International Journal of Mathematical Archive-10(1), 2019, 72-77
Available online through www.ijma.info ISSN 2229 – 5046

IFSGB-CONNECTEDNESS IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

ANGELIN TIDY.G*[1] AND FRANCINA SHALINI.A[2]

[1]Research Scholar, Department of Mathematics, Nirmala College for Women, India.
[2]Assistant Professor, Department of Mathematics, Nirmala College for Women, India.

(Received On: 25-11-18; Revised & Accepted On: 06-01-19)

ABSTRACT

In this paper we have introduced the intuitionistic fuzzy semi generalized b-connected space sand intuitionistic fuzzy semi generalized b-extremally disconnected space. We investigated some of their properties. Also we characterized the intuitionistic fuzzy semi generalized b- super connected space.

Keywords: *Intuitionistic fuzzy topology, Intuitionistic fuzzy semi generalized b-connected space, Intuitionistic fuzzy semi generalized b- super connected space.*

1. INTRODUCTION

Zadeh [11] introduced the notion of fuzzy sets. Fuzzy topological space was introduced by Chang [4]. After that there have been a number of generalizations of this fundamental concept. Atanassov [3] introduced the notion of intuitionistic fuzzy sets. Using the notion of intuitionistic fuzzy sets, Coker [5] introduced the notion of intuitionistic fuzzy topological space. Connectedness in intuitionistic fuzzy special topological spaces was introduced by Oscag and Coker [7]. Angelin Tidy and Francina Shalini [1] introduced intuitionistic fuzzy sgb-closed sets.

In this paper we have introduced intuitionistic fuzzy semi generalized b-connected space, intuitionistic fuzzy semi generalized b-super connected space, intuitionistic fuzzy semi generalized b-strongly connected space, intuitionistic fuzzy semi generalized b-extremally disconnected space and studied their properties and characterizations.

2. PRELIMINARIES

Definition 2.1: [3] Let X be a nonempty fixed set. An intuitionistic fuzzy set (briefly IFS) A is an object of the form $A = \{\langle x, \mu(x), \nu(x)\rangle : x \in X\}$, where μ and ν are degrees of membership and non-membership of each $x \in X$, respectively, and $0 \leq \mu(x) + \nu(x) \leq 1$ for each $x \in X$. A class of all the IFS's in X is denoted as IFS(X). When there is no danger of confusion, an IFS $A = \{\langle x, \mu(x), \nu(x)\rangle : x \in X\}$ may be written as $A = \langle \mu_A, \nu_A\rangle$.

Definition 2.2: [3] Let X be a nonempty set and $A = \langle \mu_A, \nu_A\rangle$, $B = \langle \mu_B, \nu_B\rangle$ IFSs in X. Then
(1) $A \subseteq B$ if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$, for all $x \in X$,
(2) $A = B$ if $A \subseteq B$ and $B \subseteq A$,
(3) $\overline{A} = \{\langle x, \nu_A(x), \mu_A(x)\rangle : x \in X\}$,
(4) $A \cap B = \{\langle x,_A \wedge \mu_B, \nu_A \wedge \nu_B\rangle : x \in X\}$ [15],
(5) $A \cup B = \{\langle x,_A(x) \vee \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle : x \in X\}$[15].

Definition 2.3: [3] IFS's $0_\sim$ and $1_\sim$ are defined as $\tilde{0} = \{\langle x, 0, 1\rangle : x \in X\}$ and $\tilde{1} = \{\langle x, 1, 0\rangle : x \in X\}$, respectively.

Definition 2.4: [5] An intuitionistic fuzzy topology (IFT for short) on a nonempty set X is a family of IFSs in X satisfying the following axioms:
(1) $\tilde{0}, \tilde{1} \in \tau$,
(2) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,
(3) $\cup G_i \in \tau$ for any arbitrary family $\{G_i : i \in J\} \subseteq \tau$.

Corresponding Author: Angelin Tidy.G*[1],
[1]Research Scholar, Department of Mathematics, Nirmala College for Women, India.

In this case, the pair (X, τ) is called an intuitionistic fuzzy topological space (briefly, IFTS) and members of τ are called intuitionistic fuzzy open (briefly, IFO) sets. The complement $\overline{A}$ of an IFO set A is called an intuitionistic fuzzy closed (IFC) set in X. Collection of all IFO (resp., IFC) sets in IFTS X is denoted as IFO(X) (resp., IFC(X)).

