

TOTAL EFFICIENT DOMINATION IN JUMP GRAPHS

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(Received On: 14-12-18; Revised & Accepted On: 05-01-19)

ABSTRACT

A set D of vertices of a jump graph $J(G)$ is a total efficient dominating set, if every vertex in $V(J(G))$ is adjacent to exactly one vertex in D . Total efficient domination number $\gamma_{te}(J(G))$ of $J(G)$ is the minimum cardinality of a total efficient dominating set of $J(G)$. In this paper the exact values of $\gamma_{te}(J(G))$ for some standard graphs are found and some bounds are obtained. Also a Nordhus-Gadum type result is obtained. In addition the total efficient domination number $d_{te}(J(G))$ of $J(G)$ is defined to be maximum order of a partition of the vertex set of $J(G)$ into total efficient dominating set of $J(G)$. Also a relation between $J(G)$ and $d_{te}(J(G))$ is established.

Keywords; Efficient dominating set, total dominating set, total efficient dominating set, total efficient domination number.

Mathematics subject classification: 05C.

INTRODUCTION

By a graph we mean a finite, undirected without loops multiple edges and isolated vertices Terms undefined here may be found in Kulli [1]

A set D of vertices in a jump graph $J(G) = (V, E)$ is called dominating set if every vertex in $V - D$ is adjacent to some vertex in D . The domination number $\gamma(J(G))$ of graph $J(G)$ is the minimum cardinality of a dominating set of $J(G)$. Recently many new domination parameter given by Venkangoud et.al.,

A dominating set D of $J(G)$ is an efficient dominating set if every vertex in $V - D$ is adjacent to exactly one vertex in D . The efficient domination number $\gamma_e(J(G))$ of $J(G)$ is the minimum cardinality of an efficient dominating set of $J(G)$.

Kulli and Patwari [28] introduced the concept of total domination.

A set D of vertices in a graph $J(G)$ is a total efficient dominating set of $J(G)$ if every vertex in V is adjacent to exactly one vertex in D . The total efficient domination number $\gamma_{te}(J(G))$ of $J(G)$ is the minimum cardinality of a total efficient dominating set of $J(G)$.

A γ_{te} -set is a minimum total efficient dominating set. Let $\Delta(J(G))$ ($\delta(J(G))$) denote the maximum (minimum) degree among the vertices of $J(G)$, let $\lceil x \rceil$ denote the least integer greater than or equal to x .

We note that $\gamma_t(J(GH))$ and $\gamma_{te}(J(G))$ are only defined for $J(G)$ with $\delta(J(G)) \geq 1$

2. TOTAL EFFICIENT DOMINATION NUMBER

We list exact values of the total efficient domination number for some standard graphs.

Proposition 1: If P_p is a path with p vertices, then

$$\gamma_{te}(J(P_p)) = \lceil \frac{p}{2} \rceil \quad \text{where } p \equiv 0 \pmod{4} \text{ and } p \equiv 3 \pmod{4}$$

Proposition 2: If C_p is a cycle with p vertices, then

$$\gamma_{te}(J(C_p)) = \frac{p}{2} \quad \text{when } p \equiv 0 \pmod{4}$$

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Proposition 3: If $K_{m,n}$ is a complete bipartite graph $1 \leq m \leq n$, then $\gamma_{te}(J(K_{m,n})) = 2$

Remark 4: Every graph $J(G)$ without isolated vertices does not contain a total efficient dominating set. It implies that $\gamma_{te}(J(G))$ does not exist.

Proposition 5: If K_p is a complete graph with $p \geq 3$ vertices, then $\gamma_{te}(J(K_p))$ does not exist.

Proposition 6: If $\gamma_{te}(J(G))$ exists, then $\gamma_t(J(G)) \leq \gamma_{te}(J(G))$ and this bound is sharp.

Proof: Clearly every efficient total dominating set is an efficient dominating set, thus the above inequality holds

The complete bipartite graphs $K_{m,n}$ $1 \leq m \leq n$ achieve this bound.

