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A NEW RANKING METHOD FOR SOLVING INTERVAL FUZZY LINEAR PROGRAMMING PROBLEM USING α-CUT OPERATION

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ABSTRACT

Linear programming problems with trapezoidal fuzzy numbers have recently attracted some interest. Some methods have been developed for solving these problems. In this paper we shall propose an algorithm to solve a fuzzy linear programming problem with trapezoidal fuzzy numbers using interval arithmetic based on α-cut. The proposed method is easy to understand and apply for fuzzy linear programming problem occurring in real life situation.

Keywords: *fuzzy linear programming problem, trapezoidal numbers, interval arithmetic, α-cut operations, two phase method.*

1. INTRODUCTION

A lot of application problems can be modeled as mathematical problem which may be formulated with uncertainty. Bellman and Zadeh [1] introduced the concept of maximizing decision in a fuzzy decision making problem. Tanaka *el.al* [7] proposed a method for solving fuzzy mathematical programming.

Linear programming problem is that branch of mathematical programming which is designed to solve optimization problem where all the constraints as well as the objective functions are expressed as linear function. Linear programming is a method for finding the fittest answer from a range of possible answers. Linear programming is in two forms: classical linear programming problem and fuzzy linear programming problem in which the variables are assessed in a fuzzy number.

The concept of fuzzy linear programming problem was first proposed by Zimmermann [9]. Vansant *et.al* [8] applied linear programming with fuzzy parameters for decision making in industrial production planning. Campos and Verdegay [2] proposed a method to solve linear programming with fuzzy coefficients in both matrix and right hand side of the constraint. Gani *et.al* [4] introduce fuzzy linear programming problem based on L-R fuzzy number. Jimenez *et.al* [6] proposed a method for solving linear programming problems where all coefficients are, in general, fuzzy numbers and using linear ranking technique. Hashem [5] introduced the decision maker in the form of non symmetrical trapezoidal fuzzy numbers and solve it by using ranking function.

In this paper, a new method is introduced to solve the fuzzy linear programming problem. Thus, it is organized as follows. In section 2 some of the preliminary concepts on fuzzy number and function principal are given. In section 3 arithmetic operations of trapezoidal fuzzy numbers. In section 4 the proposed algorithm is discussed. In section 5 the numerical example is given and in section 6 the paper is concluded.

2. PRELIMINARIES

In this section, some necessary definitions and notions of fuzzy set theory are reviewed.

2.1 Definition:
Let A be a classical set. $\mu_A(x)$ be a real valued function defined from R→[0,1]. A fuzzy set A^* with the function $\mu_A(x)$ is defined by A^*= {(x,$\mu_A(x)$); xϵ A and $\mu_A(x)$ ∈[0, 1]}. The function $\mu_A(x)$ is known as the membership function of A^*.

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2.2 Definition:
Given a fuzzy set A defined on X and any number α $\epsilon[0,1]$, the α-cut α_A is the crisp set

$$\alpha_A = \{x\ /A(x) \geq \alpha\}$$

2.3 Definition:
Given a set A defined on X and any number αϵ [0, 1], the strong α-cut α_{+A} is the crisp set $\alpha_{+A} = \{x\ /A(x) > \alpha\}$.

2.4 Definition:
A fuzzy number is a convex normalized fuzzy set of the real line R whose membership function is piecewise continuous.

2.5 Definition:
A fuzzy number $\tilde{A}$ in R is said to be a trapezoidal fuzzy number if its membership function $\mu_{\tilde{A}}(x): R \rightarrow [0,1]$ has following characteristics

$$\mu_{\tilde{A}}(x) = \begin{Bmatrix} \dfrac{x-a}{b-a} & if\ x\epsilon[a,b] \\ \dfrac{d-x}{d-c} & if\ x\epsilon[c,d] \\ 0 & otherwise \end{Bmatrix}$$

3. ARITHMETIC OPERATION

Interval Arithmetic:
Interval arithmetic, interval mathematics, interval analysis or interval computation method are an approach to putting bounds on rounding errors and measurement of errors in mathematical computation and thus developing numerical methods that yield reliable results. This represents each value as a range of possibilities.

