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EXISTENCE AND UNIQUENESS OF THE SOLUTIONS TO NEUTRAL STOCHASTIC FUNCTIONAL DIFFERENTIAL EQUATIONS WITH FINITE DELAY

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ABSTRACT

This article presents results on existence and Uniqueness of mild solutions to stochastic neutral functional differential equations (SNFDEs) with finite delays under non- Lipschitz condition with Lipschitz condition being considered as a special case and a weakened linear growth condition. The solution is constructed by the successive approximation. Some results in Govindan [3, 4] are generalized to cover a class of more general SNFDEs

Keywords: *Stochastic neutral partial functional differential equation, mild solution, existence, uniqueness.*

1. INTRODUCTION

The study of existence and uniqueness of mild solutions of SNFDEs due to their range of applications in various sciences such as physics, mechanical engineering, control theory and economics where in, quite often the future state of such systems depends not only on the present state but also on its past history (delay) leading to SNFDEs with delays. Mao [7] discussed important kind SNFDEs is the following neutral SFDEs with finite delay under the classic Lipschitz condition and linear growth condition on the coefficients;

$$\mathrm{d}\big[X(t) - G\big(X(t-\tau)\big)\big] = f\left(t, X_{t,} X(t- \tau)\right) dt + g\left(t, X_{t,} X(t- \tau)\right) dB(t)$$

Following it, kolmanovskil and myshkis [5] introduced another important kind of SFDEs

$$d[X(t) - G(X_t)] = f(t, X_t)dt + g(t, X_t)dB(t)$$

which could be used in chemical engineering and aero elasticity .Under the global Lipschitz and linear growth condition Taniguchi [10] Luo [6] considered the existence and Uniqueness of mild solutions to SPFDEs and Zhou and Xue [12] established the uniqueness theorem of solution to the neutral stochastic functional differential equations with infinite delay .

Motivated by the above papers, in this work we aim to extend the existence and Uniqueness of mild solution to cover a class of more general SNFDEs under a non –Lipschitz condition with the Lipschitz condition being regarded as a special case and a weakened linear growth condition.

2. PRELIMINARY RESULTS

Let $\{\Omega, \mathcal{F}, \wp\}$be a complete probability space equipped with some filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions (i.e., it is right continuous and $\mathcal{F}_0$ contains all p- null sets). Let h, k be two real separable Hilbert spaces and we denote by$\langle .,. \rangle_{H,}$ $\langle .,. \rangle_k$ their inner products and by $\|.\|_H \|.\|_K$ their vector norms, respectively. We denote by$\mathcal{L}(K, H)$ the set of all linear bounded operators from K into H, equipped with the dual operator$\|.\|$. In this paper, we always use the same symbol $\|.\|$ to denote norms of operators regardless of the spaces potentially involved when no confusion possibly arises. Let $\tau > 0$ and $\mathsf{C}\equiv \mathsf{C}([-\tau, 0]; H)$ denote the family of all continuous H- valued functionsφ defined on [-τ, 0] with norm$\|\varphi\|_{\mathsf{C}} = sup_{-r\leq\theta\leq 0}\|\varphi(\theta)\|_H$.

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We denote by {W(t), $t \geq 0$} a K- valued$\{\mathcal{F}_t\}_{t\geq 0}$ – Wiener process defined on the probability space $\{\Omega, \mathcal{F}, \wp\}$ with covariance operator Q. i.e.,

$$E\langle w(t), x\rangle\ E\langle w(t), x\rangle_k \langle w(t), y\rangle_k = (t \wedge s)\ \langle Qx, y\rangle_{k,} \quad \forall x, y\ \in K,$$

Where Q is a positive, self – adjoint, trace class operator o≥ 0}n K, In particular, we call such {W(t), $t \geq$ o} a K – valued Q-Wiener process relative to $[\mathcal{F}_t\}_{t\geq 0}$. According to Da, Prato [2], Proposition 4.1, P87], W(t) is defined by

$$W(t) = \sum_{n=1}^{\infty} \sqrt{\lambda_n}\ \beta_n(t) e_{n,} \quad t \geq 0$$

Where $\beta_n(t)$ (n = 1, 2, 3...) is a sequence of real standard Brownian motion s mutually independent on the probability space $\{\Omega, \mathcal{F}, \mathcal{P}\}$, $(\lambda_n, n\ \in N)$ are the eigenvalues
That $Qe_n = \lambda_n e_{n,}$, n = 1, 2, 3,….

