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FUZZY HYPONORMAL OPERATOR IN FUZZY HILBERT SPACE

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ABSTRACT

In this paper we introduce the definition of Fuzzy Hyponormal Operator acting on a Fuzzy Hilbert space (FH-space) and investigate certain properties of Fuzzy Hyponormal operator using spectrum and we have given some definitions related to Fuzzy Hyponormal operator in FH-space.

Key words: *Adjoint Fuzzy operator, Fuzzy Hilbert space, Fuzzy Normal operator, Fuzzy Hyponormal operator, Spectrum of a fuzzy bounded linear operator, Self-Adjoint Fuzzy operator.*

1. INTRODUCTION

The notion of Fuzzy Norm on a linear space is first introduced by Katsaras[3] in 1984. Then after any other mathematicians have studied fuzzy normed space from several points of view [5], [12], [15]. The definition of fuzzy inner product space (FIP-space) headmost started by R.Biswas[10] and after that according the chronological in [6], [7], [11], [4], [9]. Modulate the definition of fuzzy inner product space (FIP-space) has been inserted by M.Goudarzi and S.M.Vaezpour in [8], [14]. The definition of a fuzzy Hilbert space has been introduced by M.Goudarzi and S.M.Vaezpour [8] in 2009. The concept of adjoint fuzzy linear operators, self - adjoint fuzzy linear operators was introduced by Sudad.M.Rasheed [2] in 2007, whereas A.Radharamani *et al*. [1] defined concept of fuzzy normal operator and their properties.

In this paper we consider fuzzy normal operator in FH-space and introduce the definitions of Fuzzy Hyponormal operator, we establish some theorems from fuzzy hyponormal operator in FH-space.

The classification of this paper is as follows:

In section 2 provides some preliminary definitions, theorems and results, which are used in this paper.In section 3 we introduce the concept of Fuzzy hyponormal operator on FH-space and some properties of fuzzy hyponormal operator have been studied.

2. PRELIMINARIES

Definition 2.1: [8] A fuzzy inner product space (FIP-Space) is a triplet (X,F,*),where X is a real vector space, * is a continuous t-norm, Fis a fuzzy set on $X^2\times \mathbb{R}$ satisfying the following conditions for every x,y,z$\in X$ and s,r,t$\in R$.
FI-1: F(x, x, 0) = 0 and F(x,x,t) > 0, for each t > 0.
FI-2: F(x,x,t)$\neq H(t)$ for some t$\in R$ if and only if x$\neq$ 0,where $H(t) = \begin{cases}1, & if\ t>0\\ 0, & if\ t\le 0\end{cases}$
FI-3: F(x,y,t)= F(y,x,t)
FI-4: For any $\alpha\in\mathbb{R}$,F (αx, y, t)= $\begin{cases} F\left(x,y,\frac{t}{\alpha}\right) & \alpha>0\\ H(t) & \alpha=0\\ 1-F\left(x,y,\frac{t}{-\alpha}\right) & \alpha<0\end{cases}$
FI-5: F(x,x,t)*F(y,y,s)≤F(x+y,x+y,t+s)
FI-6: $\text{Sup}_{s+r=t}$ [F(x,z,s)* F(y,z,r)]= F(x+y,z,t)
FI-7: F(x,y,.):$\mathbb{R}\to[0,1]$ *is continuous on* $\mathbb{R}\backslash\{0\}$
FI-8: $\lim_{t\to\infty} F(x,y,t)=1$

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Definition 2.2: [14] Let (E, F,*) be probabilistic inner product space.

1. Asequence $\{x_n\} \in E$is called $\mathcal{T}$ - converges to x$\in E$,if for any $\epsilon > 0$ and $\lambda > 0$, $\exists N \in Z^+$, N= N (ϵ, λ) such that $F x_n$-x,x_n-x (ϵ) >1- λ ,whenever n>N.
2. A linear Functional f(x) defined on E is called $\mathcal{T}_F$ -continuous, if $x_n \xrightarrow{\mathcal{T}_F} x$ implies $f(x_n) \xrightarrow{\mathcal{T}_F}$ f(x) for any $\{x_n\}$, x$\in E$

Theorem 2.3: [8] Let(X,F,*) be a FIP-space, where * is strong t- norm, and for each $x, y \in X$, Sup{t$\in \mathbb{R}$:F(x,y,t)<1}<∞.Define$\langle .,. \rangle$:X×X→ $\mathbb{R}$ by $\langle x,y \rangle = S$up {t$\in \mathbb{R}$:F(x,y,t)<1}. Then (X,$\langle .,. \rangle$) is an IP- space (inner product space), so that (X,$\|.\|$)is a N-space (normed space), where$\|.\|=\langle x,x \rangle^{1/2}$, $\forall x \in X$.

Definition 2.4: [8] Let (X, F,*) be a FIP- space with IP $\langle x,y \rangle = S$up {t$\in \mathbb{R}$:F(x,y,t)<1},$\forall x, y \in X$. If X is complete in the $\|.\|$, then X is called Fuzzy Hilbert – space (FH-space).