Definition 2.5: [5] Let (X, τ) be an IFTS and A = $\langle \mu_A, \nu_A \rangle$ an IFS in X. Then the fuzzy interior and fuzzy closure of A are denoted and defined as

Cl A = ∩ {K: K is an IFC set in X and A ⊆ K},
Int A = ∪ {G: G is an IFO set in X and G ⊆ A}.

Proposition 2.6: [8] Let (X, τ) be an IFTS and A, B be intuitionistic fuzzy sets in X. Then the following properties hold:

(i) $cl(\overline{A}) = \overline{(int(A))}$,
(ii) $int(\overline{A}) = \overline{(cl(A))}$,
(iii) int(A) ⊆ A ⊆ cl(A).

Definition 2.7: [1] An IFS A = {⟨x, $\mu_A(x)$, $\nu_A(x)$ ⟩ / x ∈ X} in an IFTS (X, τ) is said to be

1) intuitionistic fuzzy b open set (IFbOS) if A ⊆ int(cl(A)) ∪ cl(int(A)),
2) intuitionistic fuzzy b- closed set (IFbCS) if cl(int(A)) ∩ int(cl(A)) ⊆ A,

Definition 2.8: [1] An IFS A is said to be an intuitionistic fuzzy semi generalized b-closed set (IFSGbCS) if bcl(A) ⊆ U whenever A ⊆ U and U is an IFSOS in (X,τ).

An IFS A is said to be an intuitionistic fuzzy semi generalized b-open set (IFSGbOS) in (X, τ) if the complement A^c is an IFSGbCS in (X, τ).

Definition 2.9: [1] Let (X, τ) be an IFTS and A = ⟨ x, μ_A, ν_A⟩ be an IFS in (X, τ). Then the intuitionistic fuzzy b closure of A (bcl(A)) and intuitionistic fuzzy b interior of A (bint(A)) are defined as

bint(A) = ∪ { G / G is an IFbOS in X and G ⊆ A},
bcl (A) = ∩ { K / K is an IFbCS in X and A ⊆ K }.

Definition 2.10: Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an

(1) intuitionistic fuzzy continuous (IF continuous) if $f^{-1}(B)$ is an IFOS in (X,τ) for every IFOS B in (Y,σ), [6]
(2) intuitionistic fuzzy semi generalized b-continuous (IFSGb continuous) if $f^{-1}(B)$ is an IFSGbOS in (X, τ) for every IFOS B in (Y,σ).[2]]

Definition 2.11: Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an

(1) intuitionistic fuzzy irresolute (IF irresolute) if $f^{-1}(B)$ IFOS in (X, τ) for every IFOS B in (Y,σ) , [2]
(2) intuitionistic fuzzy semi generalized b-irresolute(IFSGb irresolute) mapping if $f^{-1}(B)$ is an IFSGbCS B in (X, τ) for every IFSGbCS B in (Y, σ).[2]

3. TYPES OF IFSGB-CONNECTEDNESS IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

Definition 3.1: An IFTS (X, τ) is IFSGb-disconnected if there exists intuitionistic fuzzy sgb-open sets A and B in X, $A \neq 0_\sim$, $B \neq 0_\sim$ such that $A \cup B = 1_\sim$ and $A \cap B = 0_\sim$. If X is not IFSGb-disconnected then it is said to be IFSGb-connected.

Example 3.2: Let X ={a, b} , τ = {$0_\sim$, $1_\sim$, G} where G = {⟨x, (0.2, 0.3), (0.5, 0.4) ⟩; x∈X}, A = {⟨x, (0.1, 0.2), (0.6, 0.5) ⟩; x∈X}, B = {⟨x, (0.2, 0.2), (0.5, 0.5) ⟩; x∈X}, A and B are intuitionistic fuzzy sgb-open sets in X, $A \neq 0_\sim$, $B \neq 0_\sim$ and $A \cup B = A \neq 1_\sim$, $A \cap B = B \neq 0_\sim$. Hence X is IFSGb-connected.

Example 3.3: Let X ={a, b}, τ = {$0_\sim$, $1_\sim$, G} where G = {⟨x, (0.2, 0.3), (0.5, 0.4) ⟩; x∈X}, A = {⟨x, (0, 1), (1, 0)⟩; x∈X}, B = {⟨x, (1, 0), (0, 1)⟩; x∈X}, A and B are intuitionistic fuzzy sgb-open sets in X, $A \neq 0_\sim$, $B \neq 0_\sim$ and $A \cup B = 1_\sim$, $A \cap B = 0_\sim$. Hence X is IFSGb-disconnected.