In order to define stochastic integral with respect to the Q- Wiener process 9t), we introduce the subspace $K_0 = Q^{\frac{1}{2}}(t)$ of K, which endoed with the inner product,

$$\langle u, v\rangle_{k_0} = \langle Q^{-\frac{1}{2}} u\, , Q^{-\frac{1}{2}} v\rangle_k\ ,$$

is a Hilbert space. Let $\mathcal{L}_2^0 = \mathcal{L}_2\ (K_0, H)$ denote the space of all Hilbert-Schmidt operators from $K_0\ into\ H.$ It turns out to be a separable Hilbert space, equipped with the norm

$$\|\psi\|_{\mathcal{L}_2^0}^2 = tr\left(\left(\psi Q^{\frac{1}{2}}\right)\left(\psi Q^{\frac{1}{2}}\right)^*\right)$$

For any $\in \mathcal{L}_2^0$. Clearly, for any bounded operators $\psi\ \in \mathcal{L}(K, H)$, this norm reduces to $\|\psi\|_{\mathcal{L}_2^0}^2 = tr\ (\psi Q\psi^*$. Let $\emptyset : (0, \infty) \ \to \mathcal{L}_2^0$ be a predictable, $\mathcal{F}_t$ –adapted such that

$$\int_0^t E\|\emptyset(s)\|_{\mathcal{L}_2^0}^2 ds < \infty.$$

Then, we can define the H- valued stochastic integral

$$\int_0^t \emptyset(s) dw(s),$$

Which is a continuous square integrable martingale. For that construction, see Da. Prato [2], P. 90-96].

We are concerned with the SNFDE

$$d[X(t) - G(X_t)] = f(t, X_t)dt + g(t, X_t)dB(t).\ \ T \geq 0 \tag{2.1}$$

With the initial condition X(t) = $\xi(t)\ \in C_{\mathcal{F}_0}^b\ ([-\tau, 0]; H)$, which denote the family of all almost surely bounded, $\mathcal{F}_0$- measurable, $C\ ([-\tau, 0]; H) -$ valued random variables, and where $X_t = \{X(t+\theta) : -\tau\ \leq\ \theta\ \leq 0\}$ can be regarded as a $C\ ([-\tau, 0]; H) -$ valued stochastic process. Moreover, the mappings G, F: $\mathbb{R}^+ \times C\ ([-\tau, 0]; H) \to$ H and L : $\mathbb{R}^+ \times ([-\tau, 0]; H) \to \mathcal{L}(K, H)$ are measurable, respectively.

For convenience, we recall from [8] the mild solution to (2.1) as follows

Definition 2.1: A stochastic process {X (t), t ∈ $[0, T]\}$, $0\ \leq T <\ \infty.$ = is called a mild solution to (2.1) if
(i) X (t) is adapted to $\mathcal{F}_t$ and continuous in t almost surely;
(ii)For arbitrary t ∈ $[0, T]$, P$\{\omega$: $\int_0^t \|X(t)\|_H^2\ ds < \infty.\}\ = 1$ and almost surely

$$X(t) = T(t)[\xi(0) + G(0, \xi)\} - G(t, X_t) - \int_0^t T(t-s)F(s, X_s ds) - \int_0^t T(t-\ s)L(s, X_s)dw(s)$$

To guarantee the existence and uniqueness of a mild solution to (2.1), the following much weaker conditions, instead of global Lipschitz and linear growth condition, are described.

