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EFFECT OF THERMOPHORESIS ON UNSTEADY MHD CONVECTIVE HEAT AND MASS TRANSFER FLOW OF A VISCOUS ROTATING FLUID PAST A STRETCHING SURFACE WITH NON-LINEAR THERMAL RADIATION, THERMO-DIFFUSION, IN PRESENCE OF HEAT SOURCE

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ABSRACT

We analyze the combined influence of thermo-diffusion and radiation absorption effects on unsteady heat and mass transfer flow of a viscous incompressible electrically conducting fluid over a stretching sheet with thermal radiation, non-uniform heat source/sink and thermophoresis particle deposition. The conservation of mass, momentum, energy and diffusion equations were transformed into a two-point boundary value problem. We employ an extensively validation, highly efficient, variational finite-element method to study the effect of unsteadiness on heat and mass transfer flow past a semi-infinite stretching sheet. The velocity, temperature, mass concentration, Nusselt and Sherwood number have numerically evaluated for different variations.

Key words: Thermophoresis Heat and mass transfer, Rotation, Stretching Surface, Non-linear thermal radiation, Soret effect, heat source.

1. INTRODUCTION

Thermophoresis is a phenomenon by which submicron sized particles suspended in a nonisothermal gas acquire a velocity relative to the gas in the direction of decreasing temperature. The velocity acquired by the particle is known as the thermophoretic velocity and the force experienced by the suspended particles due to the temperature gradient is called the thermophoretic force. The magnitudes of the thermoporetic force and velocity are proportional to the temperature gradient. Thermophoresis has many applications in radioactive particle deposition in nuclear reactors, deposition of silicon thin films, particles impacting the blade surface of gas turbines and aerosol technology. Many authors have done good work by taking thermophoresis in the account. Recently several authors Goren [14a], Chamkha and Pop [5a], Seddeek [37a], Partha [52] was first to analyze the thermophoresis in laminar flow over a flat plate for cold and hot plate conditions.

A number of analytical and experimental papers in thermophoretic heat and mass transfer have been communicated. Several authors, Talbot *et al.* [43], Duwairi and Damseh *et al.* [12], Damseh *et al.* [8], Mahdy and Hady [23], Liu *et al.* [22], Postelnicu [32], Dinesh and Jayaraj [9], Grosan *et al.* [15], Tsai and Huang [44] have investigated the effect of thermophoresis in vertical plate, micro-channel, horizontal plate and parallel plate.

The study of flow in a rotating frame is motivated in view of its theoretical and practical significance on significance in geophysical and engineering. Prominent geophysical applications include the magma flow in earth's mantle close to earth crust and flows in geophysical formations subject to earth rotation. The engineering applications of such flows exist in chemical and food processing industry, centrifugal filtration process, rotating machinery and design of multipore distributor in a gas-solid fluidized bed, Pioneering study on the three-dimensional rotating viscous flow induced by a stretching surface was presented by Wang [49]. His problem was governed by an interesting parameter λ that signifies the ratio of the rotation to the stretching rate. He constructed series solutions for small values of parameter λ by regular perturbation approach. He found that velocity distribution (above the sheet) decreases upon increasing this parameter λ .

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Rajeswari and Nath [33] and Nazar et al. [28], Hayat et al. [18], Zaimi et al. [51], Rashidi et al. [35], Sheikholeslami et al. [38], Mustafa [27] were extended the Wang's work for unsteady case. Their results indicate a smooth transition from initial unsteady flow to final steady-state flow and reported numerical results of nanofluid flow and heat transfer in a rotating system with the consideration of magnetic field effects. Also used Cattaneo-Christov heat flux model to investigate the rotating flow of visco-elastic fluid bounded by a stretching surface

The problem of two dimensional boundary layer flow, heat and mass transfer over a continuous stretching heated surface through porous medium finds numerous and wide range of applications in many engineering and manufacturing disciplines.

