of time. The concept of string theory was developed to describe events of the early stage of the evolution of the Universe. The general relativistic treatment of strings was initially done by Stachel [28] and Letelier [15, 16]. Letelier [15] obtained the general solution of Einglein's field equations for a cloud of strings with spherical, plane and a particular case of cylindrical symmetry. Letelier [16] also obtained massive string cosmological models in Bianchi type-I and kantowski-Sachs space-times. The gravitational effects of cosmic strings have been extensively discussed by Vilenkin [30] and Gott [10] in general relatively. The string cosmological models with magnetic field are investigated by Chakraborty [5], Tikelar & Patel [29]. Banerjee *et al.* [2] have investigated an axially symmetric Bianchi type-I string dust cosmological model with and without magnetic field. Krori *et al.* [12] and Wang [31] studied the exact solutions of string cosmology for Bianchi type-II, VI0, VII and IX space-times.

Cosmic strings are important in the early stages of evolution of the universe before the particle creation. The present day observations do not rule out the possible existence of large scale networks of strings in the early universe. Reddy [21] have shown that, the cosmic strings which have received considerable attention in cosmology do not exist in the framework of Rosen's [23] bimetric theory of gravitation. Krori *et al.* [13] and Deo *et.al.* [8] have shown that, the cosmic strings do not occur in Bianchi type-V cosmology.

The role of viscosity in cosmology has been investigated by Weinbergs [32], Nightingale [18] and Heller & Klimek [11]. Heller and Klimek [11] have investigated viscous fluid cosmological model without initial singularity. They have shown that the introduction of bulk viscosity effectively removes the initial singularity. Roy and Singh [22] have investigated LRS Biancli type-V cosmological model with viscosity. Santos *et al.* [26] investigated isotropic homogeneous cosmological model with bulk viscosity assuming viscous coefficient as power function of mass density. Banerjee and Sanyal [3] have investigated Bianchi type-V cosmological models with viscosity and heat flow. Pradhan *et al.* [19] have found the integrability of cosmic string in Bianchi type-III space time in presence of bulk viscous fluid. Bianchi type-V cosmological models have been studied by several researchers such as Farnesworth [9], Collins [7], Maartens and Nel [17], Beesham [4], Camci *et al.* [6], Aydogdu and Salti [1], in different physical contexts. Following the work of Saha [24], Singh and Chaubey [27] have obtained the quadrature form of metric function for Bianchi type-V model with perfect fluid and viscous fluid.

In this paper, we have investigated non-existence of cosmic strings in bianchi type-V cosmological model with and without bulk viscous fluid. This paper is organized as follows: The metric and field equations with bulk viscous fluid are presented in Section 2. Subsection 2.1 deals with solution of the field equations. Some geometrical and physical properties of the model is presented in Subsection 2.2. Field equations without bulk viscous fluid are presented in Section 3. Subsection 3.1 deals with solution of the field equations. Some geometrical and physical properties of the model is presented in Subsection 3.2 Finally the conclusions of the paper are presented in Section 4.

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2. STRING COSMOLOGICAL MODEL WITH BULK VISCOUS FLUID

We consider a spatially homogeneous and anisotropic Bianchi type-V metric in the from

$$ds^2 = -dt^2 + A^2 dx^2 + e^{2mx}[B^2 dy^2 + C^2 dz^2], \tag{1}$$

where the metric potentials A, B and C are function of cosmic time t alone and m is a costant.

The energy-momentum tensor for bulk viscous string dust is given by Letelier [16], Landau and Lifshitz [14] as

$$T_i^j = \rho v_i v^j - \lambda x_i x^j - \xi v_{;\ell}^{\ell}(g_i^j + v_i v^j), \tag{2}$$

where ρ is the proper the energy density of the cloud of string with particle attached to them, λ is the string tension density and ξ is the coefficient of bulk viscosity. $v^i = (0,0,0,1)$ is the four velocity of the particle and x^i is a unit space-like vector representing the direction of string. The vectors v^i and x^i satisfy the conditions

$$v_i v^i = -x_i x^i = -1, \qquad v_i x_i = 0. \tag{3}$$

Choosing x^i parallel to $\frac{\partial}{\partial x}$, we have

$$x^i = \left(\frac{1}{A},\ 0,\ 0,\ 0\right). \tag{4}$$

If the particle density of the configuration is denoted by ρ_p, then

$$\rho = \rho_p + \lambda \tag{5}$$

The Einstein's field equations (in gravitational units $8\pi G = c = 1$) are given by

$$R_i^j - \frac{1}{2}Rg_i^j = -T_i^j. \tag{6}$$

The field equation (6) with Equation (2) subsequently lead to the following system of equations:

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{m^2}{A^2} = \lambda + \xi\, v_{;\ell}^{\ell}, \tag{7}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{m^2}{A^2} = \xi\, v_{;\ell}^{\ell}, \tag{8}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{C} + \frac{\dot{A}\dot{B}}{AC} - \frac{m^2}{A^2} = \xi\, v_{;\ell}^{\ell} \tag{9}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC} - \frac{3m^2}{A^2} = \rho, \tag{10}$$

$$2\frac{\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C} = 0, \tag{11}$$

where

$$\theta = v_{;\ell}^{\ell} = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}. \tag{12}$$

2.1. SOLUTION OF THE FIELD EQUATIONS

Integrating (11) and engrossing the constant of integration in B or C, without loss of generality, we obtain

$$A^2 = B\,C \tag{13}$$

Subtracting (8) from (9) and taking integral of the resulting equation two times, we get

$$\frac{B}{C} = c_2 \exp\left[c_1 \int \frac{dt}{ABC}\right], \tag{14}$$

where c_1 and c_2 are constants of integration.

Equations (7) - (11) are five independent equations in *six* unknown parameters A, B, C, λ ξ and ρ. For the complete determination of the system, we need one extra condition. Following Yadav [33], we assume the law of variation of scale factor as increasing function of time as given by

$$a = (t^k e^t)^{\frac{1}{n}}, \tag{15}$$

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where k and n are positive constants. If we put n = 2, Eq. (15) reduces to $a(t)=\sqrt{t^k e^t}$ which is used by Saha *et al.* [25] and Pradhan & Amirhashchi [20] in studying two-fluid scenario for dark energy models in an FRW universe and accelerating dark energy models in Bianchi type-V space-time respectively.

Now the spatial volume V of the model is given by

$$V = a^3 = ABC \tag{16}$$

Equations (13), (15) and (16) lead to

$$A(t) = (t^k e^t)^{\frac{1}{n}} \tag{17}$$

Inserting Equations (17) and (13) into Equation (14), we get

$$B(t) = \sqrt{c_2}\,(t^k e^t)^{\frac{1}{n}} \exp\left[\frac{c_1}{2}\int \frac{dt}{(t^k e^t)^{\frac{3}{n}}}\right], \tag{18}$$

$$C(t) = \frac{1}{\sqrt{c_2}}(t^k e^t)^{\frac{1}{n}} \exp\left[\frac{-c_1}{2}\int \frac{dt}{(t^k e^t)^{\frac{3}{n}}}\right]. \tag{19}$$

Hence the metric (1) reduces to the form

$$ds^2 = -dt^2 + (t^k e^t)^{\frac{2}{n}}\left[dx^2 + e^{2mx}\left(c_2 \exp\left(c_1\int \frac{dt}{(t^k e^t)^{\frac{3}{n}}}\right)dy^2 + \frac{1}{c_2}\exp\left(-c_1\int \frac{dt}{(t^k e^t)^{\frac{3}{n}}}\right)dz^2\right)\right] \tag{20}$$

2.2 SOME GEOMETRICAL AND PHYSICAL PROPERTIES OF THE MODEL

The proper energy density (ρ), string tension density (λ) and bulk viscosity coefficient (ξ) are respectively given by

$$\rho = \frac{3}{n^2}\left(\frac{k}{t}+1\right)^2 - \frac{c_1^2}{4\,(t^k e^t)^{\frac{6}{n}}} - \frac{3m^2}{(t^k e^t)^{\frac{2}{n}}}, \tag{21}$$

$$\lambda = 0, \tag{22}$$

$$\xi = \frac{1}{n}\left(\frac{k}{t}+1\right) + \frac{n}{3\left(\frac{k}{t}+1\right)}\left[\frac{c_1^2}{4(t^k e^t)^{\frac{6}{n}}} - \frac{m^2}{(t^k e^t)^{\frac{2}{n}}} - \frac{2k}{nt^2}\right]. \tag{23}$$