Definition 3.4: An IFTS (X, τ) is $IFSGbC_5$-disconnected if there exists IFS A in X, which is both IFSGbOS and IFSGbCS such that $A \neq 0_\sim$, and $A \neq 1_\sim$. If X is not $IFSGbC_5$- disconnected then it is said to be $IFSGbC_5$-connected.

Example 3.5: Let X ={a, b} , τ = {$0_\sim$, $1_\sim$, G} where G ={⟨x, (0.2, 0.4), (0.7, 0.5)⟩; x∈X}, A ={⟨x, (0.6, 0.7),(0.3, 0.2)⟩; x∈X} , A is an IFSGbOS in X, But A is not IFSGbCS since cl(int(A))∩int(cl(A))⊈A, and $1_\sim \neq A \neq 0_\sim$. Thus X is $IFSGbC_5$-connected.

Example 3.6: Let X ={a, b}, τ = {0~, 1~, G} where G={⟨x, (0.2, 0.3), (0.5, 0.4) ⟩; x∈X}, A={⟨x, (0.1, 0.2), (0.6, 0.5)⟩; x∈X}, A is an IFSGbOS in X, And A is also IFSGbCS since cl(int(A))⋂int(cl(A)) =0 ⊆ A. Hence there exists an IFS A in X such that 1~≠A≠0~ which is both IFSGbOS and IFSGbCS in X. Thus X is IFSGbC_5-disconnected.

Proposition 3.7: IFSGbC_5- connectedness implies IFSGb- connectedness.

Proof: Suppose that there exists nonempty intuitionistic fuzzy SGb-open sets A and B such that A⋃B=1~ and A⋂B=0~ (IFSGb-disconnected) then $\mu_A \vee \mu_B = 1$, $\nu_A \wedge \nu_B = 0$ and $\mu_A \vee \mu_B = 0$, $\nu_A \wedge \nu_B = 1$. In other words $\overline{B} = A$. Hence A is IFSGb-clopen which implies X is IFSGbC_5-disconnected.

But the converse need not be true as shown by the following example.

Example 3.8: Let X ={a, b}, τ = {0~, 1~, G} where G = {⟨x, (0.2, 0.3), (0.5, 0.4)⟩; x∈X}, A = {⟨x, (0.2, 0.3), (0.6, 0.5)⟩; x∈X}, B = {⟨x, (0.1, 0.2), (0.5, 0.4)⟩; x∈X} A is an IFSGbOS in X, And B is an IFSGbOS in X since B ⊆ int(cl(A))⋃cl(int(A)). 1~ ≠ A⋃B = {⟨x, (0.2, 0.3), (0.5, 0.4) ⟩; x∈X}, 0~ ≠A⋂B = {⟨x, (0.1, 0.2), (0.6, 0.5)⟩; x∈X}. Hence X is IFSGb-connected. Since IFS A is both IFSGbOS and IFSGbCS in X, X is IFSGbC_5 –connected.

Proposition 3.9: Let f: (X, τ) → (Y, σ) be a IFSGb-irresolute surjection, (X, τ) is an IFSGb-connected, then (Y, σ) is IFSGb-connected.

Proof: Assume that (Y, σ) is not IFSGb-connected then there exists nonempty intuitionistic fuzzy SGb-open sets A and B in (Y, σ) such that A⋃B =1~ and A⋂B = 0~. Since f is IFSGb-irresolute mapping, C = f^{-1}(A) ≠ 0~, D = f^{-1}(B) ≠ 0~ which are intuitionistic fuzzy SGb-open sets in X. And f^{-1}(A)⋃f^{-1}(B) = f^{-1}(1~) = 1~ which implies C⋂D = 1~. f^{-1}(A)⋂ f^{-1}(B) = f^{-1}(0~) = 0~ which implies C⋂D = 0~.Thus X is IFSGb-disconnected, which is a contradiction to our hypothesis. Hence Y is IFSGb-connected.

Proposition 3.10: (X, τ) is IFSGbC_5-connected iff there exists no nonempty intuitionistic fuzzy SGb-open sets A and B in X such that A = $\overline{B}$.