The following are the basic operation of interval arithmetic for two variable [a,b] and [c,d] that are subsets of the real line (-∞,∞) .

i. Addition: *[a,b] + [c,d] = [a+c ,b+d]*
ii. Subtraction: *[a,b] - [c,d] = [a-d ,b-c]*
iii. Multiplication: *[a,b] × [c.d] = [min (ac,ad,bc,bd) , max (ac,ad,bc,bd)]*
iv. Division: $[\frac{[a,b]}{[c,d]}] = [min\left(\frac{a}{c},\frac{a}{d},\frac{b}{c},\frac{b}{d}\right), max\ (\frac{a}{c},\frac{a}{d},\frac{b}{c},\frac{b}{d})]$, where 0 is not in *[c,d]*

Arithmetic Operation of Fuzzy Numbers using α-cut Method:
In this section we consider addition, subtraction, multiplication and division of fuzzy numbers using α-cut method.

Let *X = (a,b,c,d)* and *Y =(p,q,r,s)* be two fuzzy trapezoidal numbers whose membership function are defined by

$$\mu_x(X) = \begin{Bmatrix} \dfrac{x-a}{b-a}\ , & x\epsilon[a,b] \\ \dfrac{d-x}{d-c}\ , & x\epsilon[c,d] \\ 0\ , & otherwise \end{Bmatrix}$$

$$\mu_y(X) = \begin{Bmatrix} \dfrac{x-p}{q-p}\ , & x\epsilon[p,q] \\ \dfrac{s-x}{s-r}\ , & x\epsilon[r,s] \\ 0\ , & otherwise \end{Bmatrix}$$

Then $\alpha_x = [(b-a)\alpha + a, d-(d-c)\alpha]$ and $\alpha_y = [(q-p)\alpha + p, s-(s-r)\alpha]$ are the α-cuts of fuzzy numbers X and Y respectively.

Now for calculate addition, subtraction, multiplication and division of fuzzy numbers X and Y using interval arithmetic.

Addition: $\alpha_x + \alpha_y = [(b-a)\alpha + a, d-(d-c)\alpha]$ +$[(q-p)\alpha + p, s-(s-r)\alpha]$

$= [(b+q-a-p)\alpha + a + p\ ,\ \ d+s+(r+c-d-s)\alpha]$

Subtraction: $\alpha_x - \alpha_y = [(b-a)\alpha + a, d-(d-c)\alpha]$ - $[(q-p)\alpha + p, s-(s-r)\alpha]$
$= [a - s + (b + s - a - r)\alpha,\ d - p + (p + c - d - q)\alpha]$

Multiplication: $\alpha_x \times \alpha_y = (b-a)\alpha + a, d-(d-c)\alpha]$ - $[(q-p)\alpha + p, s-(s-r)\alpha]$
$= [\{(b-a)\alpha + a \times (q-p)\alpha + p\}, \{d-(d-c)\alpha \times s-(s-r)\alpha\}]$

Division: $\frac{\alpha_x}{\alpha_y} = \left[\frac{(b-a)\alpha+a}{s-(s-r)\alpha}, \frac{d-(d-c)\alpha}{(q-p)\alpha+p}\right]$

4. PROPOSED ALGORITHM

The arithmetic operations of fuzzy numbers using α-cut operations discussed in the earlier sections are used below to solve the fuzzy linear programming problem. The steps for the computation of an optimum solution are as follows:
Step-1: Find the value of α_x and α_y .

Step-2: Add the α-cuts of X and Y using interval arithmetic.

Step-3: The values obtained in step 1 and 2 is converted into a crisp linear programming problem and formulated as,

$$\text{Max } Z = \sum_{j=1}^{n} c_j x_j$$

$$\text{Subject to} \quad \sum_{j=1}^{n} a_{ij} x_j \leq, \geq b_i \qquad i=1, 2 \ldots.m$$

$$x_j \geq 0$$

The function to be maximized is called an objective function. This is denoted by Z. c= $(c_1, c_2 \ldots \ldots .. c_n)$ is called a cost vector. The matrix $[a_{ij}]$ is called an activity matrix and the vector $b_i = b_1, b_2, \ldots \ldots . b_m$ is called right hand side vector.

