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F-INDICES OF CHEMICAL NETWORKS

V. R. KULLI*
Department of Mathematics,
Gulbarga University, Gulbarga, 585106, India.

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ABSTRACT

In Chemical Sciences, a forgotten topological index or F-index has significant importance to collect information about properties of chemical compounds. In this paper, we introduce the second F-index, second hyper F-index, the general first and second F-indices, sum connectivity F-index and product connectivity F-index of a graph. We compute the first and second F-indices, first and second hyper F-indices and their polynomials of armchair polyhex, zigzag polyhex and carbon nanacone networks. Furthermore, we determine the sum connectivity F-index and product connectivity F-index of these networks.

Keywords: *F-indices, hyper F-indices, general first and second F-indices, sum connectivity F-index, product connectivity F-index, network.*

Mathematics Subject Classification: *05C05, 05C12, 05C35.*

1. INTRODUCTION

Let G be a finite, simple, connected graph with vertex set $V(G)$ and edge set $E(G)$. The degree $d_G(v)$ of a vertex v is the number of edges incident to v. For additional definitions and notations, the reader may refer to [1].

Chemical graph theory has an important effect on the development of Chemical Sciences. A molecular graph is a graph whose vertices correspond to the atoms and the edges to the bonds. A topological index is a numeric quantity from the structure of a molecule. Several topological indices have some applications in Theoretical Chemistry, especially in QSPR/QSAR study see [2, 3].

In [4] Furtula and Gutman studied the F-index and defined it as

$$F(G)=\sum_{uv\in E(G)}\left[d_G(u)^2+d_G(v)^2\right].$$

Now we call it as the first F-index and denote it by $F_1(G)$.

We introduce the second F-index of a graph, as

$$F_2(G)=\sum_{uv\in E(G)}d_G(u)^2\,d_G(v)^2.$$

In [5], Ghobadi *et al.* defined the hyper F-index of a graph G as

$$HF(G)=\sum_{uv\in E(G)}\left[d_G(u)^2+d_G(v)^2\right]^2.$$

Now we call it as the first hyper F-index and denote it by $H_1F(G)$.

We propose the second hyper F-index of a graph defined as

$$HF_2(G)=\sum_{uv\in E(G)}\left[d_G(u)^2\,d_G(v)^2\right]^2.$$

Corresponding Author: V. R. Kulli*
Department of Mathematics, Gulbarga University, Gulbarga, 585106, India.

We introduce the sum connectivity F-index and the product connectivity F-index of a graph G, defined as

$$SF(G)=\sum_{uv\in E(G)}\frac{1}{\sqrt{d_G(u)^2+d_G(v)^2}}$$

$$PF(G)=\sum_{uv\in E(G)}\frac{1}{\sqrt{d_G(u)^2 d_G(v)^2}}$$

We continue this generalization and define the general first and second F-indices of a graph G as

$$F_1^a(G)=\sum_{uv\in E(G)}\left[d_G(u)^2+d_G(v)^2\right]^a, \tag{1}$$

$$F_2^a(G)=\sum_{uv\in E(G)}\left[d_G(u)^2 d_G(v)^2\right]^a \tag{2}$$

In [5], the first F-polynomial of a graph is defined by Ghobadi *et al.* as

$$F_1(G,x)=\sum_{uv\in E(G)}x^{d_G(u)^2+d_G(v)^2}. \tag{3}$$

We propose the second F-polynomial, and the first and second hyper F polynomials of a graph G as

$$F_2(G,x)=\sum_{uv\in E(G)}x^{d_G(u)^2 d_G(v)^2}. \tag{4}$$

$$HF_1(G,x)=\sum_{uv\in E(G)}x^{\left[d_G(u)^2+d_G(v)^2\right]^2}. \tag{5}$$

$$HF_2(G,x)=\sum_{uv\in E(G)}x^{\left[d_G(u)^2 d_G(v)^2\right]^2}. \tag{6}$$

In this study, we consider the families of armchair polyhex, zigzag polyhex and carbon nanocone networks. Some degree based topological indices of these networks were studied in [6, 7, 8]. A forgotten topological index or F-index has a significant importance to collect information about properties of chemical compounds. In Chemical Graph Theory, graph polynomials related chemical graph were studied in [9, 10, 11, 12, 13, 14, 15, 16, 17, 18]. In this paper, the F-indices, F hyper indices and their polynomials of armchair polyhex nanotubes, zigzag, polyhex nanotubes and carbon nanocone networks are computed. Also the connectivity indices of these chemical networks are determined.