(H1) The mappings F (·) and L (·) satisfy the following non-Lipschitz condition: for any
$\xi, \eta \in H\ and\ t\ \geq 0, \|F(t, \xi) - F(t, \eta)\|_H^2 +\ \|L(t, \xi) - L(t, \eta)\|_{\mathcal{L}_2^0}^2\ \leq k(\|\xi - \eta\|_c^2,$
Where k (·) is a concave non decreasing function from $\mathbb{R}^+$ to $\mathbb{R}^+$ such that k(0) = 0,
k(u) > 0 for u > 0 and $\int_{0^+} \frac{du}{k(u)} =\ \infty$, e.g., k(u) $\sim u^{\alpha}, \frac{1}{2} <\ \alpha < 1.$

(H2)There is an M > 0 such that

$$\sup_{0\leq t\leq\infty}\ (\|F(t, 0)\|_H^2\ \vee\ \|L(t, 0)\|_{\mathcal{L}_2^0}^2\ \leq M.$$

(H3) The mapping G (t, x) satisfies that there exist number $\alpha \in [0,1]$ and $k_1\ \geq 0 \geq$ such that. For any $\xi, \eta \in H$ and $t\ \geq 0$, G (t, x) $\in \mathfrak{D}((-A)^{\alpha}$ and $\|(-A)^{\alpha} G(t, \xi)\ - (-A)^{\alpha} G(t, \eta)\|_H\ \leq k_1 \|\xi - \eta\|_c$.

We further assume that G (t, 0) ≡ 0 $for\ t\ \geq 0.$

Since T (t), t ≥ 0. Is an analytic semi group with the infinitesimal generator A such that 0 $\epsilon for\ any\ \rho(A)$,then under somecircumstances it is possible to define the fractional power $(-A)^{\alpha}$ for any $\alpha \epsilon [0,1]$ which is a closed operator with its domain $\mathfrak{D}((-A)^{\alpha}$

In the sequel, to show our main results the following three lemmas.

Lemma 2.1: (Caraballo [1], Lemma 1). For u, v ∈ H, and 0 < c <1,

$$\|u\|_H^2 \leq \frac{1}{1-c}\|u-v\|_H^2 + \frac{1}{c}\|v\|_H^2$$

Lemma 2.2: (Pazy [9, Theorem 6.13, p. 74]). Let the assumption (H1) hold. Then for any $\beta \epsilon (0,]$ and x $\epsilon\ \mathfrak{D}((-A)^{\beta}$, T(t) $(-A)^{\beta}x = (-A)^{\beta}$T(t)$x$ and there exists a positive constant M_β such that for any t >0

$$\left\|(-A)^{\beta}\, T(t)\right\| \leq M_\beta t^{-\beta} e^{-\gamma t}\ ,$$

Lemma 2.3: [7: Let T > 0 and c > 0. Let k: $\mathbb{R}^+$ to $\mathbb{R}^+$ be a continuous nondecreasing function such that κ(t) > 0 for all t >0. Let u (.) be a Borel measurable bounded nonnegative function n $[0, \mathrm{T}]$. If

$$u(t) \leq c + \int_0^t v(s)k\big(u(s)\big)ds \text{ for all } 0\leq t \leq \mathrm{T}.$$

Then

$$\mathfrak{u}(t) \leq J^{-1}\left(J(c) + \int_0^t \mathfrak{v}(s)ds\right),$$

holds for all such t ∈ $[0, \mathrm{T}]$ that

$$J(c) + \int_0^t \mathfrak{v}(s)ds \in \mathrm{Dom}\,(J^{-1}),$$

where $J(r) = \int_0^r ds/k(s)$, on r > 0, and J^{-1} is the inverse function of J In Particular, if, c = 0 and $\int_{0^+}^r ds/\kappa(s) = \infty$, then $\mathfrak{u}(t) = 0$ for all t ∈ $[0, \mathrm{T}]$

3. EXISTENCE AND UNIQUENESS

In this section, we start to study the existence and uniqueness of mild solutions to SNFDEs under the non-Lipschitz condition and a weakened linear growth condition. To complete our main results, we need to prepare several lemmas which will be utilized in the sequel.

