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# MORE ON δgβ-IRRESOLUTE FUNCTIONS IN TOPOLOGICAL SPACES AND RELATED GROUPS

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***ABSTRACT***

***A*** *function f; $X \to Y$ is said to be δgβ-irresolute if the inverse image of every δgβ- closed set in Y is δgβ-closed set in X. Some properties of these functions were obtained and relations with group theory has been studied.*

***Key words:*** *gβ-irresolute, δgβ-irresolute, homeomorphism group.*

***AMS Subject classification:*** *54C08.*

## 1. INTRODUCTION

Throughout the present paper, X and Y denote topological spaces. Let A be a subset of X. We denote the interior and closure of A by Int(A) and Cl(A) respectively.

A subset A of a topological space X is said to be β-open [1] or semi-preopen [3] (if $A \subseteq Cl(Int(Cl(A)))$. The complement of β-open set is β-closed. The intersection of all β-closed sets containing A is called β-closure [2]) of A and is denoted by βCl(A). Further A is said to be regular open if A=Int(Cl(A)) and it is said to be regular closed if A=Cl(Int(A)). It is said to be π-open[10] if it is finite union of regular open sets and δ-open [9] if for each $x \in A$, there exists a regular open set V such that $x \in V \subset A$. Every π-open set is δ-open. Also δ-closure [9] of A, denoted by δCl(A) is defined to be the set of all $x \in X$ such that $A \cap Int(Cl(U)) \neq \phi$ for every open neighbourhood U of x. If A=δCl(A), then A is called δ-closed. The complement of δ-closed set is δ-open. Also, A is said to be generalized semi-preclosed [6] (briefly.gsp-closed) or gβ-closed (resp.πgβ-closed [8], δgβ-closed [5]) if $\beta Cl(A) \subseteq U$, whenever $A \subseteq U$ and U is open (resp π-open, δ-open) in X.

## 2. PRELIMINARIES

**Definition 2.1:** A function f: $X \to Y$ is said to be πgβ-irresolute [8] (resp. δgβ-irresolute [5]) if the inverse image of every πgβ-closed (resp.δgβ-closed) set in Y is πgβ-closed(resp. δgβ-closed) set in X.

**Remark 2.1:** Every πgβ-irresolute function is δgβ-irresolute but not conversely as can be seen from the following example which is example 4.6 of [7]:

**Example 2.1:** Let (X,τ) be the Moore plane (also known as Niemytzki plane). Set S={(x, y): x is irrational and y=0}. Let A={ (x, y) : y<2} - S and let B = { (x, y) : $x^2+(y-4)^2=1$}. Let σ be the topology on the upper half plane generated by A and B. Now consider the identity function f: $(X,\tau) \to (X,\sigma)$.Note that in (X,τ), B is regular open and A is union of regular open sets, that is, A can be represented as the union of all open balls of radius 1 tangent to the x-axis at the rational along with corresponding rational. Note that every such set is regular open. Thus f is δgβ-irresolute. But A is regular open in (X,σ) and there is no way A can be represented as finite union of regular sets of (X,τ).Thus f is not πgβ-irresolute function.

**Theorem 2.1:** Every homeomorphism is δgβ -irresolute

**Proof:** It is obvious from the fact that every homeomorphism is πgβ-irresolute function ([8], Theorem 2.3 (iv)) and Remark 2.1.

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**Definition 2.2:** A function f: $(X,\tau)\rightarrow(Y,\sigma)$ is called $\delta g\beta c$-homeomorphism if f is a $\delta g\beta$-irresolute and $f^{-1}$ is $\delta g\beta$-irresolute.

For a topological space $(X,\tau)$, we introduce the following:

$h(X;\tau)= \{f \mid f: (X,\tau) \rightarrow(X,\tau)$ is a homeomorphism$\}$, $\delta g\beta ch(X;\tau)= \{f \mid f: (X,\tau) \rightarrow(X,\tau)$ is a $\delta g\beta c$-homeomorphism$\}$.

**Theorem 2.2:** For a topological space $(X,\tau)$, $h(X;\tau) \subseteq \delta g\beta ch(X;\tau)$.

**Proof:** Let $f\in h(X;\tau)$.Then by Theorem 2.1 and Definition 2.2, it is shown that f and $f^{-1}$ are $\delta g\beta c$-homeomorphism, that is, $f\in \delta g\beta ch(X;\tau)$.