Theorem 2.5: [8] Let(X,F,*) be a FH- space with IP $\langle x,y \rangle = S$up {t$\in \mathbb{R}$:F(x,y,t)<1},$\forall x, y \in X$ for $x_n \in X$ and $x_n \xrightarrow{\|.\|} x$, then $x_n \xrightarrow{\mathcal{T}_F} x$.

Theorem 2.6: (Riesz theorem) [8], [14]
Let(X, F,*) be FH- space. For any $\mathcal{T}_F$ – continuous functional, ∃ unique $y \in X$ such that for all $x \in X$, we have g(x)= Sup {t$\in \mathbb{R}$:F(x,y,t)<1}.

Theorem 2.7: [1] Let (E,G,*) be a FIP space, where * is strong t- norm, and Sup {x ∈ $\mathbb{R}$:G(u,v,t)<1}<∞ for all u,v $\in E$, then Sup{x$\in \mathbb{R}$:G(u+v, w, x)<1}= Sup{x$\in \mathbb{R}$:G(u,w,x)<1}+Sup {x$\in \mathbb{R}$:G(v,w,x)<1}, $\forall u, v, w \in E$.

Remark 2.8: [1] Let FB (E) the set of all fuzzy linear operators on E.

Theorem 2.9: [1] (Adjoint Fuzzy operator in FH-space)
Let (E, G,*) be a FH-space, Let S$\in FB(E)$ be $\mathcal{T}_F$ –continuous linear functional, then ∃ unique $S^* \in FB(E)$ such that $\langle Su,v \rangle = \langle u,S^*v \rangle$ }$\forall u, v \in E$.

Definition 2.10: [1] Let (E,G,*) be a FH-space with IP: $\langle u,v \rangle =$ Sup {x$\in \mathbb{R}$:G(u,v,x)<1}, $\forall u, v \in E$ and let S $\in FB(E)$,then S is self –adjoint Fuzzy operator. If S=S^* where S* is adjoint Fuzzy operator of S.

Theorem 2.11: [1] Let (E, G,*) be a FH-space with IP: $\langle u,v \rangle = S$up {x$\in \mathbb{R}$:G(u,v,x)<1} and let S $\in FB(E)$, then S is self –adjoint Fuzzy operator.

Theorem 2.12: [1] Let (E, G,*) be a FH-space with IP: $\langle u,v \rangle = S$up {x$\in \mathbb{R}$:G(u,v,x)<1},$\forall u, v \in E$ and let S^* be the adjoint Fuzzy operator of S$\in FB(E)$, then:

i. $(S^*)^*$=S
ii. $(\alpha S)^* = \alpha S^*$
iii. $(\alpha S + \beta T)^* = \alpha S^* + \beta T^*$ where α,β are scalars and T$\in FB(E)$.
iv. $(ST)^* = T^* S^*$

Theorem 2.13: [1] Let (E,G,*) be a FH-space with IP: $\langle u,v \rangle =$ Sup {x$\in \mathbb{R}$:G (u,v,x) < 1}, $\forall u, v \in E$ *and* let S $\in FB(E)$ then,$\|Su\| = \|S^*u\|$ for all u$\in E$.

Remark 2.14: Let (H, F, *) be a FH – space with IP:$\langle x, y \rangle = Sup\{u \in R: F(x, y, u) < 1\} \forall x, y \in H$ *and* If $U \in FB(H)$, it is easy to see that $U = 0 \Leftrightarrow \langle Ux, y \rangle = 0, \forall x, y \in H$.

Theorem 2.15: Let $U \in FB(H)$. Then $U = 0 \Leftrightarrow \langle Ux, x \rangle = 0, \forall x \in H$.

Proof: It is obvious that $U = 0 \Rightarrow \langle Ux, x \rangle = 0, \forall x \in H$.

Conversely, it suffices to show that $\langle Ux, y \rangle = 0,\ for\ any\ x, y \in H$. The proof of this depends on the following easily verified identity,

$$\langle U(\alpha x + \beta y), \alpha x + \beta y \rangle - |\alpha|^2 \langle Ux, x \rangle - |\beta|^2 \langle Uy, y \rangle = \alpha \bar{\beta} \langle Ux, y \rangle + \bar{\alpha} \beta \langle Uy, x \rangle \quad (1)$$

By hypothesis equation (1) holds and equals 0, for all α and β.

If we put $\alpha = 1$ and $\beta = 1$, then $\langle Ux, y \rangle + \langle Uy, x \rangle = 0$ (2)

And if we put $\alpha = i$ and $\beta = 1$, then $i \langle Ux, y \rangle - i \langle Uy, x \rangle = 0$ (3)

Dividing (3) by i and adding the result to (2), we get
$\langle Ux,y\rangle + \langle Uy,x\rangle + \langle Ux,y\rangle - \langle Uy,x\rangle = 0 \quad \Rightarrow 2\langle Ux,y\rangle = 0,$ so $\langle Ux,y\rangle = 0 \quad \Rightarrow U = 0.$

The proof is complete.