In industry, polymer sheets and filaments are manufactured by continuous extrusion f the polymer from a die. The thin polymer sheet constitutes of continuously moving surface with a non-uniform velocity through an ambient fluid. The problem of heat and mass transfer flow due to stretching sheet has been implemented on many flow situations. The problem of steady two dimensional viscous incompressible fluid caused by a stretching sheet was first examined by (Sikiadis [39], Tsou et al. [46]. Crane [7], Sikiadis [39]. Gupta et al. [17], Grubka and Bobba [16], Ali [2], Vajravelu [47]). Two cased have been discussed in this problem, (i) the sheet with prescribed sheet temperature (PST-case) and (ii) the sheet with prescribed wall flux (PHF-case)

Plumb et al. [31] was the first to examine the effect of horizontal cross flow and radiation on natural convection from vertical heated surface in a saturated porous media. Recently, the problem of free convection heat transfer characteristics in an electrically conducting fluid near an isothermal sheet to study the combined effect of buoyancy and radiation in the presence of uniform transverse magnetic field. Pal D et al. [29], Mansour and El-Shaer [25], Pal [30], Vajravelu and Rollins [48], Molla et al. [26], Abo-Eldahab and El-Gendy [1] has discussed radiation effect on hydro magnetic Darcy Forchheimer mixed convection flow over stretching sheet.

Dulal Pal et al. [10] has studied MHD non-Darcian mixed convection heat and mass transfer over a non-linear stretching sheet with Soret and Dufour effects and chemical reaction. MHD mixed convection flow with Soret and Dufour effects past a vertical plate embedded in porous medium was studied by Makinde [24], Reddy et al. [36] has presented finite element solution to the heat and mass transfer flow past a cylindrical annulus with Soret and Dufour effects. Chamkha et al. [6] has studied the influence of Soret and Dufour effects on unsteady heat and mass transfer flow over a rotating vertical cone and they suggested has temperature and concentration fields are more influenced with the values of Soret and Dufour parameter.

In all the above studies the physical situation is related to the process of uniform stretching sheet. For the development of more physically realistic characterization of the flow configuration it is very useful to introduce unsteadiness into the flow, heat and mass transfer problems. The working fluid heat generation or absorption effects are very crucial in monitoring the heat transfer in the regions, heat removal from nuclear fuel debris, underground disposal of radiative waste material, storage of food stuffs, exothermic chemical reactions and dissociating fluids in packed-bed reactors. This heat source can occurs in the form of a coil or battery. Very few studies have been found in literature on unsteady boundary flows over a stretching sheet by taking heat generation/absorption into the account. Wang CY [50] was first studied the unsteady boundary layer flow of a liquid film over a stretching surface. Later, Elbashbeshy and Bazid [13], Tsai et al. [45], Ishak et al. [19], Ishak [20], Dulal Pal [11], Dulal Pal et al. [11] has presented the heat transfer over an unsteady stretching surface.

Sreenivasa Reddy [41, 42, 42a], Aliveni et al [3] has discussed Soret and Dufour effect on convective heat and mass transfer flow of micro polar fluid in the presence of thermoporesis and effect of thermophoresis and Hall effects on unsteady convective heat and mass transfer flow of a viscous rotating fluid past a stretching surface with thermal radiation, thermo-diffusion, radiation absorption in the presence of non-uniform heat source.

In this paper we analyze the combined influence of thermo-diffusion and radiation absorption effects on unsteady heat and mass transfer flow of a viscous incompressible electrically conducting fluid over a stretching sheet with thermal radiation, non-uniform heat source/sink and thermophoresis particle deposition. Using Galerkin Finite Element equation analysis the governing equations have been solved.

2. FORMULATION OF THE PROBLEM

We analyse the transient convective heat and mass transfer flow of an electrically conducting fluid past a stretching sheet with the plane at y=0 and the flow is confined to the region y>0.A schematic representation of the physical model is exhibited in fig.1.We choose the frame of reference