Introduce the following successive approximating procedure: for each integer n = 1, 2, 3…..

$$X^n(\mathrm{t}) = \mathrm{T(t)}[\xi(0) + G(0,\xi)\} - G(t, X_t) - \int_0^t T(t-s)F(s, {X^{n-1}}_s ds) + \int_0^t T(t-s)L(s, {X^{n-1}}_s)dw(s) \quad (3.1)$$

and for n = 0,

$$x^0(t) = S(t)\xi(0), \quad \mathrm{t} \in [0,\ \mathrm{T}].$$

While for n = 1, 2, ……..

$$x^n(t) = \xi(t), \quad \mathrm{t} \in [-\tau, \mathrm{T}].$$

Lemma 3.1: Let the hypothesis (H1) - (H3) hold and K < 1. Then there is a positive constant $C_{1,}$ which is independent of n ≥ 1, such that for any t ∈ $[0, \mathrm{T}]$,

$$\mathbb{E} \sup_{0\leq t\leq T} \|x^n(t)\|^2 \leq C_1. \quad (3.2)$$

Proof: For $0 \leq t \leq T$, it follows easily from (3.1) that

$$\begin{aligned} &\mathbb{E} \sup_{0 \leq t \leq T} \|x^n(t) + G(t, X_t^n\|_H^2 \\ &\quad\leq 3\,\mathbb{E} \sup_{0 \leq t \leq T} \|T(t)(\xi(0) + G(0,\xi)\|_H^2 \\ &\qquad+3\mathbb{E} \sup_{0 \leq t \leq T} \left\|\int_0^t T(t-\ \ s)F(s, X_s^{n-1}\, ds\right\|^2 \\ &\qquad+\ 3\,\mathbb{E} \sup_{0 \leq t \leq T} \left\|\int_0^t T(t-s)L(s, X_s^{n-1})\, dw(s)\right\|_H^2 \\ &\qquad+ 3\,\mathrm{I}_1 + \mathrm{I}_2 + \mathrm{I}_3). \end{aligned} \quad (3.3)$$

Note from [11] that $(-A)^{-\alpha}\ for\ 0 < \alpha \leq 1$ is a bounded Operator. Employing the assumption (H3) results with

$$\mathrm{I}_1 \leq M_1\ (1+k)^2 \mathrm{E}\|\ \emptyset\|_C^2 \ \text{ Where } M_1 = \sup_{0 \leq t \leq T} \|\ T(t)\|^2 \quad (3.4)$$

On the other hand, in view of (H2), we obtain from the Holder's inequality that

$$\begin{aligned} \mathrm{I}_2 &\leq T\mathbb{E}_{0 \leq t \leq T}^{\ sup} \int_0^t \|T(t-s)F(s, X_s^{n-1}) - F(s,0) + F(s,0)\|^2{}_H ds \\ &\leq TK^2 \int_0^T E\|X_s^n\|_C^2\, ds\ 1) \end{aligned} \quad (3.5)$$

Next by (Liu [8, Theorem 1.2.6, p 14], together with (H1) there exists a constant $C_1 > 0$ such that

$$I_3 \le C_1 \int_0^T E\| H(s, X_s^{n-1}) - H(s,0) + H(s,0)\|^2_{\mathcal{L}_2^0}\, ds$$
$$\le 2C_1 \left[MT + \int_0^T E_k \|X_s^{n-1}\|_c^2\, ds \right] \tag{3.6}$$

Since $\kappa(u)$ is concave on $u \ge 0$, there is a pair of positive constants a, b such that

$$\kappa(u) \le a + bu. \tag{3.7}$$

Putting (3.3) to (3.6) into (3.2) yields that, for some positive constants C_2 and C_3,

$$\mathbb{E} \sup_{0 \le t \le T} \|x^n(t) + G(t, X_t^n\|_H^2$$
$$\le C_2 + C_3\, \mathbb{E} \int_0^T E\|X_s^{n-1}\|_c^2\, ds \tag{3.8}$$

