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LOGARITHMIC MEAN LABELING OF PATH RELATED GRAPHS

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ABSTRACT

A function f is called a logarithmic mean labeling of a graph $G(V,E)$ with p vertices and q edges if $f: V(G) \rightarrow \{1,2,3,\ldots,q+1\}$ is injective and the induced function $f^*: E(G) \rightarrow \{1,2,3,\ldots,q\}$ defined as

$$f^*(uv) = \left\lfloor \frac{f(v)-f(u)}{ln f(v) - ln f(u)} \right\rfloor, \; for \; all \; uv \in E(G),$$

is bijective. A graph that admits a logarithmic mean labeling is called a logarithmic mean graph. In this paper, we have discussed the logarithmic meanness of the graphs path P_n, the star graph S_n, $P_n(X_1, X_2, \ldots, X_n)$, $TW(P_n)$, the graph $P_n e S_m$,, the graph $[P_n; S_m]$, square graph of a path, total graph of a path, middle graph of a path, the graph $P(1,2,3,\ldots,n-1)$, the graph $S(P_n \circ K_1)$ and the arbitrary subdivision of S_3.

Keywords: *labeling, logarithmic mean labeling, logarithmic mean graph.*

2010 Mathematics Subject Classification: *05C78.*

1. INTRODUCTION

Throughout this paper, by a graph we mean a finite, undirected and simple graph. Let $G(V,E)$ be a graph with p vertices and q edges. For notations and terminology, we follow [4]. For a detailed survey on graph labeling we refer to [3].

Path on n vertices is denoted by P_n. $K_{1,n}$ is called a star graph and it is denoted by S_n. A tree is a connected acyclic graph. $P_n(X_1, X_2, \ldots, X_n)$ is a tree obtained from a path on n vertices by attaching X_i pendent vertices at each i^{th} vertex of the path, for $1 \leq i \leq n$. A Twig $TW(P_n), n \geq 3$ is a graph obtained from a path by attaching exactly two pendant vertices to each internal vertices of the path. If $v_1^{(i)}, v_2^{(i)}, v_3^{(i)}, \ldots, v_{m+1}^{(i)}$ and $u_1, u_2, \ldots, u_n$ be the vertices of the star graph S_m and the path P_n, then the graph $[P_n; S_m]$ is obtained from n copies of S_m and the path P_n by joining u_i with the central vertex $v_1^{(i)}$ of the i^{th} copy of S_m by means of an edge, for $1 \leq i \leq n$. The corona $G_1 \circ G_2$ is a graph obtained by taking one copy of G_1 of order p_1 and p_1 copies of G_2 and then joining the i^{th} vertex of G_1 with every vertex in the i^{th} copy of G_2. The graph $P_n \circ K_1$ is called as comb. When $G_2 = \overline{K_m}$, then the graph $G_1 \circ G_2$ is denoted as $G_1 e S_m$. Square of a graph G, denoted by G^2, has the vertex set as in G and two vertices are adjacent in G^2 if they are at a distance either 1 or 2 apart in G. The total graph $T(G)$ of a graph G is the graph whose vertex set is $V(G) \cup E(G)$ and two vertices are adjacent if and only if either they are adjacent vertices of G or adjacent edges of G or one is a vertex of G and the other one is an edge incident on it. The middle graph $M(G)$ of a graph G is the graph whose vertex set is $\{v: v \in V(G)\} \cup \{e: e \in E(G)\}$ and the edge set is $\{e_1 e_2: e_1, e_2 \in E(G)$ and e_1 and e_2 are adjacent edges of $G\}$ $\cup \{ve: v \in V(G), e \in E(G)$ and e is incident with $v\}$. For a graph G the graph $S(G)$ is obtained by subdividing each edge of G by a vertex. An arbitrary subdivision of a graph G is a graph obtained from G by a sequence of elementary subdivisions forming edges into paths through new vertices of degree 2. An arbitrary super subdivision $P(m_1, m_2, \ldots, m_{n-1})$ of a path P_n is a graph obtained by replacing each i^{th} edge of P_n by identifying its end vertices of the edge with a partition of K_{2,m_i} having 2 elements, where m_i is any positive integer.