Proof: Suppose that A and B are intuitionistic fuzzy SGb-open sets in X such that A ≠ 0~ ≠ B and A = $\overline{B}$. Since A = $\overline{B}$, $\overline{B}$ is an IFSGbOS and B is an IFSGbCS. And A ≠ 0~ implies B ≠ 1~. But this is a contradiction to the fact that X is IFSGbC_5-connected.

Conversely, let A be both IFSGbOS and IFSGbCS in X such that 0~ ≠ A ≠ 1~. Now take B = $\overline{A}$. B is an IFSGbOS and A ≠ 1~ which implies B = $\overline{A}$ ≠ 0~ which is a contradiction.

Definition 3.11: An IFTS (X, τ) is IFSGb-strongly connected if there exists no nonempty IFSGbCS A and B in X such that $\mu_A + \mu_B \subseteq 1$, $\nu_A + \nu_B \supseteq 1$.

In otherwords, an IFTS (X, τ) is IFSGb-strongly connected if there exists no nonempty IFSGbCS A and B in X such that A ⋂ B = 0~.

Proposition 3.12: An IFTS (X, τ) is IFSGb-strongly connected if there exists no IFSGbOS A and B in X, A ≠1~ ≠ B such that $\mu_A + \mu_B \supseteq 1$, $\nu_A + \nu_B \subseteq 1$.

Example 3.13: Let X ={a, b}, τ={0~, 1~, G} where G = {⟨x, (0.4, 0.4), (0.5, 0.4)⟩; x∈X}, A = {⟨x, (0.3, 0.4), (0.6, 0.5)⟩; x∈X}, B={⟨x, (0.2, 0.2), (0.8, 0.7)⟩; x∈X} A is an IFSGbOS in X, And B is an IFSGbOS in X since B⊆ int(cl(A))⋃cl(int(A)). $\mu_A + \mu_B \subseteq 1$, $\nu_A + \nu_B \supseteq 1$. Hence X is IFSGb-strongly connected.

Proposition 3.14: Let f: (X, τ)→ (Y, σ) be a IFSGb-irresolute surjection. If X is an IFSGb-strongly connected, then so is Y.

Proof: Suppose that Y is not IFSGb-strongly connected then there exists IFSGbCS C and D in Y such that C ≠ 0~, D ≠ 0~, C⋂D = 0~. Since f is IFSGb-irresolute, f^{-1}(C), f^{-1}(D) are IFSGbCSs in X and f^{-1}(C)⋂f^{-1}(D) = 0~, f^{-1}(C) ≠ 0~, f^{-1}(D) ≠ 0~. (If f^{-1}(C) = 0~ then f(f^{-1}(C)) = C which implies f(0~) = C. So C = 0~ a contradiction) Hence X is IFSGb-strongly disconnected, a contradiction. Thus (Y, σ) is IFSGb-strongly connected.

IFSGb-strongly connected does not imply IFSGbC_5-connected, and IFSGbC_5-connected does not imply IFSGb-strongly connected. For this purpose we see the following examples.

Example 3.15: Let X ={a, b}, $\tau = \{0_\sim, 1_\sim, G\}$ where G = {⟨x, (0.2, 0.3), (0.5, 0.4) ⟩; x∈X}, A = {⟨x, (0.1, 0.2), (0.6, 0.5) ⟩; x∈X} , B = {⟨x, (0.4, 0.5), (0.4, 0.5) ⟩; x∈X} A is an IFSGbOS in X, And B is an IFSGbOS in X since $B \subseteq int(cl(A)) \cup cl(int(A))$. $\mu_A + \mu_B \subseteq 1$, $\nu_A + \nu_B \supseteq 1$. Hence X is IFSGb-strongly connected. But X is not $IFSGbC_5$-connected, since A is both IFSGbOS and IFSGbCS in X.

Example 3.16: Let X ={a, b}, $\tau = \{0_\sim, 1_\sim, G\}$ where G = {⟨x, (0.2, 0.4), (0.7, 0.5)⟩; x∈X}, A={⟨x, (0.6, 0.7), (0.3, 0.2)⟩; x∈X}, B = {⟨x, (0.8, 0.9), (0.2, 0.1) ⟩; x∈X}, X is $IFSGbC_5$-connected. But X is not IFSGb-strongly connected since A and B are intuitionistic fuzzy SGb-open sets in X such that $\mu_A + \mu_B \supseteq 1$, $\nu_A + \nu_B \subseteq 1$.