While for $K < 1$ By Lemma 2.1,

$$\mathbb{E} \sup_{0 \le t \le T} \|x^n(t)\|_H^2 \le \frac{C_2}{(1-k)^2} + \left[\frac{C_2\, T}{(1-k)^2} + \frac{K}{1-k}\right] E\,\|\emptyset\|_c^2 + \frac{2C_2}{1-k} \int_0^T \sup_{0 \le \theta \le s} \|X^{n-1}(\theta)\|_H^2\, ds$$

Observing that

$$\max_{1 \le n \le \kappa} \mathbb{E} \sup_{0 \le t \le T} \|x^{n-1}(t)\|^2 \le \mathbb{E}\,\|\varphi\|_C^2 + \max_{1 \le n \le \kappa} \mathbb{E} \sup_{0 \le t \le T} \|x^n(t)\|_H^2$$

Allows for some positive constants C_3 and C_4

$$\max_{1 \le n \le \kappa} \mathbb{E} \sup_{0 \le t \le T} \|x^n(t)\|_H^2 \le C_3 + C_4 \mathbb{E} \int_0^T \max_{1 \le n \le \kappa} \mathbb{E} \sup_{0 \le \theta \le s} \|x^n(t)\|_H^2 ds$$

Now, the application of the well-known Gronwall's inequality yields that

$$\max_{1 \le n \le \kappa} \mathbb{E} \sup_{0 \le t \le T} \|x^n(t)\|^2 \le \mathrm{C3} + e^{\mathrm{C4T}}$$

Since K is arbitrary the required assertion 3.3 directly follows.

Lemma 3.2: Let the condition (H1)-(H3) be satisfied. For $\alpha \in \left(\frac{1}{2}, 1\right]$, further assume that

$$\frac{3\ \mathrm{K}_1^2\, \mathcal{M}_{1-\alpha}^2 \gamma^{-2\alpha} \Gamma^{2\alpha-1}}{1 - \mathrm{K}_1 \left\|(-A)^{-\alpha}\right\|} + \mathrm{K}_1 \left\|(-A)^{-\alpha}\right\| < 1, \tag{3.9}$$

where $\Gamma(\cdot)$ is the Gamma function and $\mathrm{M}_{1-\alpha}$ is a constant in Lemma 2.3. Then there exists a positive constant $\bar{\mathrm{C}}$ such that, for all $0 \le t \le T$ and $\mathfrak{n}, \mathfrak{m} \ge 1$

$$\mathbb{E} \sup_{0 \le s \le T} \|x^{n+m}(s) - x^n(s)\|_H^2 \le \bar{\mathrm{C}} \int_0^t \kappa\left(\mathbb{E} \sup_{0 \le s \le T} \|x^{n+m-1}(s) - x^{n-1}(s)\|_H^2\right) ds. \tag{3.10}$$

Proof: It is easy to see that for any $0 \le t \le T$,

$$\mathbb{E} \sup_{0 \le s \le T} \left\|x^{n+m}(s) - x^n(s) + G\left(s, X_s^{n+m}(s) - G(s, X_s^n(s))\right)\right\|_H^2$$
$$\le 2\mathbb{E} \sup_{0 \le s \le T} \left\|\int_0^s R(s-l)\left[F\left(l, X_l^{n+m-1}, F(l, X_l^{n-1})\right)\right] dl\right\|_H^2 +$$
$$2\mathbb{E} \sup_{0 \le s \le T} \left\|\int_0^s R(s-l)\left[F\left(l, X_l^{n+m-1}, F(l, X_l^{n-1})\right)\right] dw(l)\right\|_H^2 = J_1 + J_2$$

Moreover, Lemma 3.1 and (H3) imply that

$$\mathbb{E} \sup_{0 \le s \le T} \|x^{n+m}(s) - x^n(s)\|_H^2 \le \frac{1}{1-k}\, \mathbb{E} \sup_{0 \le s \le T} \left\|x^{n+m}(s) - x^n(s) + G\left(s, X_s^{n+m}(s) - G(s, X_s^n(s))\right)\right\|_H^2 + \mathrm{K}$$
$$\mathbb{E} \sup_{0 \le s \le T} \|x^{n+m}(s) - x^n(s)\|_H^2$$
$$\le \frac{C5}{1-k} \int_0^s \lambda\left(\mathbb{E} \sup_{0 \le s \le T} \|x^{n+m-1}(l) - x^{n-1}(l)\|_H^2\right) ds + \mathrm{K}\,\mathbb{E} \sup_{0 \le s \le T} \|x^{n+m}(s) - x^n(s)\|_H^2$$

So the desired assertion (3.10) follow from (3.9)

It is possible now to state our main result.