2. ARMCHAIR POLYHEX NANOTUBES

Carbon polyhex nanotubes are the nanotubes whose cylindrical surface is made up of entirely hexagons. These carbon nanotubes exist in nature with remarkable stability and possess very interesting electrical, thermal and mechanical properties, The armchair polyhex nanotube is denoted by $TUAC_6$ [p, q] is shown in Figure 1.

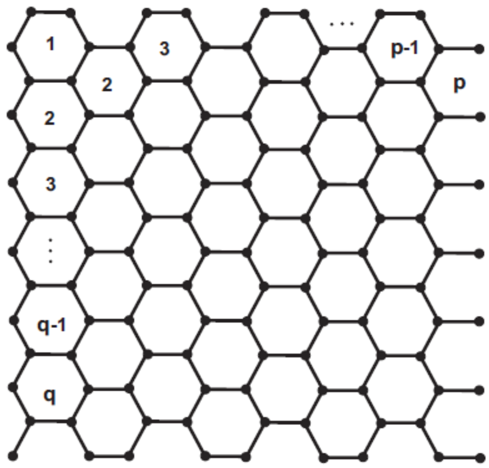

Figure-1: A 2-dimensional networks of $TUAC_6[p, q]$.

Let $G = TUAC_6\,[p, q]$. By calculation, G has $2p(q+1)$ vertices and $3pq + 2p$ edges. There are three types of edges based on degrees of end vertices of each edge. We present that the edge partition of G is given in Table 1.

$d_G(u), d_G(v) \setminus uv \in E(G)$	(2, 2)	(2, 3)	(3, 3)
Number of edges	p	$2p$	$3pq- p$

Table-1: Edge partition of $TUAC_6\,[p, q]$

In the following theorem, we determine the general first F-index of $TUAC_6\,[p, q]$.

Theorem 1: The general first F-index of $TUAC_6\,[p, q]$ is given by

$$F_1^a\left(TUAC_6[p,q]\right) = 3\times 18^a\,pq + \left(8^a + 2\times 13^a - 18^a\right)p. \tag{7}$$

Proof: By using equation (1) and Table 1, we derive

$$\begin{aligned} F_1^a\left(TUAC_6[p,q]\right) &= \sum_{uv\in E(G)} \left[d_G(u)^2 + d_G(v)^2\right]^a \\ &= \left(2^2+2^2\right)^a p + \left(2^2+3^2\right)^a 2p + \left(2^2+3^2\right)^a \left(3pq-p\right) \\ &= 3\times 18^a\,pq + \left(8^a + 2\times 13^a - 18^a\right)p. \end{aligned}$$

We obtain the following results by using Theorem 1.

Corollary 1.1: The first F-index of $TUAC_6\,[p, q]$ is

$F_1(TUAC_6\,[p, q]) = 54\,pq + 16p$.

Proof: Put $a = 1$ in equation (7), we get the desired result.

Corollary 1.2: The first hyper F-index of $TUAC_6\,[p, q]$ is

$HF_1\,(TUAC_6\,[p, q]) = 972pq + 78p$.

Proof: Put $a = 2$ in equation (7), we obtain the desired result.

Corollary 1.3: The sum connectivity F-index of $TUAC_6\,[p, q]$ is

$$SF\left(TUAC_6[p,q]\right) = \frac{1}{\sqrt{2}}\,pq + \left(\frac{1}{\sqrt{8}} + \frac{2}{\sqrt{13}} - \frac{1}{\sqrt{18}}\right)p.$$

Proof: Put $a = -\frac{1}{2}$ in equation (7), we get the desired result.

In the following theorem, we compute the general second F-index of $TUAC_6\,[p, q]$.

