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A SURVEY ON MULTI CHANNEL FUZZY QUEUEING MODEL USING α - CUT AND DSW ALGORITHM

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ABSTRACT

In this paper we study FM/FM/3 Queueing Model using DSW algorithm based on α – cut representation in triangularization method. The numerical example also given to test the efficiency of the model.

Keywords: *Inter arrival time, Service time, Membership function, Triangular fuzzy numbers, α-cuts, DSW algorithm, and Interval analysis.*

1. INTRODUCTION

Queueing theory is a branch of mathematics that studies and models the act of waiting lines. The first paper on Queueing theory, "The theory of probabilities and Telephone conversation" was published in 1909 by Anger Krarup Erlang [4], as consider the father of field.

A queue is formed whenever the demand for service exceeds the capacity to provide service at that point in time. (eg: bank, hospital, reservation counter, ration shop). A queueing system is composed of customers or units needing some kind of service who arrive at a service facility where such service is provided join a queue of service is not immediately available and eventually leave after service.

Queueing theory can be applied the imperfect matching between the customer and service facilities is caused be one's inability to predict accurately the arrival and service time of customers. It tries to answer the questions like average number of customer waiting in the system and queue and average time of customer spends in the system and queue. These questions are mainly investigated by Queueing Model

Our aim of this paper discussed about multi server queueing model and first come first served discipline using triangular fuzzy numbers under α-cut represented through DSW algorithm. Here fuzzy set decompose into distinct eleven points through α-cut method.

Fuzzy queueing model have been developed by researchers George J Klir and Bo Yuan [5] in 1995.Negi.R.S and Lee.E.S [8] in 1992, S.P. Chen [3] in 2005, R.Srinivasan[11] in 2014 and Shanmugasundram.S and Venkatesh.B[10] in 2015. Traditional Queueing Model will be more realistic if it is converted into Fuzzy queueing model.

2. PRELIMINARIES

2.1 Fuzzy Set

Let X be a nonempty set. A fuzzy set A in X is characterized by its membership function $\mu_A : X \to [0,1]$ and $\mu_A(x)$ is interpreted as the degree of membership of element x in fuzzy set A for each $x \in X$.

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2.2 α - Cuts

If a fuzzy set is defined on for any $\alpha \in [0, 1]$ the cuts is represented by the following crisp set.

Strong **α** cuts:

$$\alpha_A{}^* = \{ x \in X / \mu_A(x) > \alpha : \alpha \in [0,1]\}$$

Weak **α** cuts:

$$\alpha_A = \{ x \in X / \mu_A(x) \geq \alpha : \alpha \in [0,1]\}$$

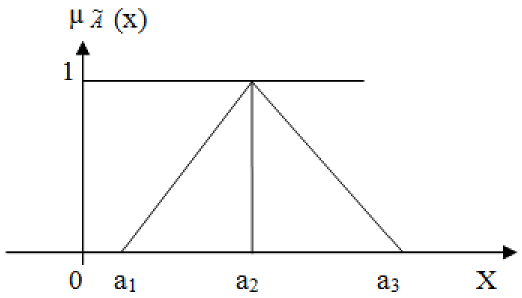
Therefore, it is inferred that fuzzy set A can be treated as crisp set α_A in which all the members have their membership values greater than or at least equal to α .

2.3 Triangular Fuzzy Number

A triangular fuzzy number $\tilde{A}$ is defined by (a_1, a_2, a_3) where a_i ƐR and $a_1 \leq a_2 \leq a_3$.Its membership function is

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1} & \text{for } x < a_1 \\ \dfrac{a_3 - x}{a_3 - a_2} & \text{for } a_2 \leq x \leq a_3 \\ 0 & \text{otherwise} \end{cases}$$

The graph of $\mu_{\tilde{A}}(x)$ is



3. INTERVAL ANALYSIS

Let I_1 and I_2 be two interval numbers defined by ordered pairs of real numbers with lower and upper bounds.

$$I_1 = [a, b],\ a \leq b, \quad I_2 = [c, d],\ c \leq d$$

To define a general arithmetic property with the symbol *, where * = [+, -, ×, ÷]

(i.e.) $I_1 * I_2 = [a,b] * [c,d]$

Where $I_1 * I_2$ represents another interval. The interval calculation depends on the magnitudes and signs of the element a, b, c, d.

$$[a, b] + [c, d] = [a + c, b + d]$$

$$[a, b] - [c, d] = [a - d, b - c]$$

$$[a, b] \,.\, [c, d] = [\min (ac, ad, bc, bd), \max (ac, ad, bc, bd)]$$

$$[a, b] \div [c, d] = [a,b]. \left[\frac{1}{d}, \frac{1}{c}\right] \text{ provided that } 0 \notin [c, d]$$

$$\alpha[a,b] = \begin{cases} [\alpha a,\ \alpha b] \text{ for } \alpha > 0 \\ [\alpha b,\ \alpha a] \text{ for } \alpha < 0 \end{cases}$$

4. DSW Algorithm

The DSW algorithm consists of the following steps:

1) Select a α cut value where $0 \leq \alpha \leq 1$.
2) Find the intervals in the input membership functions that correspond to this α.
3) Using standard binary interval operations compute the interval for the output membership function for the selectedα cut level.
4) Repeat steps 1 to 3 for different values of α to complete α cut representation of the solution

5. DESCRIPSITION OF THE MODEL

If we consider a traditional queueing system with number of server Three, calling population is infinite and queue discipline is first in first out discipline. In technically (i.e.) (M/M/C: ∞/FIFO).