O(x,y,z) such that the x-axis is along the direction of motion of the surface, the y-axis is normal to the surface and z-axis © 2019, IJMA. All Rights Reserved 59

transverse to the (x-y) plane. An uniform magnetic field of strength H0 is applied in the positive y-direction. The surface of the sheet is assumed to have a variable temperature and concentration $T_w(x)$, and $C_w(x)$ respectively, while the ambient fluid has a uniform temperature and concentration T_{∞} and C_{∞} , where $T_{w}(x) > T_{\infty}$, $C_{w}(x) > C_{\infty}$ corresponds to a heated plate and $T_w(x) < T_{\infty}$, $C_w(x) < C_{\infty}$ corresponds to a cooling plate. The flow is assumed to be confined in the region y>0.We consider a non-uniform internal heat generation/absorption source in the flow to get the temperature and concentration differences between the surface and ambient fluid. We that the velocity is proportional to its distance from the slit. We consider Hall effects into consideration and assume the electron pressure gradient, the ion-slip and the thermo-electric effects are neglegible. Using boundary layer approximation, Boussinesq's approximation the basic equations governing the flow, heat and mass transfer are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.1}$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -v \frac{\partial^2 u}{\partial y^2} + \beta_T g(T - T_\infty) + \beta_C g(C - C_\infty) + 2\Omega w - \frac{\sigma B_o^2}{\rho} u$$
(2.2)

$$\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} = v \frac{\partial^2 w}{\partial y^2} - 2\Omega u$$
(2.3)

$$\rho C_{p} \left(\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k_{f} \frac{\partial^{2} T}{\partial y^{2}} - Q_{H} (T - T_{\infty}) + \mu \left[\left(\frac{\partial u}{\partial y} \right)^{2} + \left(\frac{\partial w'}{\partial y} \right)^{2} \right] +$$

$$(2.4)$$

$$+\sigma B_o^2[(u^2+w^2] - \frac{\partial(q_R)}{\partial y}]$$

$$\partial C = \partial C = \sigma^2 C \qquad for all a constant and be constant and be constant and be a$$

$$\left(\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y}\right) = D_B \frac{\partial^2 C}{\partial y^2} - k_c (C - C_\infty) + \frac{D_m K_T}{T_m} \frac{\partial^2 T}{\partial y^2} - \frac{\partial (V_T C)}{\partial y}$$
(2.5)

The relevant boundary conditions are

$$u = U_w(x,t) + L \frac{\partial u}{\partial y}, v = V_w, T = T_w, C = C_w \qquad at \quad y = 0$$

$$u \to 0, T \to T_{\infty}, C \to C_{\infty} \qquad as \quad y \to \infty$$
(2.6)

Where T is the temperature, C is the concentration inside the boundary layer, u, v and w sre the velocity components along x,y and z-directions respectrively, Cp is the specific heat at constant pressure, Cs is the concentration susceptibility, ρ is the density of the fluid, kf is the thermal conductivity, μ is the fluid viscosity, ν is the kinematic viscosity, $T_w(x,t)$ is the stretching surface temperature, $C_w(x,t)$ is the concentration of the stretching surface, T_{∞} is the temperature far away from the stretching surface with Tw> T_{∞} , C_{∞} is the concentration far away from the stretching surface with Cw>C_∞. The term $V_W = -\sqrt{\frac{\nu U_W}{2x}}f(0)$ represents the mass transfer at the surface with V_W > 0 for

suction and $V_W < 0$ for injection.

The coefficient q''' is the rate of internal heat generation (>0) or absorption (<0). The internal heat generation /absorption q''' is modeled as

$$q''' = \frac{k_f U_w(x,t)}{x V} (A_1 (T_w - T_\infty) f' + (T - T_\infty) B_1)$$
(2.7)

where A1 and B1 are coefficients of space dependent and temperature dependent internal heat generation or absorption respectively. It is noted that the case A1>0 and B1>0, corresponds to internal heat generation and that A1<0 and B1<0, the case corresponds to internal heat absorption case.

Due to stretching of the sheet the flow is caused and it moves with the surface velocity, temperature and concentration of the form

$$U_{w}(x,t) = \frac{dx}{1-ct}, T_{w}(x,t) = T_{\infty} + \frac{dx}{1-ct}, C_{w}(x,t) = C_{\infty} + \frac{dx}{1-ct}$$
(2.8)

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where a is stretching rate and c are positive with ct < 1, $c \ge 0$. it is noticed that the stretching rate $\frac{ax}{1-ct}$ increases with time t since $a \ge 0$.

time t since a > 0.