Theorem 2: The general second F index of $TUAC_6\,[p, q]$ is given by

$$F_2^a\left(TUAC_6[p,q]\right) = 3\times 81^a\,pq + \left(16^a + 2\times 36^a - 81^a\right)p. \tag{8}$$

Proof: From equation (2) and using Table 1, we deduce

$$\begin{aligned} F_2^a\left(TUAC_6[p,q]\right) &= \sum_{uv\in E(G)} \left[d_G(u)^2\, d_G(v)^2\right]^a \\ &= \left(2^2\times 2^2\right)^a p + \left(2^2\times 3^2\right)^a 2p + \left(3^2\times 3^2\right)^a \left(3pq-q\right) \\ &= 3\times 81^a\,pq + \left(16^a + 2\times 36^a - 81^a\right)p. \end{aligned}$$

The following results are obtained by using Theorem 2.

Corollary 2.1: The second F-index of $TUAC_6\,[p, q]$ is

$F_2(TUAC_6\,[p, q]) = 243pq + 7p$.

Proof: Put $a = 1$ in equation (8), we get the desired result.

Corollary 2.2: The second hyper F-index of $TUAC_6\,[p, q]$ is

$$HF_2(TUAC_6\,[p, q]) = 19683pq - 3713p.$$

Proof: Put $a = 2$ in equation (8), we obtain the desired result.

Corollary 2.3: The product connectivity F-index of $TUAC_6\,[p, q]$ is

$$PF\left(TUAC_6\left[p,q\right]\right)=\frac{1}{3}pq+\frac{11}{36}p.$$

Proof: Put $a = -\frac{1}{2}$ in equation (8), we obtain the desired result.

Theorem 3: Let $G = TUAC_6\,[p, q]$ be an armchair polyhex nanotube. Then

(i) $F_1(G,x)= px^8 + 2px^{13} + (3pq - p)x^{18}.$

(ii) $F_2(G,x)= px^{16} + 2px^{36} + (3pq - p)x^{81}.$

(iii) $HF_1(G,x)= px^{64} + 2px^{169} + (3pq - p)x^{324}.$

(iv) $HF_2(G,x)= px^{256} + 2px^{1296} + (3pq - p)x^{6561}.$

Proof: (i) By using equation (3) and Table 1, we obtain

$$\begin{aligned} F_1(G,x) &= \sum_{uv\in E(G)} x^{d_G(u)^2+d_G(v)^2} \\ &= px^{2^2+2^2} + 2px^{2^2+3^2} + (3pq-p)x^{3^2+3^2} \\ &= px^8 + 2px^{13} + (3pq-p)x^{18} \end{aligned}$$

(ii) From equation (4) and by using Table 1, we derive

$$\begin{aligned} F_2(G,x) &= \sum_{uv\in E(G)} x^{d_G(u)^2 d_G(v)^2} \\ &= px^{2^2\times 2^2} + 2px^{2^2\times 3^2} + (3pq-p)x^{3^2\times 3^2} \\ &= px^{16} + 2px^{36} + (3pq-p)x^{81}. \end{aligned}$$

(iii) By using equation (5) and Table 1, we deduce

$$\begin{aligned} HF_1(G,x) &= \sum_{uv\in E(G)} x^{\left[d_G(u)^2+d_G(v)^2\right]^2} \\ &= px^{\left(2^2+2^2\right)^2} + 2px^{\left(2^2+3^2\right)^2} + (3pq-p)x^{\left(3^2+3^2\right)^2} \\ &= px^{64} + 2px^{169} + (3pq-p)x^{324} \end{aligned}$$

(iv) From equation (6) and using Table 1, we have

$$\begin{aligned} HF_2(G,x) &= \sum_{uv\in E(G)} x^{\left[d_G(u)^2 d_G(v)^2\right]^2} \\ &= px^{\left(2^2\times 2^2\right)^2} + 2px^{\left(2^2\times 3^2\right)^2} + (3pq-p)x^{\left(3^2\times 3^2\right)^2} \\ &= px^{256} + 2px^{1296} + (3pq-p)x^{6561} \end{aligned}$$

3. ZIGZAG POLYHEX NANOTUBES

The zigzag polyhex nanotube is denoted by $TUZC_6\,[p, q]$, where p is the number of hexagons in a row whereas q is the number of hexagons in a column. A 2-dimensional networks of $TUZC_6\,[p, q]$ is depicted in Figure 2.