Proposition 7: If $\gamma_{te}(J(G))$ exists, then $\gamma_t(J(G)) \leq \gamma_{te}(J(G))$ and this bound is sharp.

Proof: Clearly every efficient total dominating set is an efficient dominating set, thus the above inequality holds
The complete bipartite graphs $K_{m,n}$ $2 \leq m \leq n$ achieve this bound.

Theorem 8: Let $J(G)$ be a (p, q) connected graph with $p \geq 2$ vertices, Then $2(p - q) \leq \gamma_{te}(J(G))$.

Furthermore inequality holds if and only if $J(G)$ is a tree with exactly one cut vertex or exactly two cut vertices.

Proof: Let D be a γ_{te} -set of $J(G)$. Then for each vertex $u \in V - D$, there exists a vertex v in D such that $uv \in E$. Also for each vertex $x \in D$, there exists unique vertex $y \in D$ such that $xy \in E$. Then

$$q \geq \frac{|D|}{2} + |V - D|$$

or $2q \geq |D| + 2|V - D|$
 or $2q \geq \gamma_{te}(J(G)) + 2p - 2\gamma_{te}(J(G))$
 or $2(q - p) \leq \gamma_{te}(J(G))$

We prove the second part.

Suppose $J(G)$ is a tree with exactly one cut vertex or two cut vertices.
Then $\gamma_{te}(J(G)) = 2 = 2(p - q)$, since $p - q = 1$

Conversely suppose $\gamma_{te}(J(G)) = 2(p - q)$. We now prove that $J(G)$ is a tree with at most two cut vertices. Clearly for any graph without isolated vertices, $\gamma_{te}(J(G)) \geq 2$

Suppose $p < q$. Then $2(p - q)$ is negative, which is a contradiction.

Suppose $p = q$. Then $2(p - q)$ is zero, which is contradiction.

Suppose $p > q$. Since $J(G)$ is connected, it implies that $J(G)$ is a tree with exactly 3 vertices, then t Remark 4 $\gamma_{te}(J(G))$ does not exist. If $J(G)$ is a tree with at least 4 cut vertices then $\gamma_{te}(J(G)) \geq 4 \neq 2(p - q)$, since $p - q = 1$. Thus we conclude that $J(G)$ is a tree with at most two cut vertices.

Next we characterize graphs for which $\gamma_{te}(J(G)) = p$.

Theorem 9: Let $J(G)$ be a graph without isolated vertices and with $p \geq 2$ vertices. Thus $\gamma_{te}(J(G)) = p$ if and only if $J(G) = mK_2$, $m \geq 1$.

Proof: Suppose $J(G) = mK_2$, $m \geq 1$. Obviously $\gamma_{te}(J(G)) = p$

Conversely Suppose $\gamma_{te}(J(G)) = p$ We now prove that $J(G) = mK_2$, $m \geq 1$. Assume $J(G) \neq mK_2$. Then $\deg_G u \geq 2$. Let D be a $\gamma_{te}(J(G))$ -set of $J(G)$. Since $\gamma_{te}(J(G)) = p$, it implies that $|V - D| = \emptyset$. Hence $u \in D$. Since $\deg_G u \geq 2$, it implies that u is adjacent with at least two vertices in D , which is a contradiction. Suppose $\deg_G u < 1$. Then u is an isolated vertex, again a contradiction.

Thus $\deg_G u = 1$ Since u is arbitrary, it follow that $J(G) = mK_2$, $m \geq 1$.

The following theorem gives a lower bound for $\gamma_{te}(J(T))$.

Theorem 10 : Let $J(T)$ be a tree with $p \geq 3$ vertices, If $\gamma_{te}(J(T))$ exists, then

$$\gamma_{te}(J(T)) \leq \lceil \frac{m}{2} \rceil + 1$$

Where m is the number of cut vertex of $J(T)$.