**Theorem 2.3:** The collection $\delta g\beta ch(X;\tau)$ forms a group under the composition of functions.

**Proof:** A binary operation $n_X :\delta g\beta ch(X;\tau) \times \delta g\beta ch(X;\tau) \rightarrow\delta g\beta ch(X;\tau)$ is well defined by $n_X(a,b)$ = boa, where boa: $X\rightarrow X$ is a composite function of the functions a and b such that $(boa)(x) = b(a(x))$ for every point $x\in X$. Indeed by ([5], Theorem 4.12 (iii)), it is shown that for every $\delta g\beta c$-homeomorphisms a and b, the composition boa is also $\delta g\beta c$-homeomorphism. Namely, for every pair $(a,b)\in \delta g\beta ch(X;\tau)$, $n_X(a,b) = boa\in \delta g\beta ch(X;\tau)$.Then it is claimed that the binary operation $n_X : \delta g\beta ch((X;\tau) \times \delta g\beta ch(X;\tau) \rightarrow \delta g\beta ch(X;\tau)$ satisfies the axiom of group, namely, putting $a.b= n_X(a,b)$, the following properties hold in $\delta g\beta ch(X;\tau)$:

(1) $((a.b).c)=(a.(b.c))$ holds for every a, b, c $\in \delta g\beta ch((X;\tau)$.
(2) for all $a \in \delta g\beta ch((X;\tau)$ ,there exists an element $e \in \delta g\beta ch(X;\tau)$ such that a.e=a=e.a hold in $\delta g\beta ch(X;\tau)$.
(3) for each element $a\in \delta g\beta ch(X;\tau)$, there exists an element $a_1\in \delta g\beta ch(X;\tau)$ such that $a.\ a_1=e= a_1.a$ hold in $\delta g\beta ch(X,\tau)$.

Indeed, the proof of (1) is obvious, the proof of (2) is obtained by taking $e= 1_X$, where $1_X$ is the identity function on X and using the fact that identity function is always $\delta g\beta c$-irresolue. Proof of (3) is obtained by taking $a_1=a^{-1}$ for each $a\in \delta g\beta ch(X;\tau)$ and Definition 2.2, where $a^{-1}$ is inverse of a. Therefore by definition of groups, the pair $(\delta g\beta ch(X;\tau), n_X)$ forms a group under the compositions of functions.

**Theorem 2.4:** The homeomorphism group $h(X;\tau)$ is a subgroup of the group $\delta g\beta ch(X;\tau)$.

**Proof:** It is obvious that $1_X$: $(X,\tau) \rightarrow (X, \tau)$ is a homeomorphism and so $h(X;\tau)\neq \emptyset$.It follows by Theorem 2.2 that $h(X;\tau) \subseteq \delta g\beta ch(X;\tau)$. Let $a,b\in h(X;\tau)$.Then we have $n_X (a,b^{-1})= b^{-1}oa\in h(X;\tau)$, where $n_X$: $\delta g\beta ch(X;\tau)\times\delta g\beta ch(X;\tau) \rightarrow\delta g\beta ch(X;\tau)$ is a binary operation.(Theorem 2.3).Therefore, the group $h(X;\tau)$ is a subgroup of $\delta g\beta ch(X;\tau)$.

**Theorem 2.5:** If $(X, \tau)$ and $(Y, \sigma)$ are homeomorphic, then $\delta g\beta ch(X,\tau) \cong \delta g\beta ch(Y, \sigma)$.

**Proof:** It follows from the assumption that there exist a homeomorphism say f: $(X, \tau) \rightarrow (Y, \sigma)$.We define a function $f^*$: $\delta g\beta ch(X,\tau) \rightarrow \delta g\beta c(Y, \sigma)$ by $f^*(a)=foaof^{-1}$ for every $a\in \delta g\beta ch(X,\tau)$.By Theorem 2.2, the bijections $foaof^{-1}$ and $(foao\ f^{-1})^{-1}$ are $\delta g\beta$-irresolute and so is $f^*$ is well defined. The induced function $f^*$ is a homomorphism. Indeed $f^*(n_X(a, b)) = fobof^{-1}ofoaof^{-1}= (f^*(b))o(f^*(a))= n_X(f^*(b), f^*(a))$ hold. Obviously $f^*$ is bijective. Thus $f^*$ is isomorphism.

**Definition 2.3:** A function f: $X\rightarrow Y$ is said to be contra $\pi g\beta$-irresolute [4] (resp.contra-$\delta g\beta$-irresolute) if the inverse image of every $\pi g\beta$-open (resp.$\delta g\beta$-open) set in Y is $\pi g\beta$ -closed(resp. $\delta g\beta$-closed) set in X.

**Definition 2.4:** For a topological space $(X,\tau)$, we define the following collection of functions:

con-$\delta g\beta ch(X;\tau)= \{f \mid f: (X,\tau) \rightarrow(X,\tau)$ is a contra-$\delta g\beta$-irresolute bijection and $f^{-1}$ is contra-$\delta g\beta$-irresolute$\}$.

**Remark 2.2:** If f and g are contra-$\delta g\beta$-irresolute, then so is fog.

**Remark 2.3:** If f is $\delta g\beta$-irresolute and g is contra-$\delta g\beta$-irresolute, then gof is contra-$\delta g\beta$-irresolute.