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The concept of mean labeling was first introduced by S. Somasundaram and R. Ponraj [5] and it was developed in [6]. Similarly the concept of F-geometric mean labeling was first introduced by A. Durai Baskar *et al.* [1] and it was developed in [2].

Motivated by the works of so many authors in the area of graph labeling, we introduced a new type of labeling called logarithmic mean labeling. A function f is called a logarithmic mean labeling of a graph $G(V,E)$ if $f: V(G) \to \{1,2,3,\dots,q+1\}$ is injective and the induced function $f^*: E(G) \to \{1,2,3,\dots,q\}$ defined as

$$f^*(uv) = \left\lfloor \frac{f(v)-f(u)}{ln f(v) - ln f(u)} \right\rfloor, \ for \ all \ uv \in E(G),$$

is bijective. A graph that admits a logarithmic mean labeling is called a logarithmic mean graph.

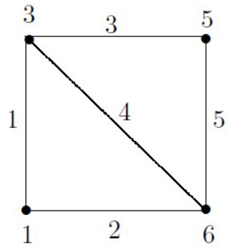

Figure-1.1: A logarithmic mean graph

In this paper, we have obtained the logarithmic meanness of the graphs path P_n, the star graph S_n, $P_n(X_1, X_2, \dots, X_n)$, $TW(P_n)$, the graph $P_n \mathrm{e} S_m$, the graph $[P_n; S_m]$, square graph of a path, total graph of a path, middle graph of a path, the graph $P(1,2,3,\dots,n-1)$, the graph $S(P_n \circ K_1)$ and the arbitrary subdivision of S_3.

2. MAIN RESULTS

Theorem 2.1: *Every path is a logarithmic mean graph.*

Proof: Let $v_1, v_2, \dots, v_n$ be the vertices of the path P_n. We define $f: V(P_n) \to \{1,2,\dots,n\}$ as follows $f(v_i) = i$, for $1 \le i \le n$. The induced edge labeling is as follows $f^*(v_i v_{i+1}) = i$, for $1 \le i \le n-1$. Hence f is a logarithmic mean labeling of the path P_n. Thus the path P_n is a logarithmic mean graph.

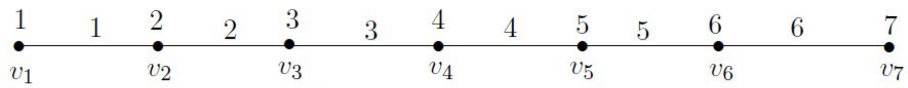

Figure-2.1: A logarithmic mean labeling of P_7.

Theorem 2.2: *Union of any two trees is not a logarithmic mean graph.*

Proof: Let G be the union of two trees S and T, then $|V(G)| = |V(S)| + |V(T)|$ and $|E(G)| = |E(S)| + |E(T)| = |V(S)| + |V(T)| - 2$. Since $|V(G)| > |E(G)| + 1$, a logarithmic mean labeling does not exist on $V(G)$.

Corollary 2.3: *Any forest is not a logarithmic mean graph.*

Proof: By the above Theorem 2.2, the result follows.

Theorem 2.4: *The star graph S_n is a logarithmic mean graph if and only if $n \le 3$.*

Proof: The number of vertices and edges of S_n are $n+1$ and n respectively. If f is a logarithmic mean labeling of S_n, then it is a bijective function from $V(S_n)$ to $\{1,2,\dots,n+1\}$. As we have to label 1 for an edge, the vertex labels of its pair of adjacent vertices are either 1 and 2 or 1 and 3. So, the central vertex of S_n is labeled as either 1 or 2 or 3. 1 can not be a label for the central vertex in case of $n \ge 2$, since two of the pendant vertices of S_n are to be labeled as 2 and 3. When $n \ge 3$, 2 cannot be the label for the central vertex, since two of its pendant vertices having the labels 3 and 4. When $n \ge 4$, the pendant vertices are labeled to be 4 and 5 if the label of central vertex is 3.

