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MIXED CONVECTIVE FLOW OF A NEWTONIAN FLUID WITH PERMEABLE WALLS BY CONSIDERING THE INFLUENCE OF ACCELERATION DUE TO GRAVITY

S. KALPANA[1] AND CH. V. RAMANAMURTHY*[2]

[1]Research Scholar, Department of Mathematics,
Krishna University, Machilipatnam – 521001, (A.P.), India.

[2]Department of Applied Mathematics,
Sri Vasavi Institute of Engineering and Technology, Pedana – 521361, (A.P.), India.

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ABSTRACT

In this paper, the situation of Mixed Convective Flow of A Non-Newtonian Fluid with permeable walls by considering the influence of acceleration due to gravity has been examined in detail. It is noticed that, as Prandtl number increases the temperature also increases. Not much of significant change is observed when the radiation parameter (R) is slightly decreased. However, a drastic change is seen when the Prandtl number changes considerably along with the radiation parameter (R). Further, it is noticed that, as the radiation parameter (R) increses the temparature in the fluid also increases. However, not much of significant change is noticed for a small change in the Prandtl number.But, there is a significant change in the profiles for larger values of Prandtl number (Pr). It is seen that, as we move far away from the lower boundary then the temperature is found to be decreasing. Further, it is observed that, as the radiation parameter (R) increases the temperature of the fluid decreases.

Key words: *Newtonian fluid, Reynolds Number, Prandtl Number, Radiation Parameter.*

INTRODUCTION

During the last several years fluid mechanics had made significant process in several areas of engineering, science and technology. An attempt has been made in this paper to explain the possibility of supporting thermal transfer in several areas of engineering, science and technology. Generally engineering systems are more complicated and experimentally confusing. It is characterized by complex systems where the fluid stream currents have a sudden change with reference to the geometry of the systems, which is not uncommon, but needs to be examined in detail.

For the last many years, extraction of geothermal energy from the deep part of the earth, oil extraction, heat removal from the nuclear debris, flow of liquids through ion exchange beds, drug permeation through human skin and glands are few such wide applications. In view of several applications in physics, chemistry and chemical technology, the problem has gained more importance, where the transfer of liquid from one container to another container is involved, the rate at which such transfer takes place at the thin film adhering to the surface of the containers needs to be taken into account. Generally in the chemical processing industry the walls of the reactor are subjected to the corrosion due to the reaction with in the vessels. Such a phenomena causes loss of production and then consuming more reaction time for the next cycle of chemical processing.

The porous medium can be considered as an ordered flow in a disordered geometry. The porous medium may be either an aggregate of large number of particles such as sand or a solid containing more capillaries such as a porous rock. When the fluid percolates through a porous material, because of the complexity of microscopic flow in the pores, the actual part of an individual fluid particle cannot be analytically analysed. However, the process can be defined in terms of equilibrium forces.

Corresponding Author: Ch. V. Ramanamurthy*[2]
[2]Department of Applied Mathematics, Sri Vasavi Institute of Engineering and Technology, Pedana – 521361 (A.P.), India.