**Theorem 2.6:** The union of two collections, $\delta g\beta ch(X;\tau)\cup$con-$\delta g\beta ch(X;\tau)$ forms a group under the composite of functions.

**Proof:** Let $B_X = \delta g\beta ch(X;\tau)\cup$ con-$\delta g\beta ch(X;\tau)$. A binary operation $W_X$: $B_X \times B_X \rightarrow B_X$ is well defined by $W_X(a,b)$ =boa where, boa:$X\rightarrow X$ is a composite function of functions a and b. Indeed let $(a, b) \in B_X$; if $a\in \delta g\beta ch(X;\tau)$ and $b\in$ con-$\delta g\beta ch(X;\tau)$, then boa:$(X, \tau) \rightarrow (X,\tau)$ is a contra $\delta$-$\beta$-irresolute bijection and $(boa)^{-1}$ is also contra $\delta$-$\beta$-irresolute and so $W_X(a,b)$ =boa$\in \delta g\beta ch(X;\tau)\subseteq B_X$ (Remark 2.3). If a, $b\in$ con-$\delta g\beta ch(X;\tau)$, then boa: $(X,\tau) \rightarrow (X,\tau)$ is a con-$\delta g\beta$ irresolute bijection and so $a\in$con-$\delta g\beta ch(X;\tau)\subseteq B_X$ (By Remark 2.2). If $a,b\in \delta g\beta ch(X;\tau)$, then boa: $(X,\tau) \rightarrow (X,\tau)$ is a $\delta g\beta$ irresolute bijection and so $a\in \delta g\beta ch(X;\tau)\subseteq B_X$ (By Remark 2.3). By similar arguments of Theorem 2.3, it is claimed that binary operation $W_X$: $B_X \times B_X \rightarrow B_X$ satisfies the axiom of group, for the identity element e of

$B_X$, $e = 1_X$: $(X,\tau) \rightarrow (X,\tau)$ (the identity function).Thus, the pair ($B_X$, $W_X$) forms a group under the composite of functions, i.e, $\delta g\beta ch(X;\tau) \cup$ con-$\delta g\beta ch(X;\tau)$ is a group.

**Theorem 2.7:** The group $\delta g\beta ch(X;\tau)$ is a subgroup of $\delta g\beta ch(X;\tau) \cup$ con-$\delta g\beta ch(X;\tau)$).

**Proof:** The group $\delta g\beta ch(X;\tau)$ is non empty from Remark 2.2.Using the binary operation in Theorem 2.6, it is shown that $W_X$ (a, $b^{-1}$)= $b^{-1}$oa $\in \delta g\beta ch(X;\tau)$ for any a,b$\in \delta g\beta ch(X;\tau)$and so $\delta g\beta ch(X;\tau)$ is a subgroup of $\delta g\beta ch(X;\tau) \cup$ con-$\delta g\beta ch(X;\tau)$.

**Theorem 2.8:** If (X, $\tau$) and (Y, $\sigma$) are homeomorphic, then there exists isomorphisms: $\delta g\beta ch(X;\tau) \cup$ con-$\delta g\beta ch(X;\tau)$.$\cong \delta g\beta ch(Y;\sigma) \cup$con-$\delta g\beta ch(Y;\sigma)$.

**Proof:** Let f:(X, $\tau$) $\rightarrow$ (Y, $\sigma$) be a homeomorphism. We put $B_X = \delta g\beta ch(X;\tau) \cup$ con- $\delta g\beta ch(X;\tau)$ (resp. $B_Y = \delta g\beta ch(Y;\sigma)$. $\cup$con-$\delta g\beta ch(Y;\sigma)$. For a topological space (X, $\tau$) (resp. (Y,$\sigma$)). First we have a well defined function f*: $B_X \rightarrow B_Y$ by f* (a)=foaof $^{-1}$ for every a$\in B_X$. Indeed by Theorem 2.2, f and f $^{-1}$ are $\delta g\beta$- irresolute ,the bijections foaof $^{-1}$ and (foaof $^{-1})^{-1}$ are $\delta g\beta$-irresolute or contra $\delta g\beta$-irresolute and so f* is well defined. The induced function f* is a homomorphism. Indeed, f*($W_X$(a,b))=fobo f $^{-1}$ofoao f $^{-1}$=(f* (b))o(f* (a))= $W_Y$(f* (a), f* (a))) hold. $W_X$: $B_X \times B_X \rightarrow B_X$ and $W_Y$: $B_Y \times B_Y \rightarrow B_Y$ are binary operations defined in Theorem 2.5, obviously f* is bijective. Thus we have the isomorphism .Also since identity function is $\delta g\beta$-irresolute, f*($1_X$) = $1_Y$ holds.

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