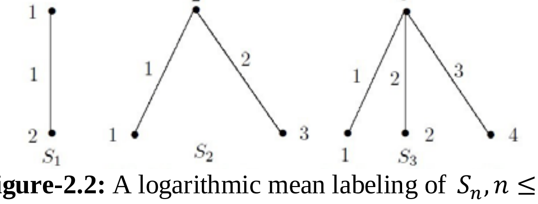

Figure-2.2: A logarithmic mean labeling of $S_n, n \le 3$.

Theorem 2.5: *$P_n(X_1,X_2,\dots,X_n)$ is a logarithmic mean graph, for $1\le X_i\le 3$ and $|X_i-X_{i+1}|\le 1$, for $1\le i\le n$.*

Proof: Let $u_1,u_2,\dots,u_n$ be the vertices of the path P_n. Let $v_1^{(i)},v_2^{(i)},\dots,v_{X_i}^{(i)}$ be the pendant vertices attached at u_i, for $1\le i\le n$.
Define $f: V(P_n(X_1,X_2,\dots,X_n))\to\{1,2,3,\dots,\sum_{i=1}^{n}X_i+n\}$ as follows:
$f(v_1^{(1)})=1, f(v_i^{(1)})=\sum_{k=1}^{i-1}X_k+i$, for $2\le i\le n$,
for $2\le i\le n$,

$$f(v_i^{(j)})=\begin{cases} f(v_i^{(1)})+2 & X_i=2 \text{ and } j=2\\ f(v_i^{(1)})+1 & X_i=3 \text{ and } j=2\\ f(v_i^{(1)})+3 & X_i=3 \text{ and } j=3\end{cases}$$

for $1\le i\le n$,

$$f(u_i)=\begin{cases} f(v_i^{(1)})+1 & X_i=1,2\\ f(v_i^{(1)})+2 & X_i=3.\end{cases}$$

The induced edge labeling is as follows:
for $1\le i\le n-1$,

$$f^*(u_iu_{i+1})=\begin{cases} f(u_i) & X_i=1\\ f(u_i)+1 & X_i=2,3,\end{cases}$$

for $1\le i\le n$,

$$f^*(v_i^{(1)}u_i)=f(v_i^{(1)}) \text{ and}$$

for $1\le i\le n$,

$$f^*(v_i^{(j)}u_i)=\begin{cases} f(u_i) & X_i=2 \text{ and } j=2\\ f(u_i)-1 & X_i=3 \text{ and } j=2\\ f(u_i) & X_i=3 \text{ and } j=3.\end{cases}$$

Hence, f is a logarithmic mean labeling of $P_n(X_1,X_2,\dots,X_n)$. Thus the graph $P_n(X_1,X_2,\dots,X_n)$ is a logarithmic mean graph for $1\le X_i\le 3$ and $|X_i-X_{i+1}|\le 1$.

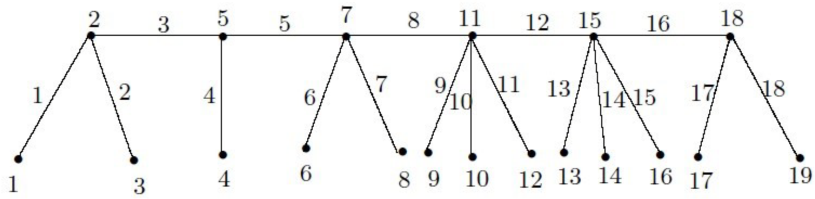

Figure-2.3: A logarithmic mean labeling of $P_6(2,1,2,3,3,2)$

Corollary 2.6: *$TW(P_n)$ is a logarithmic mean graph for $m\le 3$.*

Corollary 2.7: *$P_n\circ S_m$ is a logarithmic mean graph for $m\le 3$.*

Theorem 2.8: *$[P_n;S_m]$ is a logarithmic mean graph, for $m\le 2$ and $n\ge 1$.*

Proof: Let $u_1,u_2,\dots,u_n$ be the vertices of the path P_n and Let $v_1^{(i)},v_2^{(i)},\dots,v_{m+1}^{(i)}$ be the vertices of the star graph S_m such that $v_1^{(i)}$ is the central vertex of S_m, for $1\le i\le n$.