The resistance force is characterised by Darcy's semi-empirical law set up by Darcy [1]. The simplest model for flow through a porous medium in one dimensional model derived by Darcy [1]. Heat transfer in the porous media has become most prominent due to the exploitation of geothermal energy, nuclear waste disposal, fossil fuel identification, regenerator bed, etc. Using the method of similarity solution, Murthy and Singh [2] studied the effect of lateral mass and thermal penetration in porous media. Later, Cheng and Minkowycz [3] analyzed sustainable free convection of vertical plates on porous dynamics in the form of dissipative inequality (Clausius – Duhem), and generally accepted idea of the specific Helmholtz free power balance must be minimum in the equilibrium state. Subsequently, Dupit and Frochheimer presented empirical evidence, MacDonald *et.al* [4] and others presented the balance between speed and pressure variations, breaks down for large enough flow speed (compilation of many experimental results). This was subsequently emphasized by Joseph *et.al* [5] who stressed that Frochheimer's actions were forced to work in the opposite direction to the velocity vector. In a multi-stream flow, it follows that the momentum equations are at least predictable for each speed component derived by Frochheimer's expanded Darcy equation. The effects of the existence of solid boundary and the presence of initial forces on mass transfer in Porous Media were submitted by Vafai and Tien [6]. Later, Knupp and Lage [7] analyzed the theoretical generalization to the tensor permeability case of emphatically obtained Frochheimer's extended Darcy's unidirectional flow model. Thereafter, MacDonald *et al.* [8] presented the balance between speed and pressure contradictions - a large enough flow rate (compilation of many experimental results). Further, they had discussed the combined effect of heat exchanges and liquid injection on Darcy mixed transfer. Hussain *et al* [9] studied the effects of heat dissemination and side effects on mixed synthesis problems and established the trend of heat transfer rate from a vertical plate in porous medium and researched in drain and temperature sectors. Later, Kuznetsov [10] examined the effect of transverse thermal dispersion on forced convection in porous media and found favourable conditions for heat transfer with dispersion effects. Mohahammadien and El-Amin [11] studied the dispersion and radiation effects in fluid saturated porous medium and the effects of radiation on the heat transfer rate for both Darcy and non- Darcy Medium. Cheng and Lin [12] in their observation pointed out that the rate of unsteady heat transfer can be accelerated by thermal dispersion. Without taking MHD into consideration Chamka and Quadri [13] examined the heat and mass transfer properties in mixed convective conditions. Wang *et al* [14] applied in explicit analytical techniques namely homotopy analysis to solve the Non-Darcy natural convection over a horizontal plate with surface mass flux and thermal diffusion and obtained a totally analytic and uniformly valid solution.

In all above said examinations, several researchers have employed different techniques viz, nearly approximate solution, mixed perturbation technique and even some times the traditional methods of solving the differential equations. The novelty of the present method is to re-examine the problem by employing a simple and regular perturbation method. The results obtained when compared to the earlier investigations are found to be more accurate and sometimes even matching with their results.

FORMULATION OF THE PROBLEM

We consider the laminar mixed convection flow of a Newtonian fluid through a porous medium in a vertical permeable channel, the space between the plates h being the same. It is expected that the rate of injection at a wall is equal to the suction rate at the other wall. A rectangular coordinate system (x, y) x - axis is parallel to the gravitational acceleration vector g, the y-axis is perpendicular to the x – axis. The left wall (i.e. y = 0) is maintained at constant temperature T_1 and the right wall (at y = h) is maintained at constant temperature T_2. It is always maintained that $T_1 > T_2$. The flow is assumed to be laminar, steady and is fully developed, i.e. the transverse velocity is zero. Then the equation of continuity drops down to $\frac{\partial u}{\partial x} = 0$.

The basic equations of momentum and energy governing such a flow, subject to the Boussinesq approximation, are

$$\rho V_0 \frac{du}{dy} = -\frac{dp}{dx} + \mu \frac{d^2u}{dy^2} + \alpha_1 V_0 \frac{d^3u}{dy^3} - \frac{\mu}{k_0} u + \rho g \beta (T - T_0) \tag{1}$$

$$V_0 \frac{dT}{dy} = \alpha \frac{d^2T}{dy^2} \tag{2}$$

Where p is the pressure, ρ is the density, μ is the dynamic viscosity of the fluid, g is the acceleration due to gravity, β coefficient of thermal expansion, , α_1 is the viscoelastic parameter, k_0 is the permeability of the porous medium and V_0 is the transpiration cross flow velocity. Further, here $\frac{dp}{dx}$ is a constant.

The boundary conditions are given by

$$u(0) = u(h) = 0\,, T(0) = T_1 \quad \text{and } T(h) = T_2 \tag{3}$$

Introducing the following non-dimensional variables

$$\overline{y} = \frac{y}{h}\,, \quad \overline{u} = \frac{u}{h^2} \quad \text{and} \quad \theta = \frac{T - T_0}{T_2 - T_0}$$

into the equations (1) and (2), we obtain

$$kR\frac{d^3u}{dy^3}+\frac{d^2u}{dy^2}-R\frac{du}{dy}-\frac{1}{Da}u+\frac{Gr}{Re}\theta+A+G\sin\emptyset=0 \quad (4)$$