The physical parameters such as spatial volume (V), Hubble parameter (H), expansion scalar (θ), shear scalar (σ) and anisotropy parameters (A_m) are given by

$$V = (t^k e^t)^{\frac{3}{n}}, \tag{24}$$

$$H = \frac{1}{n}\left(\frac{k}{t}+1\right), \tag{25}$$

$$\theta = \frac{3}{n}\left(\frac{k}{t}+1\right), \tag{26}$$

$$\sigma^2 = \frac{c_1^2}{4\left(t^k e^t\right)^{\frac{6}{n}}}, \tag{27}$$

$$A_m = \frac{1}{6}\frac{c_1^2 n^2}{\left(\frac{k}{t}+1\right)^2\left(t^k e^t\right)^{\frac{6}{n}}}. \tag{28}$$

From the above results, it can be seen that the spatial volume is zero at t=0. This shows that the universe starts evolving with zero volume at t=0 and expands with cosmic time t. We also observed that at initial epoch the values of proper energy density (ρ), coefficient of bulk viscosity (ξ), Hubble factor (H), expansion parameter (A_m) are very high and these values gradually decrease with the evolution of time; that is ρ, ξ, H, θ, σ, and A_m tend to zero as $t\to\infty$. Thus our model starts with big-bang at t = 0 and the expansion in the model decreases as time increases. Since $\lim_{t\to\infty}\frac{\sigma}{\theta} = 0$.

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Hence the model isotropizes for large values of t. The model (20) has real physical singularly at t=0. Since in the presence of bulk viscosity, the string tension density λ=0. Thus the string cosmological model with bulk viscous fluid for Binachi type-V metric dose not exist.

3. STRING COSMOLOGICAL MODEL WITHOUT VISCOUS FLUID

In this section we obtain a string cosmological model without bulk viscous fluid. The field equation (6) with Equation (2) (without bulk viscous fluid), lead to the following system of equations:

$$\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{B}\dot{C}}{BC}-\frac{m^2}{A^2}=\lambda\,, \tag{29}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{C}}{AC}-\frac{m^2}{A^2}=0\,, \tag{30}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}-\frac{m^2}{A^2}=0\,, \tag{31}$$

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{A}\dot{C}}{AC}+\frac{\dot{B}\dot{C}}{BC}-\frac{3m^2}{A^2}=\rho\,, \tag{32}$$

$$\frac{2\dot{A}}{A}-\frac{\dot{B}}{B}-\frac{\dot{C}}{C}=0\,. \tag{33}$$

3.1. SOLUTION OF THE FIELD EQUATIONS

From Equations (30) and (31), we obtain

$$\left(\frac{\ddot{B}C-B\ddot{C}}{\dot{B}C-B\dot{C}}\right)+\frac{\dot{A}}{A}=0\,, \tag{34}$$

Using Equation (33) in Equation (34), we get

$$\left(\frac{\ddot{B}C-B\ddot{C}}{\dot{B}C-B\dot{C}}\right)+\frac{1}{2}\left(\frac{B\dot{C}+\dot{B}C}{BC}\right)=0. \tag{35}$$

Equation (35), after integration, reduces to

$$\left(\dot{B}C-B\dot{C}\right)=\frac{L}{\sqrt{BC}}\,, \tag{36}$$

where L is constant of integration.