Case-i. $m=1$.
Define $f: V([P_n;S_m])\to\{1,2,3,\dots,3n\}$ as follows:

$$f(u_i)=\begin{cases} 3i & 1\le i\le n \text{ and } i \text{ is odd}\\ 3i-2 & 1\le i\le n \text{ and } i \text{ is even},\end{cases}$$

$$f(v_1^{(i)})=3i-1, \text{for } 1\le i\le n \text{ and}$$

$$f(v_2^{(i)})=\begin{cases} 3i-2 & 1\le i\le n \text{ and } i \text{ is odd}\\ 3i & 1\le i\le n \text{ and } i \text{ is even}.\end{cases}$$

The induced edge labeling is as follows:

$$f^*(u_iu_{i+1})=3i, \text{ for } 1\le i\le n-1,$$

$$f^*(u_iv_1^{(i)})=\begin{cases} 3i-1 & 1\le i\le n \text{ and } i \text{ is odd}\\ 3i-2 & 1\le i\le n \text{ and } i \text{ is even}\end{cases}$$

$$\text{and } f^*(v_1^{(i)}v_2^{(i)})=\begin{cases} 3i-2 & 1\le i\le n \text{ and } i \text{ is odd}\\ 3i-1 & 1\le i\le n \text{ and } i \text{ is even}.\end{cases}$$

Case-ii. $m = 2$.
Define $f: V([P_n; S_m]) \to \{1,2,3, \dots, 4n\}$ as follows:

$$f(u_i) = \begin{cases} 4i & 1 \le i \le n \text{ and } i \text{ is odd} \\ 4i-2 & 1 \le i \le n \text{ and } i \text{ is even,} \end{cases}$$

$$f(v_1^{(i)}) = 4i-1, \text{for } 1 \le i \le n,$$
$$f(v_2^{(i)}) = 4i-3, \text{for } 1 \le i \le n \text{ and}$$
$$f(v_3^{(i)}) = \begin{cases} 4i-2 & 1 \le i \le n \text{ and } i \text{ is odd} \\ 4i & 1 \le i \le n \text{ and } i \text{ is even.} \end{cases}$$

The induced edge labeling is as follows:

$$f^*(u_i u_{i+1}) = 4i, \text{ for } 1 \le i \le n-1,$$
$$f^*(u_i v_1^{(i)}) = \begin{cases} 4i-1 & 1 \le i \le n \text{ and } i \text{ is odd} \\ 4i-2 & 1 \le i \le n \text{ and } i \text{ is even} \end{cases}$$
$$f^*(v_1^{(i)} v_2^{(i)}) = 4i-3, \text{ for } 1 \le i \le n \text{ and}$$
$$f^*(v_1^{(i)} v_3^{(i)}) = \begin{cases} 4i-2 & 1 \le i \le n \text{ and } i \text{ is odd} \\ 4i-1 & 1 \le i \le n \text{ and } i \text{ is even.} \end{cases}$$

Hence, f is a logarithmic mean labeling of $[P_n; S_m]$. Thus the graph $[P_n; S_m]$ is a logarithmic mean graph, for $m \le 2$ and $n \ge 1$.

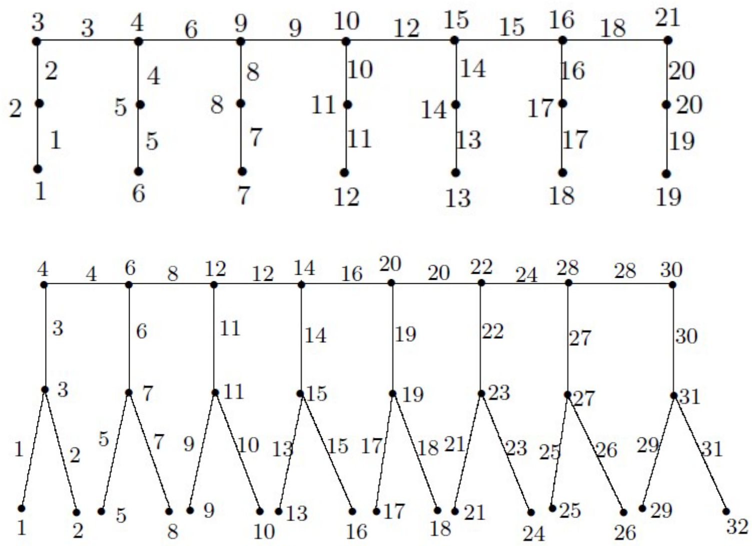

Figure-2.4: A logarithmic mean labeling of $[P_7; S_1]$ and $[P_8; S_2]$

Theorem 2.9: P_n^2 *is a logarithmic mean graph for every* $n \ge 3$.