Step-4: Now solve reduced linear programming problem by two phase simplex method.

4.1. Two Phase Simplex Method

The two phase simplex method is used to solve a given linear programming problem in which some artificial variable are involved. The solution is obtained in two phases as follows:

Phase I: In this phase, the simplex method is applied to a specially constructed auxiliary linear programming problem leading to a final simplex table containing a basic feasible solution to the original problem.

1. Assign a cost -1 to each artificial variable and a cost 0 to all other variables (in place of their original cost) in the objective function.
2. Construct the auxiliary linear programming problem in which the new objective function Z^* is to be maximized subject to the given set of constraints.
3. Solve the auxiliary problem by simplex method until either of the following three possibilities do arise.
   i. Max$Z^* < 0$ and at least one artificial vector appear in the optimum basis at a positive level. In this case given problem does not possess any feasible solution.
   ii. Max$Z^* = 0$ = and at least one artificial vector appears in the optimum basis at zero level. In this case proceed to phase II.
   iii. Max$Z^* = 0$ and no artificial vector appears in the optimum basis. In this case also proceed to phase II.

Phase II: Now assign the actual costs to the variable in the objective function and a zero cost to every artificial variable that appears in the basis at the zero level. This new objective function is now maximized by simplex method subject to the given constraints. That is, simplex method is applied to the modified simplex table obtained at the end of phase-I, until an optimum basic feasible solution (if exists) has been attained. The artificial variables which are non basic at the end of phase-I are removed.

5. NUMERICAL EXAMPLE

Using the proposed algorithm an interval arithmetic linear programming problem with trapezoidal fuzzy numbers is considered

$$\text{Max } Z = [(2,4,6,8) + (10,12,14,16)]x_1 + [(1,3,5,7) + (9,11,13,15)]x_2$$

$$\text{Subject to} \quad 3x_1 + x_2 \geq (1,2,3,4) + (5,6,7,8)$$

$$2x_1 + x_2 \leq (0,2,4,6) + (1,3,5,7)$$

$$x_1, x_2 \geq 0$$

Step-1:
Determine α_x and α_y

The α-cut of the fuzzy number (*2, 4, 6, 8*) is

$$\alpha = \frac{x-2}{2}, \frac{8-x}{2}$$

$$\alpha_x = [2\alpha + 2, 8 - 2\alpha]$$

The α-cut of the fuzzy number (*10, 12, 14, 16*) is

$$\alpha = \frac{x-10}{2}, \frac{16-x}{2}$$

$$\alpha_x = [2\alpha + 10, 16 - 2\alpha]$$

Step-2:
Adding the α-cuts of x and y using arithmetic we obtain

$$\alpha_x + \alpha_y = [2\alpha + 2, 8 - 2\alpha] + [2\alpha + 10, 16 - 2\alpha]=36$$

Similarly right hand side number b_i and in objective function c_j are

$$c_2 = [(1,3,5,7) + (9,11,13,15)] = 32$$

$$b_1 = [(1,2,3,4) + (5,6,7,8) = 18$$

$$b_2 = [(0,2,4,6) + (1,3,5,7)] = 14$$

Now the given problem become the crisp linear programming problem as

$$\text{Max Z}=36x_1 + 32x_2$$

$$\text{Subject to } 3x_1 + x_2 \geq 18$$

$$2x_1 + x_2 \leq 14$$

$$x_1, x_2 \geq 0$$

Now we will solve this linear programming problem by two phase simplex method.

Introducing the surplus variable, slack variable and artificial variable $x_3, x_4, a_1 \geq 0$ respectively, the constraints of the given problem become

$$3x_1 + x_2 - x_3 + a_1 = 18$$

$$2x_1 + x_2 + x_4 = 14$$

$$x_1, x_2, x_3, x_4, a_1 \geq 0$$

Phase-I:
Assigning a cost -1 to artificial variable a_1 and cost 0 to all other variables, the new objective function for auxiliary problem becomes,

$$\text{Max } Z^* = 0x_1 + 0x_2 + 0x_3 + 0x_4 - 1a_1$$

Subject to the above given constraints. Now apply simplex method in usual manner.