$$\frac{d^2\theta}{dy^2}-RPr\frac{d\theta}{dy}=0 \quad (5)$$

where $k=\frac{\alpha_1}{\rho h^2}$ is the viscoelastic parameter , $R=\frac{\rho V_0 h}{\mu}$ is the cross flow Reynolds number, $Gr=\frac{g\beta(T_2-T_1)h^3}{V^2}$ is the Grashof number, $Re=\frac{\rho U_0 h}{\mu}$ is the Reynolds number, $Pr=\frac{V}{\alpha}$ is the Prandtl number, $\gamma_T=\frac{T_1-T_0}{T_2-T_0}$ is the wall temperature parameter, $A=-(\frac{dp}{dx})\frac{U_0V}{h^2}$ is the constant pressure gradient.

The corresponding dimensionless boundary conditions are given by

$$u(0)=u(1)=0\,;\quad \theta(0)=\gamma_T \text{ and } \theta(1)=1 \quad (6)$$

Solution of the problem

We consider the first – order perturbation solution of the BVP (1) – (2) for smallϵ. The perturbation solution obtained by retaining the terms up to the same order of smallness of ϵ must be quite logical and reasonable. We write

$$u=u_0+\epsilon u_1 \quad (7)$$

and $$\theta=\theta_0+\epsilon\theta_1 \quad (8)$$

Using equations (7) and (8) into equations (4) and (5) and boundary conditions (6) and then equating the like powers of ϵ, we then get

Zeroth-Order system (ϵ^0) :

$$kR\frac{d^3u_0}{dy^3}+\frac{d^2u_0}{dy^2}-R\frac{du_0}{dy}-\frac{1}{D_a}u_0=-\frac{Gr}{Re}\theta_0-A-G\sin\varphi$$

Under the assumption that $\frac{d^3}{dy^3}$ is negligible the above equation reduces to

$$\frac{d^2u_0}{dy^2}-R\frac{du_0}{dy}-\frac{1}{D_a}u_0=\frac{Gr}{Re}\theta_0-A-G\sin\varphi \quad (9)$$

$$\frac{d^2\theta_0}{dy^2}-RPr\frac{d\theta_0}{dy}=0 \quad (10)$$

Together with the set of boundary conditions

$$u_0(0)=u_0(1)=0;\ \theta_0(0)=\gamma_T\ and\ \theta_0(1)=1 \quad (11)$$

First-Order system (ϵ^1) :

$$kR\frac{d^3u_1}{dy^3}+\frac{d^2u_1}{dy^2}-R\frac{du_1}{dy}-\frac{1}{D_a}u_1=\frac{Gr}{Re}\theta_1$$

Under the assumption that $\frac{d^3}{dy^3}$ is negligible the above equation reduces to

$$\frac{d^2u_1}{dy^2}-R\frac{du_1}{dy}-\frac{1}{D_a}u_1=\frac{Gr}{Re}\theta_1 \quad (12)$$

$$\frac{d^2\theta_1}{dy^2}-RPr\frac{d\theta_1}{dy}=0 \quad (13)$$

Together with the set of boundary conditions

$$u_1(0)=u_1(1)=0; \text{ and } \theta_1(0)=\theta_1(1)=0 \quad (14)$$

Zeroth-Order solution:

Solving equations (9) and (10) using the boundary conditions (11), we get

$$\theta_0=\frac{(1-\gamma_T e^{RPr})+(\gamma_T-1)e^{RPry}}{(1-e^{RPr})} \quad (15)$$

$$u_0=c_1e^{ay}+c_2e^{by}+\frac{Gr}{Re}(f_1-f_2e^{RPr\,y)})+ADa+GDa\sin\varphi \quad (16)$$

Where $a=\frac{R+\sqrt{R^2+4/Da}}{2}$ $b=\frac{R-\sqrt{R^2+4/Da}}{2}$ $f_1=\frac{(1-\gamma_T e^{RPr})Da}{(1-e^{RPr})}$

$f_2=\frac{(\gamma_T-1)}{(1-e^{RPr})(R^2Pr^2-R^2Pr-1/Da}$, $f_3=\frac{Gr}{Re}(f_1-f_2)+ADa+DaG\sin\varphi$

$f_4=\frac{Gr}{Re}(f_1-f_2e^{RPr})+ADa+DaG\sin\varphi$, $C_1=\frac{f_4-f_3e^b}{e^b-e^a}$, $C_2=\frac{f_3e^a-f_4}{e^b-e^a}$.