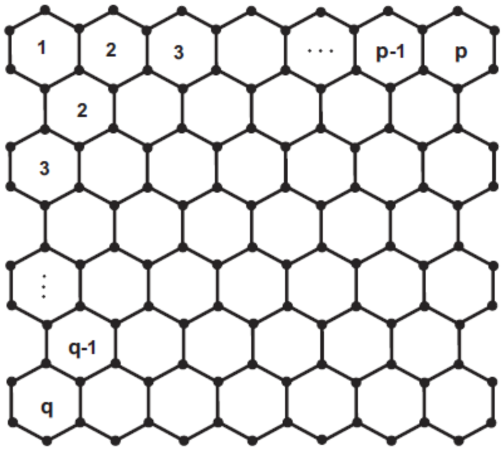

Figure-2: A 2-dimensional networks of $TUZC_6[p, q]$

Let G be a graph of a (p, q) dimensional zigzag polyhex nanotube. The graph G has $2p(q+1)$ vertices and $3pq + 2p$ edges. In G, there are two types of edges based on degrees of end vertices of each edge. By calculation, the edge partition of G is given in Table 2.

$d_G(u), d_G(v) \setminus uv \in E(G)$	(2, 3)	(3, 3)
Number of edges	$4p$	$2pq - 2p$

Table-2: Edge partition of $TUZC_6\,[p, q]$

In the following theorem, we compute the general first F-index of $TUZC_6\,[p, q]$.

Theorem 4: The general first F-index of $TUZC_6\,[p, q]$ is

$$F_1^a\left(TUZC_6[p,q]\right) = 3\times 18^a\, pq + \left(4\times 13^a - 2\times 18^a\right)p. \tag{9}$$

Proof: From equation (1) and by using Table 2, we deduce

$$F_1^a\left(TUZC_6[p,q]\right) = \sum_{uv\in E(G)}\left[d_G(u)^2 + d_G(v)^2\right]^a$$
$$= \left(2^2+3^2\right)^a 4p + \left(3^2+3^2\right)^a\left(3pq-2p\right)$$
$$= 3\times 18^a\, pq + \left(4\times 13^a - 2\times 18^a\right)p.$$

We establish the following results by using Theorem 4.

Corollary 4.1: The first F-index of $TUZC_6\,[p, q]$ is
$F_1(TUZC_6\,[p, q]) = 54pq + 16p.$

Proof: Put $a = 1$ in equation (9), we obtain the desired result.

Corollary 4.2: The first hyper F-Index of $TUZC_6\,[p, q]$ is
$HF_1\,(TUZC_6\,[p, q]) = 972\,pq + 28p.$

Proof: Put $a = 2$ in equation (9), we get the desired result.

Corollary 4.3: The sum connectivity F-index of $TUZC_6\,[p, q]$ is

$$SF\left(TUZC_6[p,q]\right) = \frac{1}{\sqrt{2}}pq + \left(\frac{4}{\sqrt{13}} - \frac{2}{\sqrt{18}}\right)p.$$

Proof: Put $a = -\frac{1}{2}$ in equation (9), we get the desired result.

Theorem 5: The general second F-index of $TUZC_6\,[p, q]$ is given by

$$F_2^a\left(TUZC_6[p,q]\right) = 3\times 81^a\, pq + \left(4\times 36^a - 2\times 81^a\right)p. \tag{10}$$

Proof: Let $G = TUZC_6\,[p, q]$. From equation (2) and using Table 2, we obtain

$$F_2^a\left(TUZC_6[p,q]\right) = \sum_{uv\in E(G)} \left[d_G(u)^2 d_G(v)^2\right]^a$$
$$= \left(2^2\times 3^2\right)^a 4p + \left(3^2\times 3^2\right)^a\left(3pq-2p\right)$$
$$= 3\times 81^a pq + \left(4\times 36^a - 2\times 81^a\right)p.$$

The following results are obtained by using Theorem 5.

