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INVERSE DOMINATION NUMBER OF ONE VERTEX UNION OF CYCLES, COMPLETE BIPARTITE GRAPH AND ONE EDGE UNION OF CYCLES

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ABSTRACT

Let G = (V, E) be a graph. Let D be a minimum dominating set in a graph G. If V-D Contains a dominating set D' of G, then D' is called an inverse dominating set with respect to D. The minimum cardinality of an inverse dominating set of a graph G is called the inverse domination number of G. In this paper we study the inverse domination number of one vertex union of cycles, complete bipartite graph and one edge union of cycles.

AMS Subject Classifications: *05C69, 05C78.*

Keywords: *Graph, Domination, Inverse domination number.*

1. INTRODUCTION

Let G (p, q) be a graph with p=$|V|$ and q=$|E|$ denote the number of vertices and edges of a graph G respectively. All the graphs considered here are finite, non-trivial, undirected and connected without loops or multiple edges. For basic terminology, we refer to Chartrand and Lesniak [5].

A set D of vertices in a graph G = (V, E) is a dominating set if every vertex in V-D is adjacent to some vertex in D. The domination number γ(G) of G is the minimum cardinality of a dominating set of G. Let D be a minimum dominating set of G. If V-D contains a dominating set D' of G, then D' is called an inverse dominating set with respect to D. The inverse domination number γ^{-1}(G) of G is the minimum cardinality of an inverse dominating set of G. This concept was first introduced by Kulli and Sigarkanthi [3] and it was studied by several graph theorists in [6][7]. As usual C_n and K_n are respectively, the cycle and complete graph of order n, $K_{r,s}$ is the complete bipartite graph with two partite sets containing r and s vertices. Any undefined term or notation in this paper can be found in [1], [2].

Definition1.1: *Let G= (V, E) be a graph. Let D be a minimum dominating set in a Graph G. If V-D Contains a dominating set D' of G, then D' is called an inverse dominating set with respect to D. the minimum cardinality of an inverse dominating set of a graph G is called the inverse domination number of G and it is denoted by* γ^{-1}(G) *studied in [3],[4].*

Example:

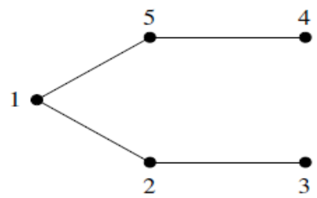

Figure 1: Inverse dominating graph

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D_1= {2,5}, D_2={2,4}, D_3={3,5} are the minimum dominating sets. Their corresponding inverse dominating sets are D_1'= {1,3,4} , D_2'={3,5} D_3'={2,4} respectively. Thus $\gamma(G) = 2$ is the domination number of G. γ^{-1}(G)=2 is the inverse domination number of G.

Remark 1.2: Every graph without isolated vertices contains an inverse dominating set, since the complement of any minimal dominating set is also a dominating set. Thus we consider a graph without isolated vertices.

Definition1.3: *A one vertex union C_n^k of k copies of cycles is the graph obtained by taking v as a common vertex such that any two cycles C_n^i and C_n^j (i,j) are disjoint and do not have any vertex in common except v it is studied in [8].*

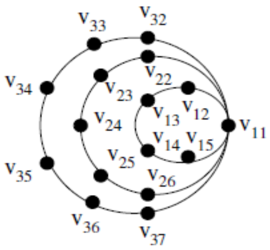

Figure 2: One vertex union of cycles

Definition1.4: *A one edge union C_n^k of k copies of cycles is the graph obtained by taking e as a common edge such that any two cycles C_n^i and C_n^j (i, j) are disjoint and do not have any vertex in common except v_1 and $v_2$it is studied in [8].*

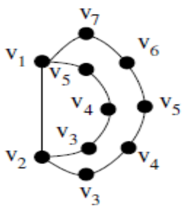

Figure 3: One edge union of cycles

2. MAIN RESULTS

Theorem 2.1: For every n the inverse domination number of attaching the cycles with one vertex as a common is $\gamma^{-1}(G) = n[\gamma^{-1}(C_n) - 1] + n$

Proof: Consider the cycle C_n on n vertices. Form a new graph G by taking m-copies of C_n.Pick an arbitrary vertex says u^i the first copy of $C_n$and select the corresponding vertex in each other copies of C_n labeled as u^2, u^3, $u^4 ... u^n$.Glue all there vertices,$u^i, 1 \le i \le n$ into one single vertex. The resulting graph is called G. Then G has the property that any cycles in G will have same vertex in common is

$$\gamma^{-1}(G) = n[\gamma^{-1}(C_n) - 1] + n$$

Theorem 2.2: For every m and n the inverse domination number of attaching the complete bipartite graph with one vertex as a common is $\gamma^{-1}(G) = n[\gamma^{-1}(k_{mn}) - 1] + n$

Proof: Consider the bipartite graph k_{mn} on (m+n) vertices. Form a new graph G by taking m-copies of k_{mn}.Pick an arbitrary vertex says u^i the first copy of $k_{mn}$and select the corresponding vertex in each other copies of k_{mn} labeled as $u^2, u^3, u^4 ... u^n$.Glue all there vertices,$u^i, 1 \le i \le n$ into one single vertex. The resulting graph is called G. Then G has the property that any bipartite graph in G will have same vertex in common is $\gamma^{-1}(G) = n[\gamma^{-1}(k_{mn}) - 1] + n$

Theorem 2.3: For every n the inverse domination number of the cycles C_n where $n = 5,7,11,...$ with one edge as common is $\gamma^{-1}(C_n) = (n+1)\ \forall n \in N$

Proof: Consider the cycle C_n $n = 5,7,11,...$ with one edge as common .Pick an arbitrary vertex says u^i the first copy of C_n where $n = 5,7,11,...$ with one edge as commonand select the corresponding vertex in each other copies of C_n where $n = 5,7,11,...$ with one edge as common. The resulting graph is called G. Then G has the property that the inverse domination number of the cycles with one edge is common is $\gamma^{-1}(C_n) = (n+1)\ \forall n \in N$

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