First Order solution:

Solving equation (13) with boundary conditions, we obtain

$$\theta_1=0 \quad (17)$$

Using equation (17) in the equation (12) and then solving the resulting equation with the corresponding conditions, we get

$$u_1 = 0.$$

RESULTS AND DISCUSSIONS

Figure 1, figure 2, figure 3 and figure 4 illustrates the variation of temperature with respect to Prandtl number for different values of radiation parameter. In each of these observations it is noticed that, as a Prandtl number increases, the temperature also increases. Not much of significant change is observed when the radiation parameter (R) is slightly decreased. However, a drastic change is noticed when the Prandtl number changes considerably along with the radiation parameter (R). The result is in agreement with Cheng *et.al* [3] and Vafai [6].

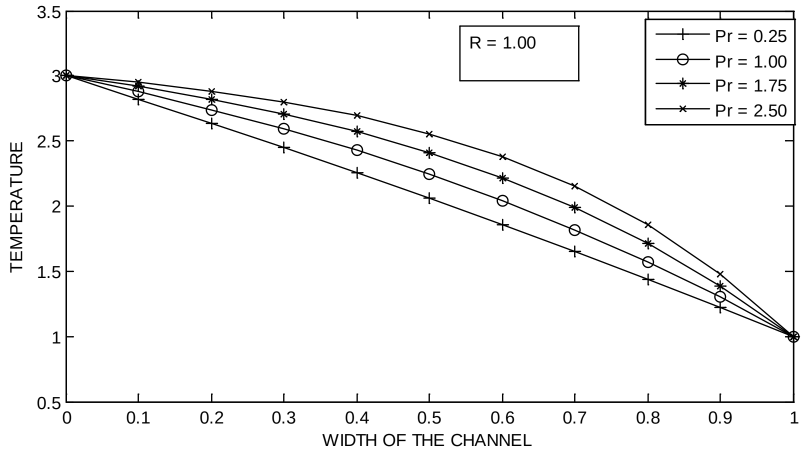

Figure-1: Temperature Profiles Along The Width Of The Channel

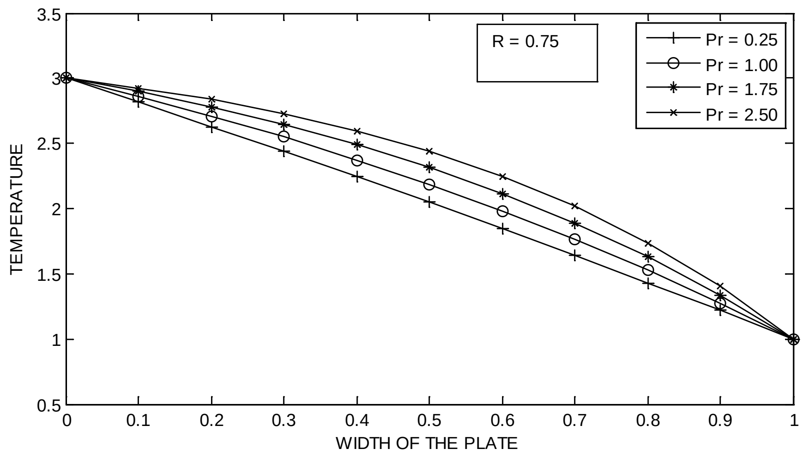