The stream function $\psi(x, t)$ is defined as:

$$u = \frac{\partial \psi}{\partial y} = \frac{ax}{(1 - ct)} f'(\eta), v = -\frac{\partial \psi}{\partial x} = \frac{av}{\overline{j(1 - ct)}} f(\eta)$$
(2.9)

On introducing the similarity variables (Dulal Pal [10]):

$$\eta = \sqrt{\frac{a}{(1-ct)}} y,$$

$$u = \frac{ax}{1-ct} f'(\eta), v = -\sqrt{\frac{va}{1-ct}} f(\eta), w = \frac{ax}{1-ct} g(\eta),$$

$$\theta(\eta) = \frac{T-T_{\infty}}{T_{w}-T_{\infty}}, \varphi = \frac{C-C_{\infty}}{C_{w}-C_{\infty}}$$

$$B^{2} = B_{\rho}^{2} (1-ct)^{-1}$$
(2.10)

By using Rosseland approximation for radiation, the radiative heat flux qr is defined as

$$q_{R} = -\frac{4\sigma^{\bullet}}{3\beta_{R}}\frac{\partial T'^{4}}{\partial y}$$
(2.11)

Where σ^{\bullet} is the Stefan-Boltzman constant, β_R is mean absorption coefficient. We assume that the temperature difference within the flow are such that the term T'^4 may be expressed as a free stream temperature T_{∞} as follows

$$T'^{4} = T_{\infty}^{4} + 4T_{\infty}^{3}(T - T_{\infty}) + 6T_{\infty}^{2}(T - T_{\infty})^{2} + \dots$$
(2.12)

Neglecting higher –order terms in the equation (2.12) beyond the first degree in $(T - T_{\infty})$.we get

$$T'^{4} \cong 4T_{\infty}^{3}T - 3T_{\infty}^{4}..$$
(2.13)

Using (2.13) equation (2.11) reduces to

$$q_{R} = -\frac{16\sigma^{\bullet}T_{\infty}^{3}}{3\beta_{R}}\frac{\partial T}{\partial y}$$
(2.14)

The non-dimensional temperature $\theta(\eta) = \frac{T - T_{\infty}}{T_{w} - T_{\infty}}$ can be simplified as

$$T = T_{\infty}(1 + (\theta_w - 1)\theta)$$

where $\theta = \frac{T_w}{T_{\infty}}$ is the temperature parameter.

and equation (2.4) becomes

$$\rho C_{p} \left(\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k_{f} \frac{\partial^{2} T}{\partial y^{2}} + \frac{k_{f} U_{w}(x,t)}{xv} (A_{1}(T_{w} - T_{\omega})f' + (T - T_{\omega})B_{1}) + \mu \left[\left(\frac{\partial u}{\partial y} \right)^{2} + \left(\frac{\partial w'}{\partial y} \right)^{2} \right] + \sigma B_{o}^{2} [u^{2} + w^{2}] + Q_{1}^{'}(C - C_{\omega}) + \frac{16\sigma^{\bullet} T_{\omega}^{3}}{3\beta_{R}} \frac{\partial^{2} T}{\partial y^{2}}$$

$$(2.15)$$

where T is the temperature and C is the concentration in the fluid. k_f is the thermal conductivity, Cp is the specific heat at constant pressure, β is the coefficient of thermal expansion, β^{\bullet} is the volumetric expansion with concentration, Q_1^1 is the radiation absorption coefficient, q_r is the radiative heat flux, kc is the chemical reaction coefficient, D_B is the molecular viscosity, D_m , K_T , T_m mean fluid temperature, k is the porous permeability parameter.