Proof: Let $v_1, v_2, \dots, v_n$ be the vertices of the path P_n.
Define $f: V(P_n^2) \to \{1,2,3, \dots, 2(n-1)\}$ as follows:

$$f(v_i) = 2i-1, \text{ for } 1 \le i \le n-1 \text{ and}$$
$$f(v_n) = 2(n-1).$$

The induced edge labeling is as follows:

$$f^*(v_i v_{i+1}) = 2i-1, \text{ for } 1 \le i \le n-1 \text{ and}$$
$$f^*(v_i v_{i+2}) = 2i, \text{ for } 1 \le i \le n-2.$$

Hence, f is a logarithmic mean labeling of the graph P_n^2. Thus the graph P_n^2 is a logarithmic mean graph, for $n \ge 3$.

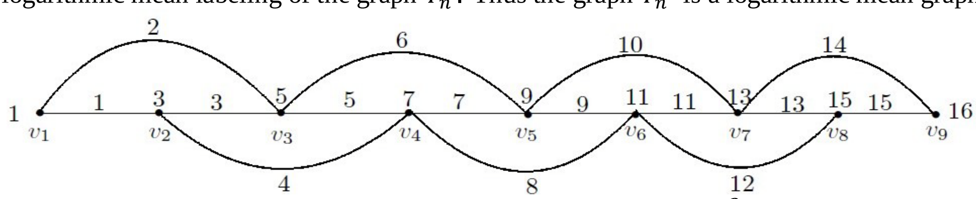

Figure-2.5: A logarithmic mean labeling of P_9^2

Theorem 2.10: $T(P_n)$ *is a logarithmic mean graph, for any* $n \ge 2$.

Proof: Let $V(P_n) = \{v_1, v_2, \dots, v_n\}$ and $E(P_n) = \{e_i = v_i v_{i+1}; 1 \le i \le n-1\}$ be the vertex set and edge set of the path P_n. Then

$$V(T(P_n)) = \{v_1, v_2, \dots, v_n, e_1, e_2, \dots, e_{n-1}\} \text{ and}$$
$$E(T(P_n)) = \{v_i v_{i+1}, e_i v_i, e_i v_{i+1}; 1 \le i \le n-1\} \cup \{e_i e_{i+1}; 1 \le i \le n-2\}.$$

Define $f: V(T(P_n)) \to \{1,2,3,\dots,4(n-1)\}$ as follows:

$$f(v_i) = 4i - 3, \ for \ 1 \le i \le n-1,$$
$$f(v_n) = 4n - 4 \ and$$
$$f(e_i) = 4i - 1, \ for \ 1 \le i \le n-1.$$

The induced edge labeling is as follows:

$$f^*(v_i v_{i+1}) = 4i - 2, \ for \ 1 \le i \le n-1,$$
$$f^*(e_i e_{i+1}) = 4i, \ for \ 1 \le i \le n-2,$$
$$f^*(e_i v_i) = 4i - 3, \ for \ 1 \le i \le n-1 \ and$$
$$f^*(e_i v_{i+1}) = 4i - 1, \ for \ 1 \le i \le n-1.$$

Hence, f is a logarithmic mean labeling of the graph $T(P_n)$. Thus the graph $T(P_n)$ is a logarithmic mean graph, for $n \ge 2$.