Lemma 3.17: [10] (i) $A \cap B = 0_\sim \Rightarrow A \subseteq \overline{B}$.
(ii) $A \nsubseteq \overline{B} \Rightarrow A \cap B \neq 0_\sim$.

Definition 3.18: A and B are non-zero intuitionistic fuzzy sets in (X, τ). Then A and B are said to be
(i) IFSGb-weakly separated if $SGb\text{-}cl(A) \subseteq \overline{B}$ and $SGb\text{-}cl(B) \subseteq \overline{A}$
(ii) IFSGb-q-separated if $(SGb\text{-}cl(A)) \cap B = 0_\sim = A \cap (SGb\text{-}cl(B))$.

Definition 3.19: An IFTS (X, τ) is said to be $IFSGbC_S$-disconnected if there exists IFSGb-weakly separated non-zero intuitionistic fuzzy sets A and B in (X, τ) such that $A \cup B = 1_\sim$.

Example 3.20: Let X ={a, b}, $\tau = \{0_\sim, 1_\sim, G\}$ where G = {⟨x, (0.4, 0.3), (0.5, 0.6) ⟩; x∈X}, A = {⟨x, (1,0), (0,1) ⟩; x∈X}, B = {⟨x, (0,1), (1,0) ⟩; x∈X}, A and B are intuitionistic fuzzy SGb-open sets in X, $SGb\text{-}cl(A) \subseteq \overline{B}$ and $SGb\text{-}cl(B) \subseteq \overline{A}$. Hence A and B are IFSGb-weakly separated and $A \cup B = 1_\sim$. So X is $IFSGbC_S$-disconnected.

Definition 3.21: An IFTS (X, τ) is said to be $IFSGbC_M$-disconnected if there exists IFSGb-q-separated non-zero IFS's A and B in (X, τ) such that $A \cup B = 1_\sim$.

Example 3.22: Let X ={a, b}, $\tau=\{0_\sim, 1_\sim, G\}$ where G = {⟨x, (0.5, 0.6), (0.4, 0.3) ⟩; x∈X}, A = {⟨x, (1,0), (0,1) ⟩; x∈X}, B = {⟨x, (0,1), (1,0)⟩; x∈X},}, A and B are intuitionistic fuzzy SGb-open sets in X, $SGb\text{-}cl(A) \cap B = 0_\sim$ and $A \cap SGb\text{-}cl(B) = 0_\sim$. Which implies A and B are IFSGb-q-separated and $A \cup B = 1_\sim$. So X is $IFSGbC_M$-disconnected.

Remark 3.23: An IFTS (X, τ) is be $IFSGbC_S$-connected if and only if (X, τ) is $IFSGbC_M$-connected.

Definition 3.24: An intuitionistic fuzzy semi generalized b-open set A is called an intuitionistic fuzzy regular semi generalized b-open set if A = SGb-int(SGb-cl(A)).
The complement of an intuitionistic fuzzy regular semi generalized b-open set is called an intuitionistic fuzzy regular semi generalized b-closed set.

Definition 3.25: An IFTS (X, τ) is said to be IFSGb-super disconnected if there exists an intuitionistic fuzzy semi generalized b-open set A in X such that $0_\sim \neq A \neq 1_\sim$. X is called IFSGb-super connected if X is not IFSGb-super disconnected.

Example 3.26: Let X ={a, b} , $\tau = \{0_\sim, 1_\sim, G\}$ where G = {⟨x, (0.5, 0.4), (0.2, 0.3) ⟩; x∈X}, A = {⟨x, (1,0), (0,1) ⟩; x∈X}, B = {⟨x, (0,1), (1,0) ⟩; x∈X},}, A and B are intuitionistic fuzzy SGb-open sets in X, SGb-int(SGb-cl(A)) =A. This implies A is an intuitionistic fuzzy semi generalized b-open set in X. Hence X is an IFSGb-super disconnected.