Using Eqution (33) in Equation (30), we get

$$\frac{2\ddot{B}}{B}+\frac{6\ddot{C}}{C}-\frac{\dot{B}^2}{B^2}+\frac{\dot{C}^2}{C^2}+\frac{4\dot{B}\dot{C}}{BC}-\frac{4m^2}{BC}=0\,. \tag{37}$$

Let $BC=\alpha$ and $\frac{B}{C}=\beta$. (38)

Using these in Equations (36) and (37), we obtain

$$\frac{\dot{\beta}}{\beta}=\frac{L}{\alpha^{3/2}} \tag{39}$$

and
$$\frac{4\ddot{\alpha}}{\alpha}-\frac{2\ddot{\beta}}{\beta}-\frac{\dot{\alpha}^2}{\alpha^2}+\frac{3\dot{\beta}^2}{\beta^2}-\frac{3\dot{\alpha}\dot{\beta}}{\alpha\beta}-\frac{4m^2}{\alpha}=0\,. \tag{40}$$

Using Equation (39) in Equation (40), we get

$$4\alpha\ddot{\alpha}+\frac{L^2}{\alpha}-\dot{\alpha}^2-4\alpha m^2=0\,,$$

which implies that

$$\ddot{\alpha}-\frac{\dot{\alpha}^2}{4\alpha}=m^2-\frac{L^2}{4\alpha^2}\,. \tag{41}$$

Let $\dot{\alpha} = f(\alpha)$ which implies that $\ddot{\alpha} = ff'$, where $f' = \frac{df}{d\alpha}$. Hence Equation (41) takes the from

$$\frac{d}{d\alpha}\left(f^2\right) - \frac{1}{2\alpha}\left(f^2\right) = 2m^2 - \frac{L}{2\alpha^2}, \tag{42}$$

which after integration, reduces to

$$f^2 = 4m^2\alpha + \frac{L^2}{3\alpha} + M\sqrt{\alpha}, \tag{43}$$

where M is an integrating constant.

Eqauctions (39) and (43) lead to

$$\log\beta = \int \frac{L\alpha^{-3/2}d\alpha}{\sqrt{4m^2\alpha + \frac{L^2}{3\alpha} + M\sqrt{\alpha}}}. \tag{44}$$

Thus the metric (1) reduces to

$$ds^2 = -\frac{dT^2}{\left[4m^2T + \frac{L^2}{3T} + M\sqrt{T}\right]} + TdX^2 + \beta Te^{2mX}dY^2 + \frac{T}{\beta}e^{2mX}dZ^2, \tag{45}$$

where x =X, y=Y, z=Z, $\alpha = \text{T}$ and β can be determined by Equation (44).

3.2. SOME GEOMETRICAL AND PHYSICAL PROPERTIES OF THE MODEL

The energy density (ρ), the string tension (λ), the scalar of expansion (θ), the magnitude of shear (σ^2), proper volume (V) and Hubble's parameter (H) for the model (45) are given by

$$\rho = \frac{3}{4}\frac{M}{T^{3/2}} \tag{46}$$

$$\lambda = 0 \tag{47}$$

$$V = T^{3/2} \tag{48}$$

$$\sigma^2 = \frac{L^2}{4T^3} \tag{49}$$

$$\theta = \frac{3}{2}\sqrt{\frac{4m^2}{T} + \frac{L^2}{3T^3} + \frac{M}{T^{3/2}}} \tag{50}$$

$$\text{H} = \frac{1}{2}\sqrt{\frac{4m^2}{T} + \frac{L^2}{3T^3} + \frac{M}{T^{3/2}}} \tag{51}$$

From Equations (48) and (50), we observe that the spatial volume is zero at T= 0 and the expansion scalar is infinite, which shows that the universe starts evolving with zero volume with infinite rate of expansion which is big -bang scenario. The physical quantities such as proper energy density (ρ), shear scalar (σ) and Hubble factor (H) diverge at T=0. As T$\rightarrow\infty$, volume becomes infinite where as ρ, θ, σ, H approach to zero. Since $\lim_{T\to\infty}\frac{\sigma}{\theta} = 0$. Hence the model approachs isotropy for large values of T. Since in the absence of bulk viscosity the string tension density λ=0. Hence the string cosmological model in the absence of bulk viscosity for Bianchi type-V metric does not exist.

4. CONCLUTION

We have presented two categories of Bianchi type-V cosmological solutions of the field equations, one with bulk viscous fluid and other without bulk viscous fluid. In both the categories of the cosmological models, we have shown that cosmic strings do not exist in Bianchi type-V cosmology. We have also discussed some physical and geometrical properties of the cosmological models.

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