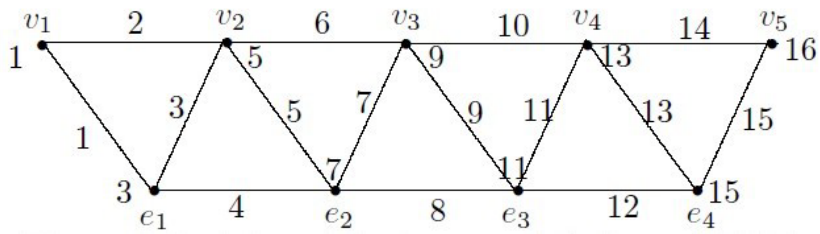

Figure-2.6: A logarithmic mean labeling of $T(P_5)$

Theorem 2.11: *The middle graph of a path is a logarithmic mean graph.*

Proof: Let $V(P_n) = \{v_1, v_2, \dots, v_n\}$ and $E(P_n) = \{e_i = v_i v_{i+1}; 1 \le i \le n-1\}$ be the vertex set and edge set of the path P_n.

Then $V(M(P_n)) = \{v_1, v_2, \dots, v_n, e_1, e_2, \dots, e_{n-1}\}$ and

$$E(M(P_n)) = \{v_i e_i, e_i v_{i+1}; 1 \le i \le n-1\} \cup \{e_i e_{i+1}; 1 \le i \le n-2\}.$$

Define $f: V(M(P_n)) \to \{1,2,\dots,3n-3\}$ as follows:

$$f(v_i) = 3i - 2, \ for \ 1 \le i \le n-1,$$
$$f(v_n) = 3n - 3 \ and$$
$$f(e_i) = 3i - 1, \ for \ 1 \le i \le n-1.$$

The induced edge labeling is as follows:

$$f^*(v_i e_i) = 3i - 2, \ for \ 1 \le i \le n-1,$$
$$f^*(e_i v_{i+1}) = 3i - 1, \ for \ 1 \le i \le n-1 \ and$$
$$f^*(e_i e_{i+1}) = 3i, \ for \ 1 \le i \le n-2.$$

Hence, f is a logarithmic mean labeling of the middle graph $M(P_n)$. Thus the middle graph $M(P_n)$ is a logarithmic mean graph.

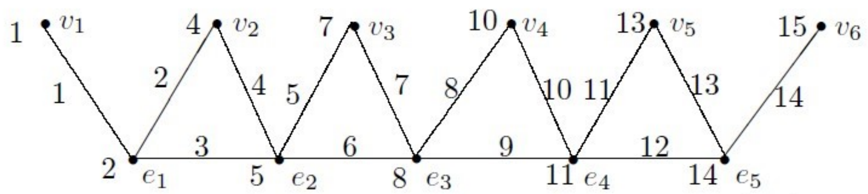

Figure-2.7: A logarithmic mean labeling of $M(P_6)$

Theorem 2.12: *For any* $n \ge 2, P(1,2,3,\dots,n-1)$ *is a logarithmic mean graph.*

Proof: Let $v_1, v_2, \dots, v_n$ be the vertices of the path P_n and let u_{ij} be the vertices of the partition of K_{2,m_i} with cardinality $m_i, 1 \le i \le n-1$ and $1 \le j \le m_i$.

Define $f: V(P(1,2,\dots,n-1)) \to \{1,2,3,\dots,n(n-1)+1\}$ as follows:

$$f(v_i) = i(i-1) + 1, \quad \text{for } 1 \le i \le n \quad \text{and}$$
$$f(u_{ij}) = i(i-1) + 2j, \quad \text{for } 1 \le j \le i \quad \text{and } 1 \le i \le n-1.$$

The induced edge labeling is as follows:

$$f^*(v_i u_{ij}) = i(i-1) + j, \quad \text{for } 1 \le j \le i \quad \text{and } 1 \le i \le n-1 \quad \text{and}$$
$$f^*(u_{ij} v_{i+1}) = i^2 + j, \quad \text{for } 1 \le j \le i \quad \text{and } 1 \le i \le n-1.$$

Hence, f is a logarithmic mean labeling of the graph $P(1,2,\dots,n-1)$. Thus the graph $P(1,2,\dots,n-1)$ is a logarithmic mean graph.