Figure-2: Variation Of Temperature Along The Width Of The Channel

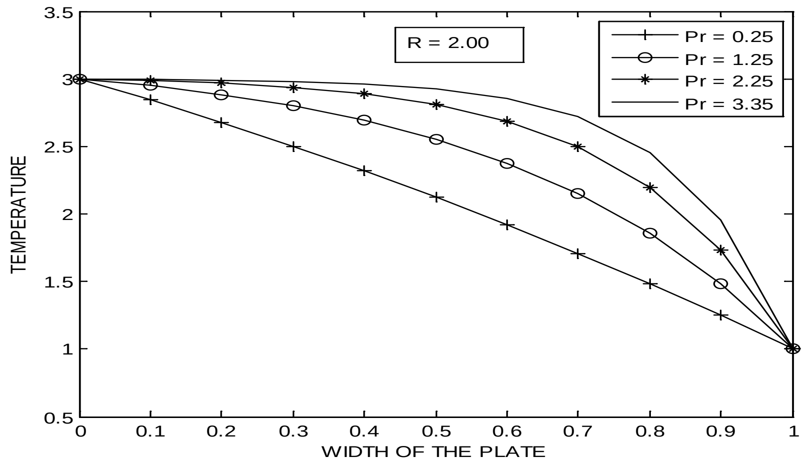

Figure-3: Distribution Of Temperature Profiles Along The Width Of The Channel

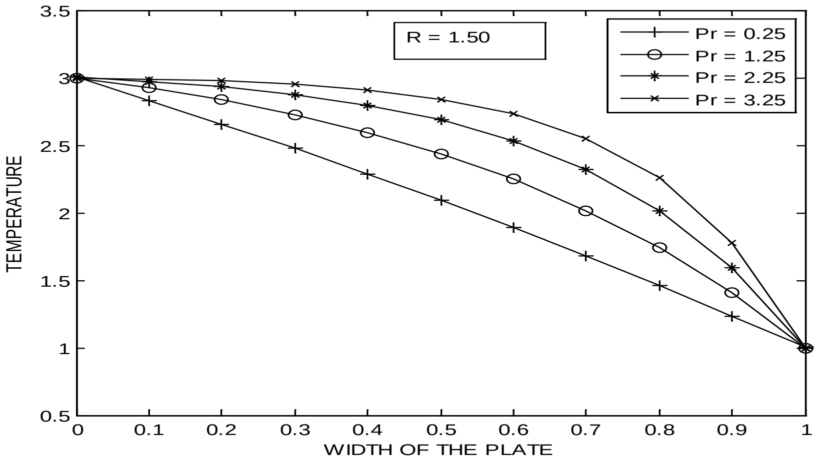

Figure-4: Variation Of Temperature Profiles Along The Width Of The Channel

The nature of temperature distribution along the width of the channel for different values of the Prandtl number with respect to the radiation parameter has been shown graphically in figure 5, figure 6, figure 7, figure 8 and figure 9. In each of these illustrations it is observed that, as the radiation parameter (R) increses the temparature in the fluid also increases. Not much of significant change is noticed for a small change in the Prandtl number. However, there is a considerable change in the profiles for larger values of Prandtl number (Pr). The illustations are in good agreement with the results obtained by Cheng *et al* [12] and Chamka *et al* [13].