Theorem 3.27: Let (X, τ) be an IFTS, then the following are equivalent.
(i) (X, τ) is an IFSGb-super connected space.
(ii) For every non-zero intuitionistic fuzzy regular semi generalized b-open set A, $SGb\text{-}cl(A) = 1_\sim$.
(iii) For every intuitionistic fuzzy regular semi generalized b-closed set A with $A \neq 1_\sim$, $SGb\text{-}int(A) = 0_\sim$.
(iv) There exists no intuitionistic fuzzy regular semi generalized b-open sets A and B in (X, τ) such that $A \neq 0_\sim \neq B$, $A \subseteq B^c$.
(v) There exists no intuitionistic fuzzy regular semi generalized b-open sets A and B in (X, τ) such that $A \neq 0_\sim \neq B$, $B = (SGb\text{-}cl(A))^c$, $A = (SGb\text{-}cl(B))^c$.
(vi) There exists no intuitionistic fuzzy regular semi generalized b-closed sets A and B in (X, τ) such that $A \neq 1_\sim \neq B$, $B = (SGb\text{-}int(A))^c$, $A = (SGb\text{-}int(B))^c$.

Proof:
(i) ⇒ (ii): Assume that there exists an intuitionistic fuzzy regular semi generalized b-open set A in (X, τ) such that $A \neq 0_\sim$ and $SGb\text{-}cl(A) \neq 1_\sim$. Now let $B = SGb\text{-}int(SGb\text{-}cl(A))^c$. Then B is a proper intuitionistic fuzzy regular semi generalized b-open set in (X, τ). But this is a contradiction to the fact that (X, τ) is an IFSGb-super connected space. Therefore $SGb\text{-}cl(A) = 1_\sim$.

(ii) ⇒ (iii): Let $A \neq 1_{\sim}$ be an intuitionistic fuzzy regular semi generalized b-closed set in (X, τ). If $B = A^c$, then B is an intuitionistic fuzzy regular semi generalized b-open set in (X, τ) with $B \neq 0_{\sim}$. Hence SGb-cl(B) $= 1_{\sim}$. This implies $(\text{SGb-cl}(B))^c = 0_{\sim}$. That is $\text{SGb-int}(B^c) = 0_{\sim}$. Hence $\text{SGb-int}(A) = 0_{\sim}$.

(iii) ⇒ (iv): Let A and B be two intuitionistic fuzzy regular semi generalized b-open sets in (X, τ) such that $A \neq 0_{\sim} \neq B$, $A \subseteq B^c$. Since B^c is an intuitionistic fuzzy regular semi generalized b-closed set in (X, τ) and $B \neq 0_{\sim}$ implies $B^c \neq 1_{\sim}$, $B^c = \text{SGb-cl}(\text{SGb-int}(B^c))$ and we have $\text{SGb-int}(B^c) = 0_{\sim}$. But $A \subseteq B^c$. Therefore $0_{\sim} \neq A = \text{SGb-int}(\text{SGb-cl}(A)) \subset \text{SGb-int}(\text{SGb-cl}(B^c)) = \text{SGb-int}(\text{SGb-cl}(\text{SGb-cl}(\text{SGb-int}(B^c)))) = \text{SGb-int}(\text{SGb-cl}(\text{SGb-int}(B^c))) = \text{SGb-int}(B^c) = 0_{\sim}$. A contradiction arises. Therefore (iv) is true.

(iv) ⇒ (i): Let $0_{\sim} \neq A \neq 1_{\sim}$ be an intuitionistic fuzzy regular semi generalized b-open set in (X, τ). If we take $B = (\text{SGb-cl}(A))^c$, then B is an intuitionistic fuzzy regular semi generalized b-open set, since $\text{SGb-int}(\text{SGb-cl}(B)) = \text{SGb-int}(\text{SGb-cl}(\text{SGb-cl}(A))^c) = \text{SGb-int}(\text{SGb-int}(\text{SGb-cl}(A)))^c = \text{SGb-int}(A^c) = (\text{SGb-cl}(A))^c = B$. Also we get $B \neq 0_{\sim}$, since otherwise, we have $B = 0_{\sim}$ and this implies $(\text{SGb-cl}(A))^c = 0_{\sim}$. This is $\text{SGb-cl}(A) = 1_{\sim}$. Hence $A = \text{SGb-int}(\text{SGb-cl}(A)) = \text{SGb-int}(1_{\sim}) = 1_{\sim}$. This is $A = 1_{\sim}$, which is a contradiction. Therefore $B \neq 0_{\sim}$ and $A \subseteq B^c$. But this is a contradiction to (iv). Therefore (X, τ) is an IFSGb- super connected space.

