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MORE ON δgβ-IRRESOLUTE FUNCTIONS IN TOPOLOGICAL SPACES AND RELATED GROUPS

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ABSTRACT

***A** function f; X→Y is said to be δgβ-irresolute if the inverse image of every δgβ- closed set in Y is δgβ-closed set in X. Some properties of these functions were obtained and relations with group theory has been studied.*

Key words: *gβ-irresolute, δgβ-irresolute, homeomorphism group.*

AMS Subject classification: *54C08.*

1. INTRODUCTION

Throughout the present paper, X and Y denote topological spaces. Let A be a subset of X. We denote the interior and closure of A by Int(A) and Cl(A) respectively.

A subset A of a topological space X is said to be β-open [1] or semi-preopen [3] (if A⊆Cl(Int(Cl(A))). The complement of β-open set is β-closed. The intersection of all β-closed sets containing A is called β-closure [2]) of A and is denoted by βCl(A). Further A is said to be regular open if A=Int(Cl(A)) and it is said to be regular closed if A=Cl(Int(A)). It is said to be π-open[10] if it is finite union of regular open sets and δ-open [9] if for each x∈A, there exists a regular open set V such that x∈ V ⊂A. Every π-open set is δ-open. Also δ-closure [9] of A, denoted by δCl(A) is defined to be the set of all x∈X such that A∩Int(Cl(U))≠ϕ for every open neighbourhood U of x. If A=δCl(A), then A is called δ-closed. The complement of δ-closed set is δ-open. Also, A is said to be generalized semi-preclosed [6] (briefly.gsp-closed) or gβ-closed (resp.πgβ-closed [8], δgβ-closed [5]) if βCl(A) ⊆U, whenever A⊆U and U is open (resp π-open, δ-open) in X.

2. PRELIMINARIES

Definition 2.1: A function f: X→Y is said to be πgβ-irresolute [8] (resp. δgβ-irresolute [5]) if the inverse image of every πgβ-closed (resp.δgβ-closed) set in Y is πgβ-closed(resp. δgβ-closed) set in X.

Remark 2.1: Every πgβ-irresolute function is δgβ-irresolute but not conversely as can be seen from the following example which is example 4.6 of [7]:

Example 2.1: Let (X,τ) be the Moore plane (also known as Niemytzki plane). Set S={(x, y): x is irrational and y=0}. Let A={ (x, y) : y<2} - S and let B = { (x, y) : $x^2+(y-4)^2=1$}. Let σ be the topology on the upper half plane generated by A and B. Now consider the identity function f: (X,τ)→(X,σ).Note that in (X,τ), B is regular open and A is union of regular open sets, that is, A can be represented as the union of all open balls of radius 1 tangent to the x-axis at the rational along with corresponding rational. Note that every such set is regular open. Thus f is δgβ-irresolute. But A is regular open in (X,σ) and there is no way A can be represented as finite union of regular sets of (X,τ).Thus f is not πgβ-irresolute function.

Theorem 2.1: Every homeomorphism is δgβ -irresolute

Proof: It is obvious from the fact that every homeomorphism is πgβ-irresolute function ([8], Theorem 2.3 (iv)) and Remark 2.1.

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Definition 2.2: A function f: $(X,\tau)\to(Y,\sigma)$ is called δgβc-homeomorphism if f is a δgβ-irresolute and f^{-1} is δgβ-irresolute.
For a topological space (X,τ), we introduce the following:
$h(X;\tau)=\{f \mid f: (X,\tau)\to(X,\tau)$ is a homeomorphism$\}$, $\delta g\beta ch(X;\tau)=\{f \mid f: (X,\tau)\to(X,\tau)$ is a δgβc-homeomorphism$\}$.

Theorem 2.2: For a topological space (X,τ), $h(X;\tau)\subseteq \delta g\beta ch(X;\tau)$.

Proof: Let $f\in h(X;\tau)$.Then by Theorem 2.1 and Definition 2.2, it is shown that f and f^{-1} are δgβc-homeomorphism, that is, $f\in \delta g\beta ch(X;\tau)$.

Theorem 2.3: The collection $\delta g\beta ch(X;\tau)$ forms a group under the composition of functions.