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The effect of thermophoresis is usually prescribed by means of average velocity acquired by small particles to the gas velocity when ex posed to a temperature gradient. Inboundary layer flow, the temperature gradient in the y-direction is very much larger than in the x-direction and therefore only the thermophoresis velocity in y-direction is considered. As a consequence, the thermophoresis velocity V_T , which appears in equation (4) is expressed as

$$V_T = -\frac{k_1 v}{T_r} \frac{\partial T}{\partial y}$$
(2.16)

In which k_1 is the thermophoresis coefficient and Tr is the reference temperature. A thermophoresis parameter τ is given by the relation

$$\tau = -\frac{k_1 (T_w - T_\infty)}{T_r}$$
(2.17)

Where the typical values of τ are 0.01,0.1 and 1.0 corresponding to approximate values of $k_1(T_w - T_\infty)$ equal to 3,30,300K for a reference temperature of T=300K

Using Equations (2.9), (2.10) & (2.17) into equations (2.2), (2.3), (2.5) and (2.15) we get

$$f''' + ff'' - (f')^{2} - S(f' - 0.5\eta f'') - M^{2}f' + G(\theta + N\phi) + 2Rg = 0$$
(2.18)

$$g'' + fg' - fg - S(g' + 0.5g'') - 2Rf' = 0$$
(2.19)

$$\left(1 + \frac{4Rd}{3}\right)\theta'' + \Pr(f\theta' - f'\theta) - \Pr S(\theta + 0.5\eta\theta') + (A_1f' + B_1\theta)$$
(2.20)

+ Pr
$$Ec((f'')^2 + (g')^2) + EcM^2((f')^2 + g^2) + Q1\varphi = 0$$

 $\phi'' + -Sc(2f'\phi - f\phi') - ScS(\phi + 0.5\eta\phi') + ScSr\theta'' - \tau(\theta'\phi' + \theta''\phi) - Sc\gamma\phi = 0$
(2.21)

 $\phi'' + -Sc(2f'\phi - f\phi') - Sc S(\phi + 0.5\eta\phi') + ScSr\theta'' - \tau(\theta'\phi + \theta'\phi) - Sc$ Where S=c/a is the unsteadiness parameter. $M = \frac{\sigma B_0^2}{\rho a}$ is the magnetic parameter,

$$D^{-1} = \frac{v}{ak}$$
 is the inverse Darcy parameter, $G = \frac{\beta g(T_w - T_{\infty})}{U_w v_w^2}$ is the thermal buoyancy parameter,

$$N = \frac{\beta^* (C_w - C_\infty)}{\beta (T_w - T_\infty)}$$
 is the buoyancy ratio, $\Pr = \frac{\mu C_p}{k_f}$ is the Prandtl number, $Ec = \frac{U_w^2}{C_p (T_w - T_\infty)}$ is the Eckert

number, $Q_1 = \frac{vQ_1}{v_w^2}$ is the Radiatiuon absorption parameter, $Sc = \frac{v}{D_B}$ is the Schmidt number, $m = \omega_e \tau_e$ is the Hall parameter, $\gamma = \frac{k_o v}{v_w^2}$ is the chemical reaction parameter and $Sr = \frac{D_m K_T (T_w - T_\infty)}{vT_m (C_w - C_\infty)}$ is the Soret parameter.

It is pertinent to mention that $\gamma > 0$ corresponds to a degenerating chemical reaction while $\gamma < 0$ indicates a generation chemical reaction.

The transformed boundary conditions (2.6) & (2.10) reduce to

$$f'(0) = 1 + Af''(0), f(0) = fw, \theta(0) = 1, \phi(0) = 1$$

$$f'(\infty) \to 0, g(\infty) \to 0, \theta(\infty) \to 0, \phi(\infty) \to 0$$

(2.22)

Where $fw = \frac{v_w}{\sqrt{av}}$ is the mass transfer coefficient such that fw>0 represents suction and fw<0 represents injection at

the surface.

3. METHOD OF SOLUTION

The equations (2.18 to 2.21) have been solved by employing finite element technique with three nodded approximation functions. The Local Stiffness Matrices have been assembled by using inter element continuity, equilibrium and boundary conditions. The resulting global matrices have been solved by using iteration procedure. The process in continued until the convergence is reached.