Corollary 5.1: The second F-index of $TUZC_6\,[p, q]$ is given by
$F_2(TUZC_6\,[p, q]) = 243pq - 18p.$

Proof: Put $a = 1$ in equation (10), we have the desired result.

Corollary 5.2: The second hyper F-index of $TUZC_6\,[p, q]$ is given by
$HF_2\,(TUZC_6\,[p, q]) = 19683pq - 7938p.$

Proof: Put $a = 2$ in equation (10), we obtain the desired result.

Corollary 5.3: The product connectivity F-index of $TUZC_6\,[p, q]$ is

$$PF\left(TUZC_6[p,q]\right) = \frac{1}{3}pq + \frac{4}{9}p.$$

Proof: Put $a = -\frac{1}{2}$ in equation (10), we get the desired result.

Theorem 6: Let $G = TUZC_6\,[p, q]$ be a zigzag polyhex nanotube. Then

(i) $F_1\left(TUZC_6[p,q],x\right) = 4px^{13} + \left(3pq-2p\right)x^{18}.$

(ii) $F_2\left(TUZC_6[p,q],x\right) = 4px^{36} + \left(3pq-p\right)x^{81}.$

(iii) $HF_1\left(TUZC_6[p,q],x\right) = 4px^{169} + \left(3pq-2p\right)x^{324}.$

(iv) $HF_2\left(TUZC_6[p,q],x\right) = 4px^{1296} + \left(3pq-2p\right)x^{6561}.$

Proof: Let $G = TUZC_6\,[p, q]$.

(i) By using equation (3) and Table 2, we deduce

$$F_1\left(TUZC_6[p,q],x\right) = \sum_{uv\in E(G)} x^{d_G(u)^2 + d_G(v)^2}$$
$$= 4px^{2^2+3^2} + \left(3pq-2p\right)x^{3^2+3^2}$$
$$= 4px^{13} + \left(3pq-p\right)x^{18}.$$

(ii) From equation (4) and by using Table 2, we obtain

$$F_2\left(TUZC_6[p,q],x\right) = \sum_{uv\in E(G)} x^{d_G(u)^2 d_G(v)^2}$$
$$= 2px^{2^2\times 3^2} + \left(3pq-2p\right)x^{3^2\times 3^2}$$
$$= 4px^{36} + \left(3pq-2p\right)x^{81}.$$

(iii) By using equation (5) and Table 2, we derive

$$HF_1\left(TUZC_6[p,q],x\right) = \sum_{uv\in E(G)} x^{\left[d_G(u)^2 + d_G(v)^2\right]^2}$$
$$= 4px^{\left(2^2+3^2\right)^2} + \left(3pq-2p\right)x^{\left(3^2+3^2\right)^2}$$
$$= 4px^{169} + \left(3pq-2p\right)x^{324}$$

(iv) From equation (6) and by using Table 2, we have

$$HF_2\left(TUZC_6[p,q],x\right)=\sum_{uv\in E(G)} x^{\left[d_G(u)^2 d_G(v)^2\right]^2}$$

$$=4px^{\left(2^2\times 3^2\right)^2}+\left(3pq-2p\right)x^{\left(3^2\times 3^2\right)^2}$$

$$=4px^{1296}+\left(3pq-2p\right)x^{6561}.$$

4. CARBON NANOCONE NETWORKS

An n-dimensional one-pentagonal nanocone is denoted by CNC_5 [n], where n is the number of hexagons layers encompassing the conical surface of the nanocone and 5 denotes that there is a pentagon on the tip called its core. A 6-dimensional one-pentagonal nanocone network is depicted in Figure 3.