Proof: A binary operation $n_X:\delta g\beta ch(X;\tau)\times\delta g\beta ch(X;\tau)\to\delta g\beta ch(X;\tau)$ is well defined by $n_X(a,b) = boa$, where boa: $X\to X$ is a composite function of the functions a and b such that $(boa)(x) = b(a(x))$ for every point $x\in X$. Indeed by ([5], Theorem 4.12 (iii)), it is shown that for every δgβc-homeomorphisms a and b, the composition boa is also δgβc-homeomorphism. Namely, for every pair $(a,b)\in \delta g\beta ch(X;\tau)$, $n_X(a,b) = boa\in \delta g\beta ch(X;\tau)$.Then it is claimed that the binary operation $n_X : \delta g\beta ch((X;\tau) \times \delta g\beta ch(X;\tau) \to \delta g\beta ch(X;\tau)$ satisfies the axiom of group, namely, putting $a.b= n_X(a,b)$, the following properties hold in $\delta g\beta ch(X;\tau)$:

(1) $((a.b).c)=(a.(b.c))$ holds for every a, b, $c \in \delta g\beta ch((X;\tau)$.
(2) for all $a \in \delta g\beta ch((X;\tau)$,there exists an element $e\in \delta g\beta ch(X;\tau)$ such that a.e=a=e.a hold in $\delta g\beta ch(X;\tau)$.
(3) for each element $a\in \delta g\beta ch(X;\tau)$, there exists an element $a_1\in \delta g\beta ch(X;\tau)$ such that a. a_1=e= a_1.a hold in $\delta g\beta ch(X,\tau)$.

Indeed, the proof of (1) is obvious, the proof of (2) is obtained by taking e= 1_X, where 1_X is the identity function on X and using the fact that identity function is always δgβc-irresolue. Proof of (3) is obtained by taking $a_1=a^{-1}$ for each $a\in \delta g\beta ch(X;\tau)$ and Definition 2.2, where a^{-1} is inverse of a. Therefore by definition of groups, the pair $(\delta g\beta ch(X;\tau), n_X)$ forms a group under the compositions of functions.

Theorem 2.4: The homeomorphism group $h(X;\tau)$ is a subgroup of the group $\delta g\beta ch(X;\tau)$.

Proof: It is obvious that 1_X: $(X,\tau) \to (X, \tau)$ is a homeomorphism and so $h(X;\tau)\neq \emptyset$.It follows by Theorem 2.2 that $h(X;\tau) \subseteq \delta g\beta ch(X;\tau)$. Let $a,b\in h(X;\tau)$.Then we have $n_X (a,b^{-1})= b^{-1}oa\in h(X;\tau)$, where n_X: $\delta g\beta ch(X;\tau)\times\delta g\beta ch(X;\tau) \to\delta g\beta ch(X;\tau)$ is a binary operation.(Theorem 2.3).Therefore, the group $h(X;\tau)$ is a subgroup of $\delta g\beta ch(X;\tau)$.

Theorem 2.5: If (X, τ) and (Y, σ) are homeomorphic, then $\delta g\beta ch(X,\tau) \cong \delta g\beta ch(Y, \sigma)$.

Proof: It follows from the assumption that there exist a homeomorphism say f: $(X, \tau) \to (Y, \sigma)$.We define a function f^*: $\delta g\beta ch(X,\tau) \to \delta g\beta c(Y, \sigma)$ by $f^*(a)=foaof^{-1}$ for every $a\in \delta g\beta ch(X,\tau)$.By Theorem 2.2, the bijections $foaof^{-1}$ and $(foao\ f^{-1})^{-1}$ are δgβ-irresolute and so is f* is well defined. The induced function f* is a homomorphism. Indeed $f^*(n_X(a, b)) = fobof^{-1}ofoaof^{-1}= (f^*(b))o(f^*(a))= n_X(f^*(b), f^*(a))$ hold. Obviously f* is bijective. Thus f* is isomorphism.

Definition 2.3: A function f: $X\to Y$ is said to be contra πgβ-irresolute [4] (resp.contra-δgβ-irresolute) if the inverse image of every πgβ-open (resp.δgβ-open) set in Y is πgβ -closed(resp. δgβ-closed) set in X.

Definition 2.4: For a topological space (X,τ), we define the following collection of functions:
$\text{con-}\delta g\beta ch(X;\tau)=\{f \mid f: (X,\tau)\to(X,\tau)$ is a contra-δgβ-irresolute bijection and f^{-1} is contra-δgβ-irresolute$\}$.

Remark 2.2: If f and g are contra-δgβ-irresolute, then so is fog.

Remark 2.3: If f is δgβ-irresolute and g is contra-δgβ-irresolute, then gof is contra-δgβ-irresolute.