4. IN FRICTION, NUSSELT NUMBER and SHERWOOD NUMBER

The physical quantities of engineering interest in this problem are the skin friction coefficient Cf, the Local Nusselt number (Nu_x) , the Local Sherwood number (Sh_x) which are expressed as

$$\frac{1}{2}C_f \overline{R_{ex}} = f''(0), \frac{1}{2}C_{fz} \overline{R_{ez}} = g'(0), \quad Nux / \overline{R_{ex}} = 1/\theta(0), \quad Shx / \overline{R_{ex}} = 1/\phi(0)$$

where $\mu = \frac{k}{\rho C_p}$ is the dynamic viscosity of the fluid and Rex is the Reynolds number.

For the computational purpose and without loss of generality ∞ has been fixed as 8. The whole domain is divided into 11 line elements of equal width, each element being three nodded.

5. RESULTSAND DISCUSSION

Comprehensive numerical commutations are conducted for different values of the parameters that describe the flow characteristics and the results are illustrated graphically and in tabular form. Selected graphical profiles are presented in figs.2-11.

Figs.2a-2d show the variation of velocity, temperature and concentration with buoyancy ratio (N). It can be seen from the velocity profiles that when the molecular buoyancy force dominates over the thermal buoyancy force the velocity components enhance when the buoyancy forces are in the same direction and for the forces acting in opposite directions, they depreciate in the boundary layer.(figs.2a &2b).From figs.2c and 2d we find that the temperature reduces and mass concentration enhances with N > 0 when the buoyancy forces are in the same direction and a reversed effect is noticed for the buoyancy forces acting in opposite directions.

Figs.3a-3d represent the variation of velocity, temperature and concentration with rotation parameter (R). It can be seen from the velocity profiles that the primary velocity (f') reduces with higher values of rotation parameter (R). while the secondary velocity (g) reduces in the flow region. An increase in R enhances the temperature and reduces the mass concentration .This is attributed to the fact that the thickness of the thermal boundary layer increases while the solutal boundary layer thickness decreases with increase in m.

Figs.4a-4d display the influence of chemical reaction parameter (γ) on the velocity, temperature and concentration. It is observed that increasing the chemical reaction parameter reduces the primary velocity and enhances the secondary velocity in the degenerating chemical reaction case (γ >0) a reversed effect is noticed in the generating chemical reaction case (γ >0). Also an increase in γ > 0, increases thickness of thermal and solutal boundary layer and decreases with γ < 0.

The variation of Soret parameter (Sr) and Dufour parameter (Du) on velocities, temperature and concentration are plotted in figs.5a-5d.It is seen from the profiles that the primary increases and secondary velocity reduces with increasing Soret parameter(or decreasing Dufour parameter Du). Higher the thermo-diffusion effects (or lesser the diffusion-thermo effects) larger the temperature and mass concentration (figs.5c & 5d). This may be attributed to the fact the thickness of the thermal and solutal boundary layers increase with increasing Sr.

The influence of unsteadiness parameter (S) on the velocity components, temperature and concentration profiles is shown in figs.6a-6d.It can be seen that the velocity components, temperature decelerates with increase in the values of unsteadiness parameter (S). This is because of the fact that, the motion is generated by the stretching of the sheet and the stretching sheet velocity and temperature is greater than the free stream velocity and temperature, so, the thermal boundary layer thickness decreases with increase in the values of S as shown in fig.6c.The concentration profiles also decreases in the flow region and is shown in fig.6d.It is also observed that the temperature profiles decreases smoothly in the absence of unsteadiness parameter (S=0) whereas temperature profiles continuously decreases with the increasing values of unsteadiness parameter. This shows that the rate of cooling is much faster for the higher values of unsteadiness parameter and it takes longer time for cooling in the steady flows.

The variation of thermal radiation parameter (Rd) on the velocity, temperature and concentration is depicted in figs.7a-7d. It is observed that there is a significant rise in the primary and secondary velocities in the presence of thermal radiation throughout the boundary layer. The radiation parameter is found to increase the hydrodynamic boundary layer along the x and y-directions. The presence of thermal radiation is very significant on the variation of temperature. It is seen that the temperature increases rapidly in the presence of thermal radiation parameter throughout the thermal boundary layer. This may be attributed to the fact that as the Rosseland radiative absorption parameter R^* diminishes the corresponding heat flux diverges and thus rising the rate of radiative heat transfer to the fluid causing a rise in the temperature of the fluid. Also an increase in Rd enhances the concentration profiles.