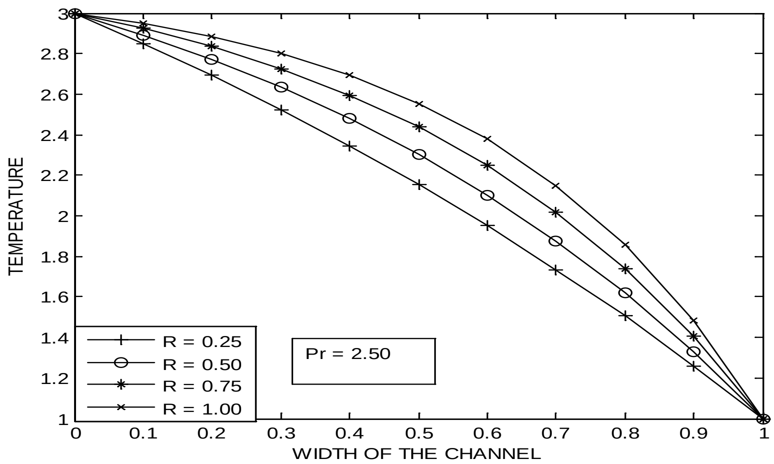

Figure-5: Temperature Profiles Along The Width Of The Channel

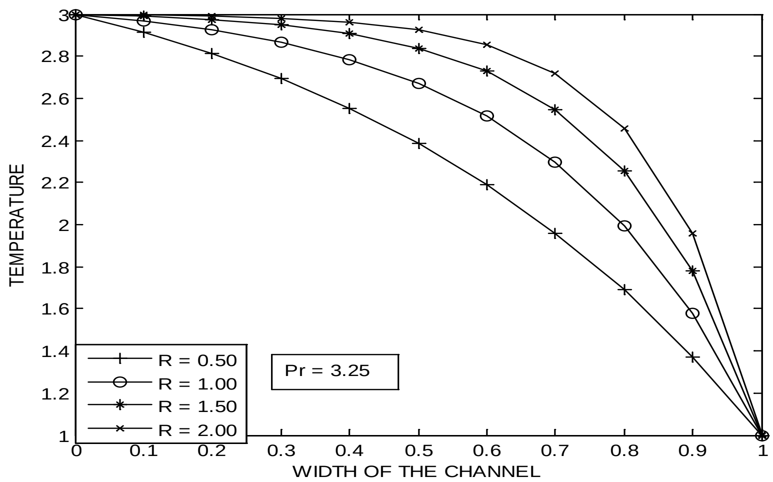

Figure-6: Temperature Profiles Along The Width Of The Channel

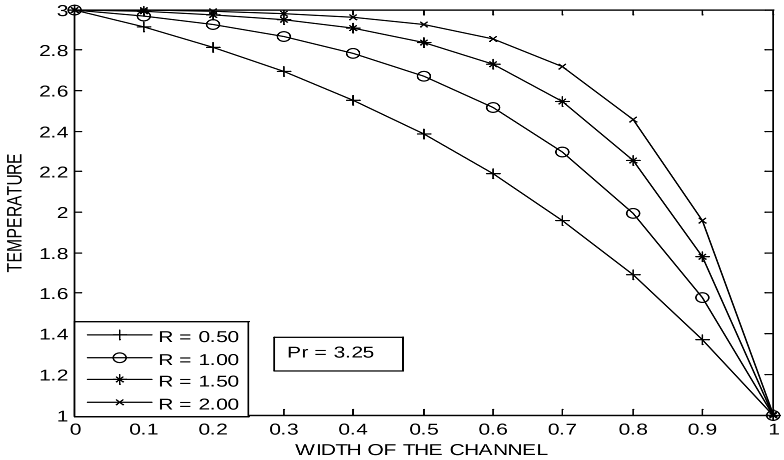

Figure-7: Temperature Profiles Along The Width Of The Channel

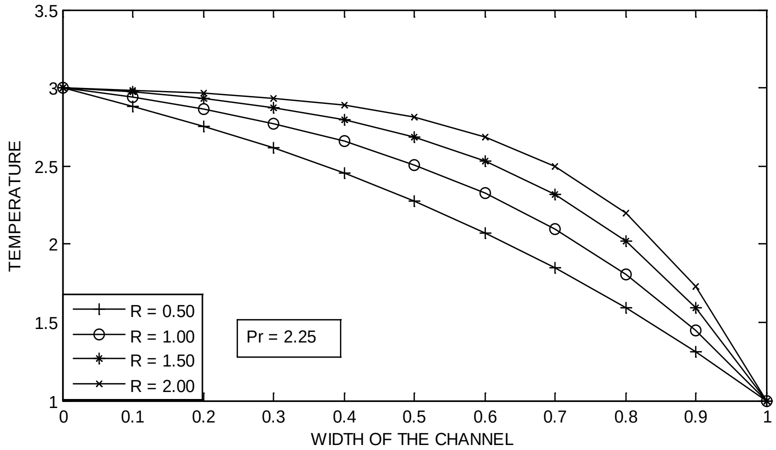

Figure-8: Temperature Profiles Along The Width Of The Channel

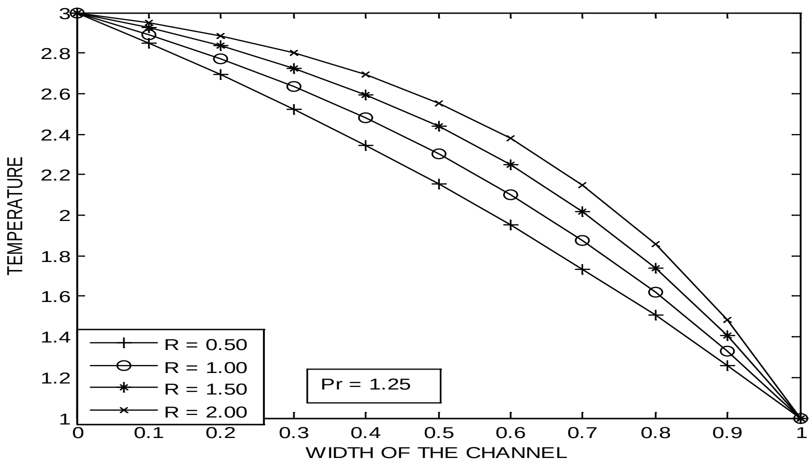

Figure-9: Temperature Profiles Along The Width Of The Channel