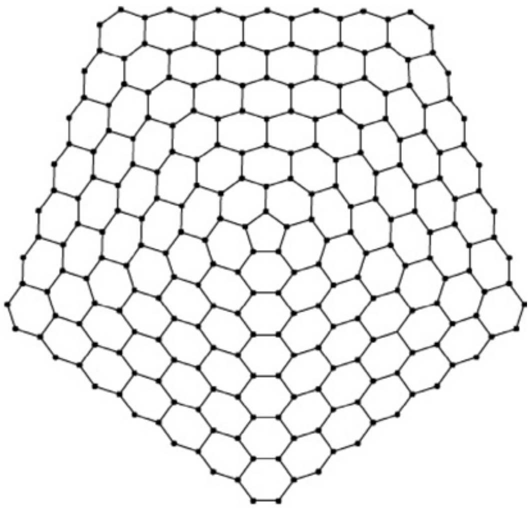

Figure-3: A 6-dimensional one-pentagonal nanocone network

Let G be an n-dimensional one-pentagonal nanocone network $CNC_5[n]$, $n \geq 2$. Then G has $5(n+1)^2$ vertices and $\frac{15}{2}n^2+\frac{25}{2}n+5$ edges. In G, there are three types of edges based on degrees of end vertices of each edge. By algebraic method, this edge partition is given in Table 3.

$d_G(u), d_G(v) \setminus uv \in E(G)$	(2, 2)	(2, 3)	(3, 3)
Number of edges	5	$10n$	$\frac{15}{2}n^2+\frac{5}{2}n$

Table-3: Edge partition of $CNC_5[n]$

In the following theorem, we compute the general first F-index of $CNC_5[n]$.

Theorem 7: The general first F-index of $CNC_5[n]$ is given by

$$F_1^a\left(CNC_6[n]\right)=\frac{15}{2}\times 18^a\times n^2+\left(10\times 13^a+\frac{15}{2}\times 18^a\right)n+5\times 8^a. \tag{11}$$

Proof: Let $G = CNC_5[n]$, By using equation (1) and Table 3, we derive

$$F_1^a\left(CNC_5[n]\right)=\sum_{uv\in E(G)}\left[d_G(u)^2+d_G(v)^2\right]^a$$

$$=\left(2^2+2^2\right)^a 5+\left(2^2+3^2\right)^a 10n+\left(3^2+3^2\right)^a\left(\frac{15}{2}n^2+\frac{5}{2}n\right)$$

$$=\frac{15}{2}\times 18^a\times n^2+\left(10\times 13^a+\frac{15}{2}\times 18^a\right)n+5\times 8^a.$$

The following results are obtained by using Theorem 7.

Corollary 7.1: The first F-index of $CNC_5[n]$ is given by

$$F_1(CNC_5[n]) = 135n^2 + 175n + 40.$$

Proof: put $a = 1$ in equation (11), we get the desired result.

Corollary 7.2: The first hyper F-index of $CNC_5[n]$ is given by

$$HF_1(CNC_5[n]) = 2430n^2 + 2500n + 320.$$

Proof: Put $a = 2$ in equation (11), we obtain the desired result.

Corollary 7.3: The sum connectivity F-index of $CNC_5[n]$ is given by

$$SF\left(CNC_5[p,q]\right) = \frac{5}{2\sqrt{2}}n^2 + \left(\frac{10}{\sqrt{13}} + \frac{6}{6\sqrt{2}}\right)n. + \frac{5}{2\sqrt{2}}$$

Proof: Put $a = -\frac{1}{2}$ in equation (11), we get the desired result.

In the following theorem, we determine the general second F-index of $CNC_5[n]$.

Theorem 8: The general second F-index of $CNC_5[n]$ is given by

$$F_2^a\left(CNC_5[n]\right) = \frac{15}{2} \times 18^a \times n^2 + \left(10 \times 36^a + \frac{5}{2} \times 81^a\right)n + 5 \times 8^a. \tag{12}$$

Proof: Let $G = CNC_5[n]$. By using equation (2) and Table 3, we deduce

$$F_1^a\left(CNC_5[n]\right) = \sum_{uv \in E(G)} \left[d_G(u)^2 d_G(v)^2\right]^a$$

$$= \left(2^2 \times 2^2\right)^a 5 + \left(2^2 \times 3^2\right)^a 10n + \left(3^2 \times 3^2\right)^a \left(\frac{15}{2}n^2 + \frac{5}{2}n\right)$$

$$= \frac{15}{2} \times 81^a \times n^2 + \left(10 \times 36^a + \frac{5}{2} \times 81^a\right)n + 5 \times 8^a.$$

We obtain the following results by using Theorem 8.