			0	0	0	0	-1	
Basic variable	C_B	X_B	X_1	X_2	X_3	X_4	A_1	Min ratio $\frac{X_B}{X_K}$
a_1	-1	18	**3**	1	-1	0	1	$\frac{18}{3}$=6
x_4	0	14	2	1	0	1	0	$\frac{14}{2}$=7
	$\Delta_j \rightarrow$		-3 ↑	-1	1	0	0 ↓	
x_1	0	6	1	1/3	-1/3	0	1/3	
x_4	0	2	0	1/3	2/3	1	-2/3	
	$\Delta_j \rightarrow$		0	0	0	0	1	

Since all $\Delta_j \geq 0$ and no artificial variable appears in the basis, an optimum solution to the auxiliary problem has been attained.

Phase –II:
In this phase, now consider the actual costs associated with the original variables, the objective function has becomes

$$\text{Max } Z = 36x_1 + 32x_2 + 0x_3 + 0x_4$$

Now apply simplex method in the usual manner.

Basic variable	C_B	X_B	X_1	X_2	X_3	X_4	Min ratio = $\frac{X_B}{X_K}$
x_1	36	6	1	1/3	-1/3	0	$\frac{6}{1/3}$=18
x_4	0	2	0	**<u>1/3</u>**	2/3	1	$\frac{2}{1/3}$=6
	$\Delta_j \rightarrow$		0	-20 $\uparrow$	-12	0 $\downarrow$	
x_1	36	4	1	0	-1	-1	
x_2	32	6	0	1	2	3	
	$\Delta_j \rightarrow$		0	0	28	60	

Since all $\Delta_j \geq 0$, hence an optimum basic feasible solution can be attained.
The optimum solution is $x_1 = 4, x_2 = 6$
Max Z= 4×36 + 6×32 =336

6. CONCLUSION

In this paper interval arithmetic linear programming problem with trapezoidal fuzzy number is solved by using α-cut operation without converting them into a classical linear programming problem. This is an easy approach for solving fuzzy linear programming problem using interval arithmetic when compared to the earlier approaches. This also can be applied in triangular, hexagonal, octagonal fuzzy numbers and also can be extended to multi objective linear programming and fully fuzzy linear programming problems.

REFERENCES

1. Bellman R. E and L. A. Zadeh, Decision Making in A Fuzzy Environment, Management Science, 17(1970), 141-164.
2. Campos Land, Verdegay J. L., Linear Programming Problems and Ranking of Fuzzy Numbers, Fuzzy Sets and Systems, 32(1989), 1-11.
3. Dutta Palash, Boruah Hrishikesh and Ali Tazid, Fuzzy arithmetic with and without using α-cut method, International Journal of Latest Lrends in Computing, 2(2011), 99-107.
4. Gani A.N., Duraisamy C. and Veeramani C., A note on fuzzy linear programming problem using L-R fuzzy number,International Journal of Algorithms, Computing and Mathematics, 2 (3) (2009), 93-106.
5. Hashem, H.A., Solving fuzzy linear programming problems with fuzzy nonsymmetrical trapezoidal fuzzy numbers, Journal of Applied Sciences Research, (9) (6) (2013), 4001-4005.
6. Jimenez M., Arenas M., Bilbao A. and Rodriguez M.V., Linear programming with fuzzy parameters, An interactive method resolution, European Journal of Operational Research, (177) (2007), 1599-1609.
7. Tanaka H., Okuda T. and Asai K., On Fuzzy Mathematical Programming, Journal of Cybernetics and Systems, 3(1973), 37-46.
8. Vansant P., Nagarajan, and Yacab S., Decision making in industrial production planning using fuzzy linear programming, Journal of Management Mathematics, 15(2004), 53-65.
9. Zimmermann, H.J., Fuzzy programming and linear programming with several objective functions, Fuzzy sets and system, 1(1978), 45-55.

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