Figure 10 and figure 11 demonstrates the distribution of temperature profiles in the fluid medium for different values of Prandtl number (Pr). In both of these illustrations it is observed that as we move far away from the lower boundary the temperature decreases. Further it is observed that as the radiation parameter (R) increases the temperature of the fluid decreases.

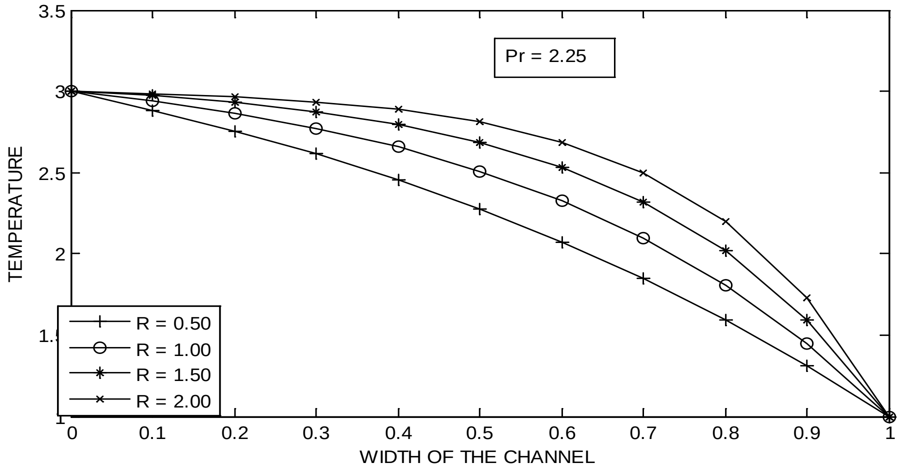

Figure-10: Variation Of Temperature Profiles

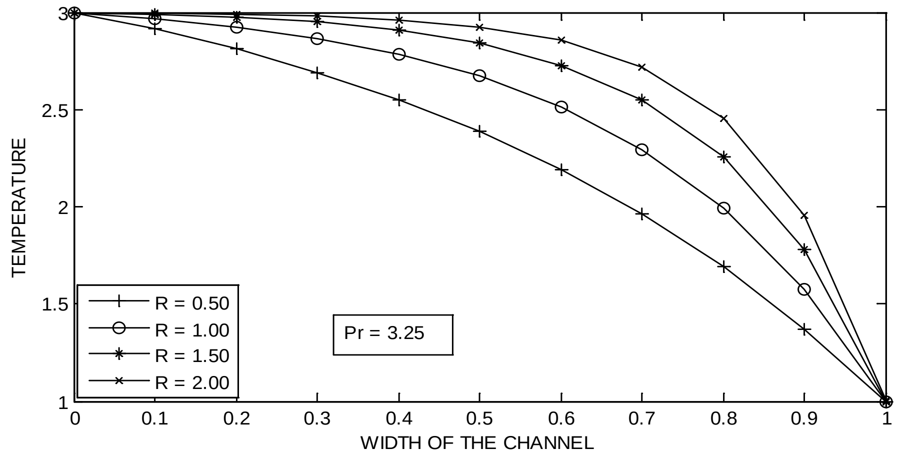

Figure-11: Distribution Of Temperature Profiles

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