(i) ⇒ (v): Let A and B be two intuitionistic fuzzy regular semi generalized b-open sets in (X, τ) such that $A \neq 0_{\sim} \neq B$, $B = (\text{SGb-cl}(A))^c$ and $A = (\text{SGb-cl}(B))^c$. Now we have $\text{SGb-int}(\text{SGb-cl}(A)) = \text{SGb-int}(B^c) = (\text{SGb-cl}(B))^c = A$, $A \neq 0_{\sim}$ and $A \neq 1_{\sim}$, since if $A = 1_{\sim}$, then $1_{\sim} = (\text{SGb-cl}(B))^c \Rightarrow \text{SGb-cl}(B) = 0_{\sim} \Rightarrow B = 0_{\sim}$. But $B \neq 0_{\sim}$. Therefore $A \neq 1_{\sim}$) A is proper intuitionistic fuzzy regular semi generalized b-open set in (X, τ), which is a contradiction to (i). Hence (v) is true.

(v) ⇒ (i): Let A be an intuitionistic fuzzy regular semi generalized b-open set in (X, τ) such that A = SGb-nit(SGb-cl(A)) and $0_{\sim} \neq A \neq 1_{\sim}$. Now take $B = (\text{SGb-cl}(A))^c$. In this case we get $B \neq 0_{\sim}$ and B is intuitionistic fuzzy regular semi generalized open set in (X, τ), $B = (\text{SGb-cl}(A))^c$ and $(\text{SGb-cl}(B))^c = (\text{SGb-cl}(\text{SGb-cl}(A))^c)^c = \text{SGb-int}(\text{SGb-cl}(A)^c)^c = \text{SGb-int}(\text{SGb-cl}(A)) = A$. But this is a contradiction to (v). Therefore (X, τ) is an IFSGb-super connected space.

(v) ⇒ (vi): Let A and B be two intuitionistic fuzzy regular semi generalized b-closed sets in (X, τ) such that $A \neq 1_{\sim} \neq B$, $B = (\text{SGb-int}(A))^c$ and $A = (\text{SGb-int}(B))^c$. Taking $C = A^c$ and $D = B^c$, C and D become intuitionistic fuzzy regular semi generalized b-open sets in (X, τ) with $C \neq 0_{\sim} \neq D$, $D = (\text{SGb-cl}(C))^c$ and $C = (\text{SGb-cl}(D))^c$, which is a contradiction to (v). Hence (vi) is true.

(vi) ⇒ (v): can be easily proved by the similar way as in (v) ⇒ (vi).

Proposition 3.28: Let f : (X, τ) → (Y, τ) be a IFSGb-irresolute surjection. If X is an IFSGb-super connected, then so is Y.

Proof: Suppose that Y is IFSGb-super disconnected. Then there exists IFSGbOS's C and D in Y such that $C \neq 0_{\sim} \neq D$, $C \subseteq \overline{D}$. Since f is IFSGb-irresolute, $f^{-1}(C)$ and $f^{-1}(D)$ are IFSGbOSs in X and $C \subseteq \overline{D}$ implies $f^{-1}(C) \subseteq f^{-1}(\overline{D}) = f^{-1}(D)$. Hence $f^{-1}(C) \neq 0_{\sim} \neq f^{-1}(\overline{D})$ which means that X is IFSGb-super disconnected which is a contradiction.

Definition 3.29: An IFTS (X, τ) is said to be an intuitionistic fuzzy GO-connected (IFGO-connected) space if the only IFSs which are both intuitionistic fuzzy generalized open and intuitionistic fuzzy generalized closed are $0_{\sim}$ and $1_{\sim}$.

Theorem 3.30: Every IFSGb-connected space is an IFGO-connected space but not conversely.

Proof: Let (X, τ) be an intuitionistic fuzzy semi generalized b-connected space. Suppose (X, τ) is not an intuitionistic fuzzy GO-connected space, then there exists a proper IFS A which both intuitionistic fuzzy g-open and intuitionistic fuzzy g-closed in (X, τ). That is A is both intuitionistic fuzzy SGb-open and intuitionistic fuzzy SGb-closed in (X, τ). This implies that (X, τ) is not an IFSGb-connected space. This is a contradiction. Therefore (X, τ) is an IFGO-connected space.

Definition 3.31: Let (X, τ) be any IFTS. X is called IFSGb- extremally disconnected if the SGb-closure of every IFSGbOS in X is IFSGbOS.