Corollary 8.1; The second F-index of $CNC_5[n]$ is given by

$$F_2\left(CNC_5[n]\right) = \frac{1215}{2}n^2 + \frac{1125}{2}n + 40.$$

Proof: Put $a = 1$ in equation (12), we get the desired result.

Corollary 8.2: The second hyper F-index of $CNC_5[n]$, is given by

$$HF_2\left(CNC_5[n]\right) = \frac{98415}{2}n^2 + \frac{32805}{2}n + 320.$$

Proof: Put $a = 2$ in equation (12), we obtain the desired result.

Corollary 8.3: The product connectivity F-index of $CNC_5[n]$, is

$$PF\left(CNC_5[n]\right) = \frac{5}{6}n^2 + \frac{35}{18}n + \frac{5}{\sqrt{8}}.$$

Proof: Put $a = -\frac{1}{2}$ in equation (12), we obtain the desired result.

Theorem 9: Let $G = CNC_5[n]$ be a carbon nanocone network. Then

(i) $F_1\left(CNC_5[n], x\right) = 5x^8 + 10nx^{13} + \left(\frac{15}{2}n^2 + \frac{5}{2}n\right)x^{18}.$

(ii) $F_2\left(CNC_5[n], x\right) = 5x^{16} + 10nx^{36} + \left(\frac{15}{2}n^2 + \frac{5}{2}n\right)x^{81}.$

(iii) $HF_1\left(CNC_5[n],x\right)=5x^{64}+10nx^{169}+\left(\frac{15}{2}n^2+\frac{5}{2}n\right)x^{324}.$

(iv) $HF_2\left(CNC_5[n],x\right)=5x^{256}+10nx^{1296}+\left(\frac{15}{2}n^2+\frac{5}{2}n\right)x^{6561}.$

Proof: Let $G = CNC_5[n]$.

(i) From equation (3) and Table 3, we obtain

$$\begin{aligned}F_1\left(CNC_5[n],x\right)&=\sum_{uv\in E(G)}x^{d_G(u)^2+d_G(v)^2}\\&=5x^{2^2+2^2}+10nx^{2^2+3^2}+\left(\frac{15}{2}n^2+\frac{5}{2}n\right)x^{3^2+3^2}\\&=5x^{8}+10nx^{13}+\left(\frac{15}{2}n^2+\frac{5}{2}n\right)x^{18}.\end{aligned}$$

(ii) By using equation (4) and Table 3, we deduce

$$\begin{aligned}F_2\left(CNC_5[n],x\right)&=\sum_{uv\in E(G)}x^{d_G(u)^2d_G(v)^2}\\&=5x^{2^2\times2^2}+10nx^{2^2\times3^2}+\left(\frac{15}{2}n^2+\frac{5}{2}n\right)x^{3^2\times3^2}\\&=5x^{16}+10nx^{36}+\left(\frac{15}{2}n^2+\frac{5}{2}n\right)x^{81}.\end{aligned}$$

(iii) From equation (5) and by using Table 3, we derive

$$\begin{aligned}HF_1\left(CNC_5[p,q],x\right)&=\sum_{uv\in E(G)}x^{\left[d_G(u)^2+d_G(v)^2\right]^2}\\&=5x^{\left(2^2+2^2\right)^2}+10nx^{\left(2^2+3^2\right)^2}+\left(\frac{15}{2}n^2+\frac{5}{2}n\right)x^{\left(3^2+3^2\right)^2}\\&=5x^{64}+10nx^{169}+\left(\frac{15}{2}n^2+\frac{5}{2}n\right)x^{324}.\end{aligned}$$

(iv) From equation (6) and by using Table 3, we have

$$\begin{aligned}HF_2\left(CNC_5[n],x\right)&=\sum_{uv\in E(G)}x^{\left[d_G(u)^2d_G(v)^2\right]^2}\\&=5x^{\left(2^2\times2^2\right)^2}+10nx^{\left(2^2\times3^2\right)^2}+\left(\frac{15}{2}n^2+\frac{5}{2}n\right)x^{\left(3^2\times3^2\right)^2}\\&=5x^{256}+10nx^{1296}+\left(\frac{15}{2}n^2+\frac{5}{2}n\right)x^{6561}.\end{aligned}$$

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