Fig.8a-8d demonstrate the influence of Eckert number on velocity, temperature and concentration .It is pointed that the presence of Eckert number increases the temperature. This is because of the fact that thermal energy is reserved in the fluid on account of friction heating. Hence, the temperature distribution rises in the entire boundary layer. The primary and secondary velocity components rise with increasing values of Ec owing to the energy release which increases the momentum boundary layer thickness. However, the mass concentration diminishes marginally with increase in Ec.

The effect of thermophoresis parameter (τ) on velocity components, temperature and concentration profiles is exhibited in figs.9a-9d. It can be seen from figs.9a & 9b that the velocity components depreciate with increase in τ . This is due to the fact that an increase in τ decreases the thickness of the momentum boundary layer. From fig.9c we find that the temperature profiles experience a, enhancement with increase in τ . This is because of the fact that the particles near the hot surface create a thermophoretic force. Fig.9d exhibits the impact of τ on concentration profiles, it is noticed that the concentration profiles decreases with increase in thermophoretic parameter (τ). This is because of the fact that the fluid moves from hot surface to the cold surface, then the values of thermophoretic parameter have been taken positive. From these two figures we conclude that the imposition of thermophoretic particles deposition into the flow increases the thickness of thermal layer and decreases the solutal boundary layer.

Figs.10a-10d demonstrate the influence of slip parameter (A11) on the velocity, temperature and concentration .It can be seen from the profiles that an increase in the slip parameter (A11) reduces the primary velocity and mass concentration. The secondary velocity reduces in the region(1.0,3.0) and increases in the remaining flow region. The temperature rises with increasing values of A11.

Figs.11a-11d represent the effect of temperature parameter (A) ($=\theta$ w) on f, g, θ and C. It can be seen from the profiles that an increase in the temperature parameter increases the velocities, temperature and reduces the mass concentration. This may be attributed to the fact that the thickness of the momentum boundary layer and thermal boundary layer grows and that of the solutal boundary layer decreases with increase in temperature parameter (A).

The skin friction components(τx), (τy), Nusselt number (Nu)and Sherwood number (Sh) for different N, Sr/Du, Rd, γ , Ec,, τ , A11, A and S. From the tabular values we find that a increase in G reduces the skin friction component τx and Sherwood number while τy and Nusselt number increase on the wall. An increase in rotation parameter (R) enhances the skin friction component τx , τy , Sh and reduces Nusselt number on the wall. When the molecular buoyancy force dominates over the thermal buoyancy force the skin friction component τx and Sh reduces while τy and Nu increases when the buoyancy forces are in the same direction and for the forces acting in opposite directions, τx , Sh increase, while τy , Nu decrease on the wall. With respect to chemical reaction parameter (γ), we find that τx enhance while τy reduces on the wall in both degenerating and generating chemical reaction cases. Nu and Sh enhance on the wall in the degenerating case, Nu reduces, Sh enhances on the wall. Higher the thermophoretic parameter (τ) larger τx , Sh and smaller τy , Nu on the wall. An increase in the slip parameter (A11) reduces the skin friction components and Nusselt number and enhances the Sherwood number on the wall. An increase in temperature parameter (A) reduces τx , Nu and enhances τy , Sh on the wall. The skin friction component τx , rate of heat and mass transfer on the wall experience an enhancement with increasing values of unsteadiness parameter S while τy reduces with S.