Theorem 3.1: Under the conditions of Lemma 3.2, then Eq. (1.1) admits a unique mild solution.

Proof:
Uniqueness: Let x and y be two mild solutions to equation (1.1). In the same way as Lemma 3.3 was done, we can show that for some $\overline{\mathrm{K}} > 0$

$$\mathbb{E} \sup_{0 \le t \le T} \|x(s) - y(s)\|_H^2 \le \overline{\mathrm{K}} \int_0^t \kappa \mathbb{E} \sup_{0 \le t \le T} \|x(s) - y(s)\|_H^2 ds$$

This together with Lemma 3.1 leads to

$$\mathbb{E} \sup_{0 \le t \le T} \|x(s) - y(s)\|_H^2 = 0.$$

Which further implies x (t) = y (t) almost surly for any $0 \le t \le T$

Existence: By Lemma 3.2 there exists a positive $\overline{\mathrm{C}}$ such that $0 \le t \le T$ $\mathrm{n}, \mathrm{m} \ge 1$,

$$\mathbb{E} \sup_{0 \le s \le t} \|x^{\mathrm{n}+1}(s) - x^{m+1}(s)\|_H^2 \le \overline{\mathrm{C}} \int_0^t \kappa \left(E \sup_{0 \le s \le t} \|x^{\mathrm{n}}(u) - x^m(u)\|_H^2\right) ds.$$

Integrating both sides and applying Jensen's inequality gives that

$$\int_0^t \mathbb{E} \sup_{0 \le l \le s} \|x^{\mathrm{n}+1}(s) - x^{m+1}(s)\|_H^2 ds \le \overline{\mathrm{C}} \int_0^t \int_0^s \kappa \left(\mathbb{E} \sup_{0 \le u \le s} \|x^{\mathrm{n}}(u) - x^m(u)\|_H^2\right) dlds.$$

$$= \overline{\mathrm{C}} \int_0^t s \int_0^s \kappa \left(\mathbb{E} \sup_{0 \le u \le s} \|x^{\mathrm{n}}(u) - x^m(u)\|_H^2\right) \frac{1}{s} \, dlds.$$

$$\le \overline{\mathrm{C}}\, t \int_0^t \kappa \left(\int_0^s \mathbb{E} \sup_{0 \le u \le s} \|x^{\mathrm{n}}(u) - x^m(u)\|_H^2 \frac{1}{s} \, d\, l\right) ds.$$

Then

$$h_{n+1,m+1}(t) \le \overline{\mathrm{C}} \int_0^t \kappa\left(h_{n,m}(s)\right) ds,$$

Where $h_{n,m}(t) = \frac{\int_0^t \mathbb{E} \sup_{0 \le l \le s} \|x^{\mathrm{n}+1}(l) - x^{m+1}(l)\|_H^2 ds}{t}$

,

While by Lemma 2.3, it is easy to see that
$\sup_{n,m} h_{n,m}(t) < \infty$, so letting h (t): = $\lim sup_{n,m \to \infty} h_{n,m}(t)$ and taking into account the Fatou's lemma,we yield that $h(t) \le \overline{\mathrm{C}} \int_0^t \kappa(h(s))$.