Theorem 2.6: The union of two collections, $\delta g\beta ch(X;\tau)\cup \text{con-}\delta g\beta ch(X;\tau)$ forms a group under the composite of functions.

Proof: Let $B_X = \delta g\beta ch(X;\tau)\cup \text{con-}\delta g\beta ch(X;\tau)$. A binary operation W_X: $B_X \times B_X \to B_X$ is well defined by $W_X(a,b)$ =boa where, boa:$X\to X$ is a composite function of functions a and b. Indeed let $(a, b) \in B_X$; if $a\in \delta g\beta ch(X;\tau)$ and $b\in \text{con-}\delta g\beta ch(X;\tau)$, then boa:$(X,\tau) \to (X,\tau)$ is a contra δ-β-irresolute bijection and $(boa)^{-1}$ is also contra δ-β-irresolute and so $W_X(a,b)$ =boa$\in \delta g\beta ch(X;\tau)\subseteq B_X$ (Remark 2.3). If a, $b\in \text{con-}\delta g\beta ch(X;\tau)$, then boa: $(X,\tau) \to (X,\tau)$ is a con-δgβ irresolute bijection and so $a\in\text{con-}\delta g\beta ch(X;\tau)\subseteq B_X$ (By Remark 2.2). If $a,b\in \delta g\beta ch(X;\tau)$, then boa: $(X,\tau) \to (X,\tau)$ is a δgβ irresolute bijection and so $a\in \delta g\beta ch(X;\tau)\subseteq B_X$ (By Remark 2.3). By similar arguments of Theorem 2.3, it is claimed that binary operation W_X: $B_X \times B_X \to B_X$ satisfies the axiom of group, for the identity element e of

B_X, $e = 1_X: (X,\tau) \rightarrow (X,\tau)$ (the identity function).Thus, the pair (B_X, W_X) forms a group under the composite of functions, i.e, $\delta g\beta ch(X;\tau) \cup$con-$\delta g\beta ch(X;\tau)$ is a group.

Theorem 2.7: The group $\delta g\beta ch(X;\tau)$ is a subgroup of $\delta g\beta ch(X;\tau) \cup$con-$\delta g\beta ch(X;\tau)$).

Proof: The group $\delta g\beta ch(X;\tau)$ is non empty from Remark 2.2.Using the binary operation in Theorem 2.6, it is shown that W_X (a, b^{-1})= b^{-1}oa $\in \delta g\beta ch(X;\tau)$ for any $a,b \in \delta g\beta ch(X;\tau)$ and so $\delta g\beta ch(X;\tau)$ is a subgroup of $\delta g\beta ch(X;\tau) \cup$ con-$\delta g\beta ch(X;\tau)$.

Theorem 2.8: If (X, τ) and (Y, σ) are homeomorphic, then there exists isomorphisms: $\delta g\beta ch(X;\tau) \cup$ con-$\delta g\beta ch(X;\tau)$.$\cong \delta g\beta ch(Y;\sigma) \cup$con-$\delta g\beta ch(Y;\sigma)$.

Proof: Let $f:(X, \tau) \rightarrow (Y, \sigma)$ be a homeomorphism. We put $B_X = \delta g\beta ch(X;\tau) \cup$ con- $\delta g\beta ch(X;\tau)$ (resp. $B_Y = \delta g\beta ch(Y;\sigma)$. $\cup$con-$\delta g\beta ch(Y;\sigma)$. For a topological space (X, τ) (resp. (Y,σ)). First we have a well defined function f*: $B_X \rightarrow B_Y$ by f* (a)=foaof $^{-1}$ for every $a \in B_X$. Indeed by Theorem 2.2, f and f $^{-1}$ are $\delta g\beta$- irresolute ,the bijections foaof $^{-1}$ and (foaof $^{-1})^{-1}$ are $\delta g\beta$-irresolute or contra $\delta g\beta$-irresolute and so f* is well defined. The induced function f* is a homomorphism. Indeed, f*(W_X(a,b))=fobo f $^{-1}$ofoao f $^{-1}$=(f* (b))o(f* (a))= W_Y(f* (a), f* (a))) hold. W_X: $B_X \times B_X \rightarrow B_X$ and W_Y: $B_Y \times B_Y \rightarrow B_Y$ are binary operations defined in Theorem 2.5, obviously f* is bijective. Thus we have the isomorphism .Also since identity function is $\delta g\beta$-irresolute, f*(1_X) = 1_Y holds.

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