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 $\begin{array}{l} \mbox{Fig.161 Effect of A on Concentration } \varphi \left(\eta \right) \\ \mbox{R=0.5 , N=0.5, Rd=0.5 Ec=0.01, } \gamma \mbox{=}0.5, \\ \mbox{Sr=0.3, Du=0.1, A11=1.01, } \tau \mbox{=}0.1 \end{array}$

Parameter		τx (0)	τz(0)	Nu(0)	Sh(0)
R	0.5	-0.795059	-0.203224	1.2123	0.841957
	1	-0.873329	-0.551124	1.18802	0.844952
	1.5	-0.976932	-0.801595	1.157	0.848676
	2	-1.07754	-0.984497	1.12882	0.852033
N	1	-0.795059	-0.203224	1.2123	0.841957
	2	-0.663526	-0.222362	1.23917	0.838405
	-0.5	-0.859867	-0.193522	1.19866	0.843719
	-1.5	-0.994097	-0.172821	1.16955	0.847391
γ	0.5	-0.795059	-0.203224	1.2123	0.841957
	1.5	-0.800555	-0.201797	1.21057	1.30999
	-0.5	-0.775067	-0.209675	1.21942	-0.192261
	-1.5	-0.810425	-0.194073	1.20414	0.490418
Sr/Du	0.06/0.1	-0.795059	-0.203224	1.2123	0.841957
	0.1/0.06	-0.794145	-0.203453	1.21258	0.757202
	1.5/0.04	-0.79323	-0.203681	1.21286	0.672402
	2.0/0.03	-0.792315	-0.203909	1.21314	0.587555
Rd	0.5	-0.795059	-0.203224	1.2123	0.841957
	1.5	-0.768455	-0.209676	0.972484	0.848625
	3.5	-0.744641	-0.215665	0.784511	0.853053
	5	-0.733421	-0.218544	0.703013	0.854709
τ	0.2	-0.795059	-0.203224	1.2123	0.841957
	0.4	-0.796612	-0.202877	1.21185	1.03205
	0.6	-0.79796	-0.202581	1.21147	1.21034
	0.8	-0.799247	-0.202305	1.21111	1.39355
Ec	0.01	-0.795059	-0.203224	1.2123	0.841957

Parameter		$\tau x(0)$	τz(0)	Nu(0)	Sh(0)
	0.03	-0.794619	-0.203322	1.20727	0.842123
	0.05	-0.79419	-0.203417	1.20238	0.842285
	0.07	-0.793878	-0.203487	1.19882	0.842402
S	0.2	-0.589246	-0.274447	1.02106	0.689101
	0.4	-0.645832	-0.253644	1.07191	0.730444
	0.6	-0.699278	-0.234653	1.12099	0.769863
	0.8	-0.795059	-0.203224	1.2123	0.841957
A11	1.01	-0.795059	-0.203224	1.2123	0.841957
	1.5	-0.61481	-0.187679	1.18377	0.845831
	2	-0.511055	-0.178279	1.16655	0.84816
	2.5	-0.437791	-0.171423	1.15399	0.849851
A1	0.2	-0.795059	-0.203224	1.2123	0.841957
	0.4	-0.788748	-0.204395	1.10223	0.846155
	0.6	-0.77287	-0.207522	0.89016	0.853848
	0.8	-0.753558	-0.211734	0.719172	0.859104

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6. CONCLUSIONS

The non-linear equations governing the flow heat and mass transfer have been solved by using Runge-Kutta Shooting technique .From the graphical representations and tabular values we find that

- An increase in Rotation parameter (R) reduces the primary velocity and concentration while the secondary velocity, temperature enhance in the flow region. The rate of heat and mass transfer experience an enhancement with increase in R.
- Higher the thermo-diffusion effect larger the velocities, and concentration and smaller the temperature in the flow region. The rate of heat and mass transfer reduces with So.
- Higher the dissipation larger the velocities, temperature and smaller the concentration. The Nusselt number enhances and the Sherwood number reduces with increase in Ec.
- Higher the radiatve heat flux larger the velocities, temperature and concentration in the flow region. The Nusselt and Sherwood number depreciates on the wall with increase in Rd.
- An increase in Thermophores is parameter (τ) reduces velocities, temperature and concentration. The Nusselt and Sherwood number on the wall enhance with increase in τ .
- An increase in slip parameter (A11) reduces the velocities, concentration and enhances the temperature. The Nusselt number reduces and the Sherwood number enhances with increase in A11.
- > An increase in the temperature parameter increases the velocities, temperature and reduces the mass concentration τx , Nu reduces and enhances τy , Sh on the wall with increase in temperature parameter (A).

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