Arrival rate and service rate are fuzzy numbers denoted by $\tilde{\lambda}$ and $\tilde{\mu}$. The inter arrival time (A) and service times (S) are represented by the following fuzzy sets.

$$A = \{(a, \tilde{\mu}_A(a)) / a \in X\}$$

$$S = \{(s, \tilde{\mu}_s(s)) / s \in Y\}$$

Where X and Y are crisp universal sets of the inter arrival time and service time.

The membership functions of A and S are

$$A(\alpha) = \{ a \in X / \tilde{\mu}_A(a) \geq \alpha \}$$

$$S(\alpha) = \{ s \in Y / \tilde{\mu}_s(s) \geq \alpha \}$$

Where $A(\alpha)$ and $S(\alpha)$ are crisp sets using α cuts, the inter arrival time and service time are represented by different levels of confidence intervals.

The triangular membership function P(A,S) is

$$\mu_{P(A,S)}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1} & \text{for } x < a_1 \\ \dfrac{a_3 - x}{a_3 - a_2} & \text{for } a_2 \leq x \leq a_3 \\ 0 & \text{otherwise} \end{cases}$$

Multi server Fuzzy queue infinite calling source and First In First Out discipline. In technically (FM/FM/3): (FIFO/∞/∞)

The queue has two or more service facility in parallel providing identical service. The customer in the waiting line can get service at any one of three servers. The arrival and service distribution follows Fuzzy Markovian distribution.

The performance of the Multi server queueing system is,

1. The Expected number of customers in the system

$$Ls = \frac{\lambda\mu\left(\frac{\lambda}{\mu}\right)^3}{2!(3\mu-\lambda)^2} P_O + \frac{\lambda}{\mu}$$

2. Expected number of customers waiting in the queue

$$Lq = \frac{\lambda\mu\left(\frac{\lambda}{\mu}\right)^3}{2!\,(3\mu-\lambda)^2} P_O$$

3. Average waiting time of a customer spends in the system

$$Ws = \frac{\mu\left(\frac{\lambda}{\mu}\right)^3}{2!(3\mu-\lambda)^2} P_O + \frac{1}{\mu}$$

4. Average waiting time of a customer in the queue

$$Wq = \frac{\mu\left(\frac{\lambda}{\mu}\right)^3}{2!\,(3\mu-\lambda)^2} P_O$$

$$Here\ P_O = \frac{1}{\sum_{n=o}^{2}\left(\frac{\lambda/\mu}{n!}\right)^n \left(\frac{\lambda/\mu}{3!}\right)^3 \cdot \frac{3\mu}{3\mu-\lambda}}$$

6. NUMERICAL EXAMPLE

6.1 Triangular fuzzy number

Consider (FM/FM/3) queue, where the both the arrival rate and service rate are triangular fuzzy numbers represented by $\tilde{\lambda}$ = [15, 16, 17] and $\tilde{\mu}$ = [6, 7, 8]. The number of server is Three. The interval of confidence at possibility level α as [15+ , 17− α] and [6+ α , 8−α]

Where x = [15+α, 17− α] and y = [6+ α, 8−α]

Table: The α-cuts of P_0, Ls, Lq, Ws, Wq

α	P_0	L_s	L_q	W_s	W_q
0	[0.1322,0.0131]	[2.5205,18.0295]	[0.6455,15.1962]	[0.1680,1.0606]	[0.0430,0.8939]
0.1	[0.1259,0.0185]	[2.6204,13.1165]	[0.7090, 10.3460	[0.1735 ,0.7761]	[0.0470,0.6122]
0.2	[0.1195,0.0240]	[2.7284,10.3849]	[0.7796 , 7.6752	[0.1795, 0.6181]	[0.0513,0.4569]
0.3	[0.1132,0.0296]	[2.8468, 8.6432]	[0.8598, 5.9924]	[0.1861, 0.5176]	[0.0562,0.3588]
0.4	[0.1069,0.0352]	[2.9768, 7.4203]	[0.9505, 4.8266	[0.1933, 0.4470]	[0.0617,0.2908]
0.5	[0.1007,0.0408]	[3.1211 , 6.5149]	[1.0544, 3.9765]	[0.2014, 0.3948]	[0.0680,0.2410]
0.6	[0.0945,0.0466]	[3.2812, 5.8321]	[1.1731, 3.3472]	[0.2103, 0.3556]	[0.0752,0.2041]
0.7	[0.0883,0.0524]	[3.4602 , 5.2861]	[1.3095, 2.8532]	[0.2204, 0.3243]	[0.0834,0.1750]
0.8	[0.0822,0.0583]	[3.6632, 4.8438]	[1.4688, 2.4614]	[0.2318, 0.2990]	[0.0930,0.1519],
0.9	[0.0762,0.0642]	[3.8960 , 4.4742]	[1.6566 , 2.1409]	[0.2450, 0.2779]	[0.1042, 0.1330]
1	[0.0701,0.0701]	[4.1608, 4.1608]	[1.8751, 1.8751]	[0.2601, 0.2601]	[0.1172, 0.1172]