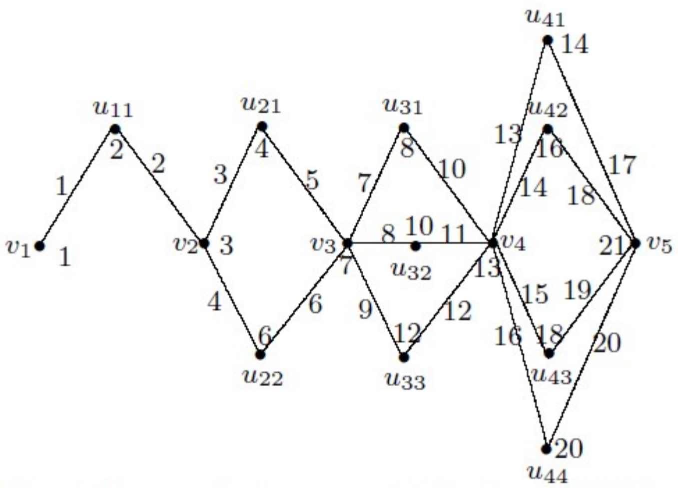

Figure-2.8: A logarithmic mean labeling of $P(1,2,3,4,5)$

Theorem 2.13: $S(P_n \circ K_1)$ *is a logarithmic mean graph, for* $n \geq 2$.

Proof: Let $V[P_n \circ K_1] = \{u_i, v_i : 1 \leq i \leq n\}$. Let x_i be the vertex which divides the edge $u_i v_i$, for $1 \leq i \leq n$ and y_i be the vertex which divides the edge $u_i v_{i+1}$, for $1 \leq i \leq n-1$. Then

$$V[S(P_n \circ K_1)] = \{u_i, v_i, x_i, y_j; 1 \leq i \leq n, 1 \leq j \leq n-1\} \text{ and}$$

$$E[S(P_n \mathrm{e} K_1)] = \{u_i x_i, v_i x_i; 1 \leq i \leq n\} \cup \{u_i y_i, y_i u_{i+1}; 1 \leq i \leq n-1\}.$$

We define $f: V[S(P_n \mathrm{e} K_1)] \to \{1,2,3,\ldots,4n-1\}$ as follows:

$$f(u_i) = 4i-1, \ for \ 1 \leq i \leq n,$$
$$f(y_i) = 4i+1, \ for \ 1 \leq i \leq n-1,$$
$$f(x_i) = 4i-2, \ for \ 1 \leq i \leq n \ and$$
$$f(v_i) = \begin{cases} 1 & i = 1 \\ 4i-4 & 2 \leq \ i \leq \ n. \end{cases}$$

The induced edge labeling is as follows:

$$f^*(u_i y_i) = 4i-1, \ for \ 1 \leq i \leq n-1$$
$$f^*(y_i u_{i+1}) = 4i+1, \ for \ 1 \leq i \leq n-1$$
$$f^*(u_i x_i) = 4i-2, \ for \ 1 \leq i \leq n \ and$$
$$f^*(v_i x_i) = \begin{cases} 1 & i = 1 \\ 4i-4 & 2 \leq \ i \leq \ n. \end{cases}$$

Hence f is a logarithmic mean labeling of $S(P_n \circ K_1)$. Thus the graph $S(P_n \circ K_1)$ is a logarithmic mean graph, for $n \geq 2$.

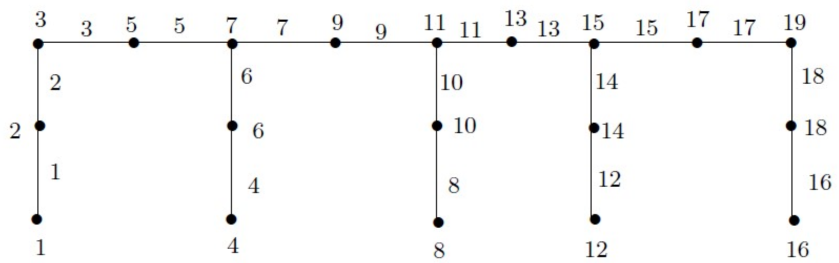

Figure-2.9: A logarithmic mean labeling of $S(P_5 \circ K_1)$.

Theorem 2.14: *Arbitrary subdivision of* S_3 *is a logarithmic mean graph.*

Proof: Let G be a graph of arbitrary subdivision of S_3. Let v_0, v_1, v_2 and v_3 be the vertices of G in which v_0 is the central vertex and v_1, v_2 and v_3 are the pendant vertices of S_3. Let the edges $v_0 v_1, v_0 v_2$ and $v_0 v_3$ of S_3 be subdivided by p_1, p_2 and p_3 number of vertices respectively.