Now, applying the Lemma 2.3 immediately reveals h (t) = 0 for any t $\in [0, T]$. This further means $\{x^n(t), n \in \mathbb{N}\}$ is a Cauchy sequence in L^2. so there is a x $\in L^2$ such that

$$\lim_{n\to\infty} \int_0^t \mathbb{E} \sup_{0 \le l \le s} \|x^{\mathrm{n}}(s) - x(s)\|_H^2 ds = 0.$$

Moreover by Lemma 2.2 it is easy to conclude that $\mathrm{E}\|x(t)\|_H^2 \le \mathrm{C}$. Hence in what follows we claim that x (t) is a mild solution to equation 3.1. Indeed on one hand (H2), the Holders inequality, according Liu [8], Theorem 1.2.6, P 14, and letting $n \to \infty$, for $0 \le t \le T$, we can also claim, for $t \in [0, T]$, that

$$\left\|\int_0^t T(t-s)[F(s, X_s^{n-1} - F(s, X_s)]ds\right\|_H^2 \to 0,$$

$$\mathrm{E}\left\|\int_0^t T(t-s)[F(s, X_s^{n-1} - F(s, X_s)]ds\right\|_H^2 \to 0,$$

On the other hand by applying (H3), we can also claim, for $t \in [0, T]$, that

$$\mathrm{E}\|G(s, X_s^n) - G(s, X_s)\|_H^2 \le K^2 E \sup_{0 \le l \le s} \|x^{\mathrm{n}}(s) - x(s)\|_H^2 \to 0$$

Now taking limits in both sides of (3.1) leads for t ≥ 0, to

$$\mathrm{X(t)=T(t)}[\xi(0) + G(0, \xi)\} - G(t, X_t) - \int_0^t T(t-s) F(s, X^{n-1}{}_s ds) + \int_0^t T(t-s) L(s, X^{n-1}{}_s) dw(s)$$

This is an illustration that X is a mild solution to of equation (3.1) on [0.T].

Remark 3.1: If G =0, that is, K_1=0, then, obviously, the condition (3.11) must be satisfied. Consequently, our results can be reduced in [3]. In other words, in this special case, we generalize [3].

Remark 3.2: In this work, we consider the existence and uniqueness of mild solutions to SNFDEs under a non-Lipschitz condition with the Lipschitz condition being regarded as a special case and a weakened linear growth assumption. Therefore, some of the results [4] are improved to cover a class of more general SNFDEs.

REFERENCES

1. T. Caraballo, J.Real and T.Taniguchi, The exponential stability of neutral Stochastic delay partial differential equations, Discrete Contin. Dyn. Syst., 18 (2007), 295-313.
2. G.Da Prato, J.Zaczyk, Stochastic Equations in Infinite Dimensions. Cambridge University Press, 1992
3. T.E. Govindan, Stability of mild solutions to Stochastic evolution equations with variabledelay, Stoch. Anal. Appl . 21 (2003) 1059 - 1077
4. T.E Govindan, Almost sure exponential Stability for Stochastic neutral Partial functional differential equations Stochastic 77 (2005) 139 -154
5. V.B Kolmanovski, A.Myshkis, Applied Theory of Functional Differential Equations, Kluwer Academic Publishers, 1992.
6. J.Luo, K. Liu, Stability of infinite dimensional Stochastic evolution equations with Memory and markovian Jumps, Stochatic process. Appl. 118 (2008)
7. X.Mao, Stochastic Differential Equation and Applications, Horwood, New York, 1997.
8. K.Liu, Stability of Infinite Dimensional Stochastic Differential Equations and Application, Chapman and Hall, CRC, London, 2004.
9. A.Pazy, Semigroups of Linear Operators and Applications to Partial Differential Equations, Springer-Velag, New York (1992).
10. T.Taniguchi, K. Liu, A. Truman, Existence, Uniqueness and asymptotic behaviour of mild Solutions to Stochastic functional differential equations in Hilbert space, J. Differential Equations, (2002) 72 – 91.
11. T.Yamada, On the Successive approximations of Solutions of Stochastic differential equations, J. Math. Univ. 21 (1981) 506 - 515.
12. S.Zhou, M, Xue, The existence and uniqueness of the solutions for neutral stochastic functional differential equations with infinite delay, Math, Appl, 21 (2008) 75-83.

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