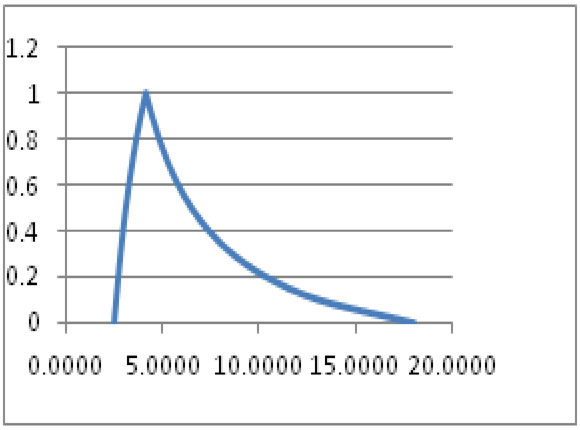


Fig 1: Expected number of customers in the system (Ls)

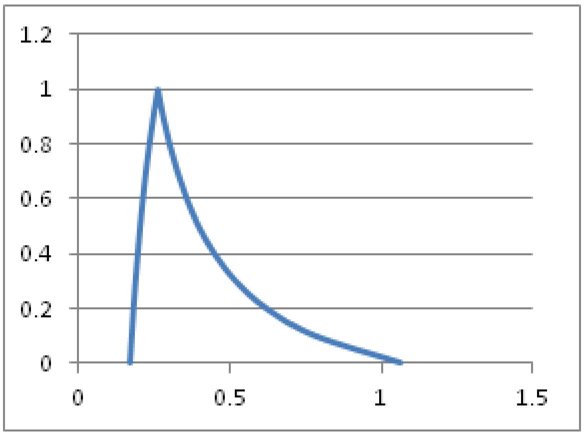
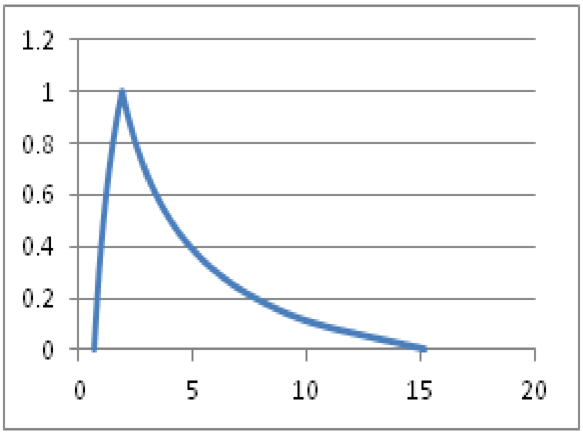


Fig 2: Average waiting time of a customer in the system (Ws)



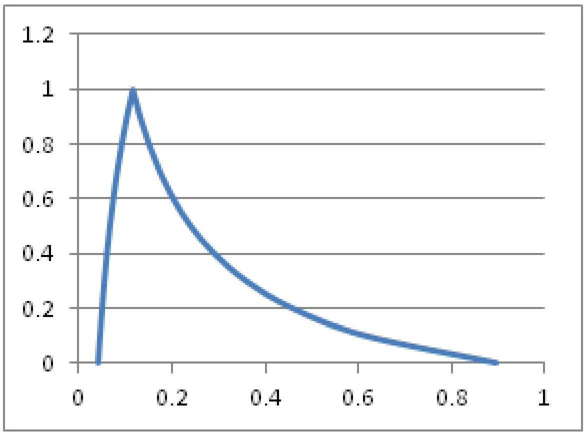


Fig 3: Expected number of customer in the queue (Lq). **Fig 4:** Average wating time of a customer in the queue (Wq)

We find the arrival rate and service rate by taking α as 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1. From the above table:

1) Expected number of customers in the system is 4.1608 (α =1) which is in the interval [2.5205, 18.0295]
2) Expected number of customers in the queue is 1.8751 (α =1) which is in the interval [0.6455, 15.1962]
3) Average waiting time of a customer in the system is 0.2601 (α =1) which is in the interval [0.1680, 0.0606]
4) Average waiting time of a customer in the queue is 0.1172 (α =1) which is in the Interval [0.0430, 0.8939]

7. CONCLUSION

The arrival time and service time follows fuzzy variable. In the numerical example we calculate the values of P_0, L_s, L_q, W_s, W_q at the point 0.0701, 4.1608, 1.8751, 0.2601, 0.1172 using DSW algorithm. It shows the efficiency of the model.

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