Let $v_0, v_1^{(1)}, v_2^{(1)}, v_3^{(1)}, \ldots, v_{p_1+1}^{(1)} (= v_1), v_0, v_1^{(2)}, v_2^{(2)}, v_3^{(2)}, \ldots, v_{p_2+1}^{(2)} (= v_2)$ and $v_0, v_1^{(3)}, v_2^{(3)}, v_3^{(3)}, \ldots, v_{p_3+1}^{(3)} (= v_3)$ be the vertices of G and $v_0 = v_0^{(i)}$, for $1 \leq i \leq 3$.

Let $e_j^{(i)} = v_{j-1}^{(i)} v_j^{(i)}, 1 \leq j \leq p_i + 1$ and $1 \leq i \leq 3$ be the edges of G and it has $p_1 + p_2 + p_3 + 4$ vertices and $p_1 + p_2 + p_3 + 3$ edges with $p_1 \leq p_2 \leq p_3$.

Define $f: V(G) \rightarrow \{1,2,3, \dots, p_1 + p_2 + p_3 + 4\}$ as follows:

$$f(v_0) = p_1 + p_2 + 3,$$
$$f(v_i^{(1)}) = p_1 + p_2 + 4 - 2i,\ for\ 1 \le i \le p_1 + 1,$$
$$f(v_i^{(2)}) = \begin{cases} p_1 + p_2 + 3 - 2i & 1 \le\ i \le\ p_1 + 1 \\ p_2 + 2 - i & p_1 + 2 \le\ i \le\ p_2 + 1\ \ and \end{cases}$$
$$f(v_i^{(3)}) = p_1 + p_2 + 3 + i,\ for\ 1 \le i \le p_3 + 1.$$

The induced edge labeling is as follows:

$$f^*(v_i^{(1)} v_{i+1}^{(1)}) = p_1 + p_2 + 2 - 2i,\ for\ 1 \le i \le p_1,$$
$$f^*(v_i^{(2)} v_{i+1}^{(2)}) = \begin{cases} p_1 + p_2 + 1 - 2i & 1 \le\ i \le\ p_1 \\ p_2 + 1 - i & p_1 + 1 \le\ i \le\ p_2, \end{cases}$$
$$f^*(v_i^{(3)} v_{i+1}^{(3)}) = p_1 + p_2 + 3 + i,\ for\ 1 \le i \le p_3,$$
$$f^*(v_0 v_1^{(1)}) = p_1 + p_2 + 2,$$
$$f^*(v_0 v_2^{(1)}) = p_1 + p_2 + 1\ and$$
$$f^*(v_0 v_3^{(1)}) = p_1 + p_2 + 3.$$

Hence f is a logarithmic mean labeling of G. Thus the arbitrary subdivision of S_3 is a logarithmic mean graph.

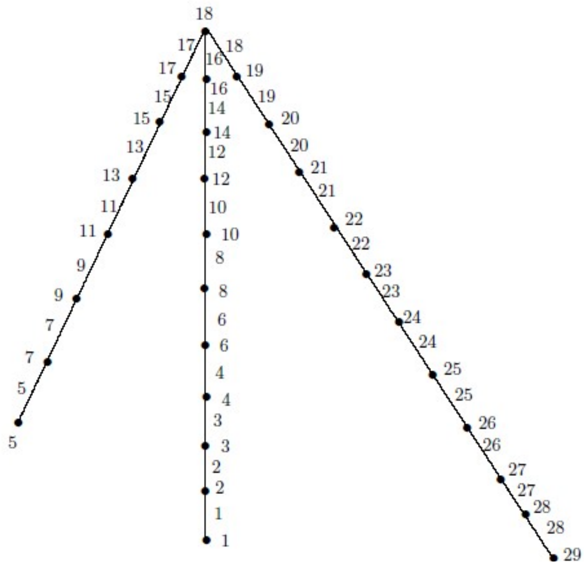

Figure-2.10: A logarithmic mean labeling of arbitrary subdivision of S_3.

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