Theorem 3.32: For an IFTS (X, τ) the following are equivalent:

(i) (X, τ) is an IFSGb-extremally disconnected space.
(ii) For each IFSGbCS A, SGb-int(A) is an IFSGbCS.
(iii) For each IFSGbOS A, SGb-cl(A) = $\overline{\text{SGb}-\text{cl}(\overline{\text{SGb}-\text{cl}(A)})}$
(iv) For each IFSGb-open sets A and B with SGb-cl(A) = $\overline{B}$, SGb-cl(A) = $\overline{\text{SGb}-\text{cl}(B)}$.

Proof:
(i) ⇒ (ii): Let A be any IFSGbCS. Then $\overline{A}$ is an IFSGbOS. So SGb-cl($\overline{A}$) = $\overline{SGb-int(A)}$ is an IFSGbOS. Thus SGb-int(A) is an IFSGbCS in (X, τ).

(ii) ⇒ (iii): Let A be an IFSGbOS. Then SGb-cl($\overline{SGb-cl(A)}$) = SGb-cl(SGb-int($\overline{A}$)). $\overline{SGb-cl(\overline{SGb-cl(A)})}$ = $\overline{SGb-cl(SGb-int(\overline{A}))}$. Since A is an IFSGbOS, $\overline{A}$ is an IFSGbCS. So by (ii) SGb-int($\overline{A}$) is an IFSGbCS.. That is SGb-cl(SGb-int($\overline{A}$)) = SGb-int($\overline{A}$). Hence $\overline{SGb-cl(SGb-int(\overline{A}))}$ = $\overline{SGb-int(\overline{A})}$ = SGb-cl(A).

(iii) ⇒ (iv): Let A and B be any two intuitionistic fuzzy SGb-open sets in (X, τ) such that SGb-cl(A) = $\overline{B}$. (iii) implies SGb-cl(A) = $\overline{SGb-cl(\overline{SGb-cl(A)})}$ = $\overline{SGb-cl(\overline{\overline{B}})}$ = $\overline{SGb-cl(B)}$.

(iv) ⇒ (i): Let A be any IFSGbOS in (X, τ). Put B = $\overline{SGb-cl(A)}$. Then SGb-cl(A) = $\overline{B}$.
Hence by (iv) SGb-cl(A) = $\overline{SGb-cl(B)}$. Therefore SGb-cl(A) is IFSGbOS in (X, τ). That is (X, τ) is an IFSGb-extremally disconnected space.

REFERENCES

1. Angelin Tidy G and Francina Shalini A, "On intuitionistic fuzzy sgb-closed sets in intuitionistic fuzzy topologicl spaces," International journal of research trends and innovation, vol. 3, issue 7, 2018.
2. Angelin Tidy G and Francina Shalini A, "On intuitionistic fuzzy sgb-continuous mappings in intuitionistic fuzzy topologicl spaces, (communicated)
3. K.T.Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems, 20(1986), 87-96.
4. C.L. Chang, Fuzzy topological spaces, J. Math. Anal. Appl., 24(1968), 182-190.
5. D. Coker, An introduction to intuitionistic fuzzy topological spaces, Fuzzy Sets and Systems, 88(1997), 81-89.
6. H. Gurcay, D. Coker and A.H. Es, On fuzzy continuity in intuitionistic fuzzy topological spaces, J. Fuzzy Math., 5(1997), 365-378.
7. S.Ozcag and D. Coker, On connectedness in intuitionistic fuzzy special topological spaces, Inter. J. Math. Math. Sci. 21 (1998) 33-40.
8. R. Renuka and V. Seenivasan, On β-Connectedness in Intuitionistic Fuzzy Topological Spaces, Gen. Math. Notes, Vol. 19, No. 1, November, 2013, pp.16-27.
9. R. Santhi and D. Jayanthi, Generalised semi-pre connectedness in intuitionistic fuzzy topological spaces, Annals of Fuzzy Mathematics and Informatics, 3(2) (2012), 243-253.
10. N. Turnali and D. Coker, Fuzzy connectedness in intuitionistic fuzzy topological spaces, Fuzzy Sets and Systems, 116(2000), 369-375.
11. L.A.Zadeh, Fuzzy Sets, Information and Control, 8(1965), 338-353.

Source of support: Nil, Conflict of interest: None Declared.