Proof: Let $J(T)$ be a tree with $p \geq 3$ vertices Suppose $\gamma_{te}(J(T))$ exists. We now prove that $\gamma_{te}(J(T)) \leq \lceil \frac{m}{2} \rceil + 1$. On the contrary, assume $\gamma_{te}(J(T)) \leq \lceil \frac{m}{2} \rceil + 1$. Then there exist 3 cut vertices u, v, w in D such that uv, vw are edges of $J(T)$ whee D is a γ_{te} -set of $J(T)$. By remark 4 $\gamma_{te}(J(T))$ does not exist which is a contradiction. This prove that $\gamma_{te}(J(T)) \leq \lceil \frac{m}{2} \rceil + 1$.

Nordhaus-Gaddum type results were obtained for many parameters for example, in [30, 31, 32, 33, 34, 35, 36].

Now we establish Nordhaus-Gaddum type result.

Theorem 11: Let $J(G)$ and $J(\bar{G})$ have no isolated vertices. If both $\gamma_{te}(J(G))$ and $\gamma_{te}(J(\bar{G}))$ exist, then $4 \leq \gamma_{te}(J(G)) + \gamma_{te}(J(\bar{G})) \leq p+3$.

Proof: Let $J(G)$ and $J(\bar{G})$ have no isolated vertices. If both $\gamma_{te}(J(G))$ and $\gamma_{te}(J(\bar{G}))$ exist, then $\gamma_{te}(J(G)) \geq 2$ and $\gamma_{te}(J(\bar{G})) \geq 2$ Therefore $4 \leq \gamma_{te}(J(G)) + \gamma_{te}(J(\bar{G}))$.

We have $\gamma_{te}(J(G)) \leq p - \Delta(J(G)) + 1$

Therefore $\gamma_{te}(J(G)) \leq p - \delta(J(G)) + 1$

Also we have $\gamma_{te}(J(\bar{G})) \leq p - \Delta(\bar{G}) + 1$

Thus
$$\begin{aligned} \gamma_{te}(J(G)) + \gamma_{te}(J(\bar{G})) &\leq 2p - [\delta(J(G)) + \Delta(\bar{G})] + 2 \\ &\leq p - (p - 1) + 2 \\ &\leq p + 3 \end{aligned}$$

The graph P_4 achieves the lower bound.

3. TOTAL EFFICIENT DOMATIC NUMBER

Definition 12: The total efficient dogmatic number $d_{te}(J(G))$ for some standard graphs.

Proposition 13: For any cycle $C_{4n}, n \geq 1$

$$d_{te}(J(C_{4n})) = 2$$

Proposition 14: For any complete bipartite graph $K_{m,n}, 1 \leq m \leq n$

$$d_{te}(J(K_{m,n})) = m$$

Proposition 15: For any tree T with $p \geq 2$ vertices,

$$d_{te}(J(T)) = 1$$

Proposition 16: Let $J(G)$ be a graph without isolated vertices, If $\gamma_{te}(J(G))$ exists, then

$$d_{te}(J(G)) \leq \frac{p}{\gamma_{te}(J(G))}$$

Proposition 17: Let $J(G)$ be a graph without isolated vertices If $d_{te}(J(G))$ exists, then

$$d_{te}(J(G)) \leq \delta(J(G)).$$

Proposition 18: If $J(G)$ is a graph without isolated vertices and if $\gamma_{te}(J(G))$ exists, then

$$\gamma_{te}(J(G)) + d_{te}(J(G)) \leq p + 1.$$

Furthermore, equality holds if $J(G) = mK_2, m \geq 1$

Proof: By theorem 11 we have

$$\gamma_{te}(J(G)) \leq p - \Delta(J(G)) + 1$$

 Or
$$\gamma_{te}(J(G)) \leq p - \delta(J(G)) + 1$$

By proposition 17 we have $\gamma_{te}(J(G)) \leq \delta(J(G))$.

Hence $\gamma_{te}(J(G)) + d_{te}(J(G)) \leq p + 1$.

We prove the second part.

If $J(G) = mK_2$, $m \geq 1$ then by theorem 9, $\gamma_{te}(J(G)) = p$. Also $d_{te}(J(G)) = 1$

Thus $\gamma_{te}(J(G)) + d_{te}(J(G)) = p + 1$.

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Source of support: Nil, Conflict of interest: None Declared.

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