Definition 2.16: [1] Fuzzy Normal Operator
Let $(X,F,*)$ be a FH-space with IP:$\langle u,v\rangle = \sup\{x \in R: F(u,v,x) < 1\}\, \forall u,v \in X$ and let $S \in FB(X)$. Then S is a fuzzy normal operator if it commutes with its (fuzzy) adjoint i.e.$SS^* = S^*S$.

Definition 2.17: [18] Fuzzy Invariant
Let $(X,F,*)$ be a fuzzy normed linear space, let $T \in FB(X)$, A subspace M of a fuzzy normed linear space X is said to be fuzzy invariant under T, if $TM \subset M$.

Theorem 2.18: [18] Let T be a normal operator on a finite dimensional fuzzy Hilbert space H over R then,

(i) T-λI *is fuzzy normal.*
(ii) Every eigen vector of T is also eigen vector for T^*

Theorem 2.19: [18] Let (H,F,$*$) be a fuzzy Hilbert space and T$\in FB(H)$ then T=0 iff

$$\sup\{x \in R: F(Tu,Tu,x) < 1\} = 0\ \forall\, u \in H.$$

Theorem 2.20: [18] A Closed subspace M of a fuzzy Hilbert space H reduces an operator T iff M is invariant under T and T^*.

Definition 2.21: [18] Let M be a closed subspace of a fuzzy Hilbert space H, and let T$\in FB(H)$, we say that T is reduced by M if both M and $M^{\perp}$ are invariant under T if T is reduced by M, then we also say that M reduces T.

Theorem 2.22: [18] A closed subspace M of a fuzzy Hilbert space H reduces an operator T iff M is invariant under both T and T^*.

Definition 2.23: [18] Let M be a closed of a fuzzy Hilbert space X and x$\notin M$ *is said that projection of* $x \in X$ *onto* M *if* $z \in M$, $N(x-z,t) = \sup\left\{\frac{t}{t+\|x-y\|}; y \in M, t > 0\right\}$, *we write* $y = P_M(x)$.

Theorem 2.24: [18] Let M is subspace of a fuzzy Hilbert space X, then
X=M $\oplus M^{\perp}$ *i.e each* $x \in X$ *can be uniqully decomposed from* $x = y + z$ *with* $y \in M, z \in M^{\perp}$.

Theorem 2.25: [18] Let M be a closed subspace of a fuzzy Hilbert space X over F ,and let T$\in FB(H)$, then M is invariant under T iff $M^{\perp}$ *is invariant under* T^*.

Definition 2.26: [18] Let (X,N, $*$) be a fuzzy normed linear space over the field $\mathbb{C}$,
where $x \neq \{0\}$ *and* $N: X \times (0,\infty) \to [0,1]$ *and* $T: (X,N,*) \to (X,N,*)$ *bea linear operator,*

A regular value (eigen value) λ *of T is complex number* $\ni$ (i).$\mathcal{R}_\lambda$(T) exist (ii). $\mathcal{R}_\lambda$(T) is fuzzy bounded linear operator on range of T_λ= T-λI (iii). $\mathcal{R}_\lambda$(T) is defined on a set which is dence in X ,where $\mathcal{R}_\lambda$(T)= $(T_\lambda)^{-1}$ =$(T-\lambda I)^{-1}$ is called resolvent operator of T. it is denoted by $\rho(T)$ and the resolvent set $\rho(T)$ of T is the set of all regular value λ *of T*. Its complement $\sigma(T) = \mathbb{C} - \rho(T)$ in complex plane $\mathbb{C}$ *is called spectrum of T, a* $\lambda \in \sigma(T)$ is called spectral value of T.

3. MAIN RESULTS

In this section we introduce the definition of fuzzy hyponormal operator in FH-space as well as some elementary properties of fuzzy hyponormal operator in FH-space are presented.

Definition 3.1: Let $(H,F,*)$ be a FH – space with IP: $\langle x,y\rangle = Sup\,\{x \in R: F(u,v,x) < 1\}\ \forall\, u,v \in H$ *and let* $T \in FB(H)$. Then T is a fuzzy hyponormal operator if $\|T^*u\| \le \|Tu\|\ \forall\, u \in H$ or equivalently $T^*T - TT^* \ge 0$.

Note: The term operator will mean fuzzy bounded linear transformation.

Theorem 3.2: Let $(H,F,*)$ be a FH – space with IP: $\langle x,y\rangle = Sup\,\{x \in R: F(u,v,x) < 1\}\ \forall\, u,v \in H$ *and let* $T \in FB(H)$ be a fuzzy hyponormal operator on H. Then $\|(T-zI)u\| \ge \|(T^* - \bar{z}I)u\|\ \ \forall\, u \in H$, i.e. $T - zI$ is FHN.

Proof: Given T be a FHN – operator on H.

Let $\|(T-zI)u\|^2 = \langle (T-zI)u, (T-zI)u \rangle$
$= \sup\{x \in R: F\big((T-zI)u, (T-zI)u, x\big) < 1\}$
$= \sup\{x \in R: F(u, (T-zI)^*(T-zI)u, x) < 1\}$
$\geq \sup\{x \in R: F(u, (T-zI)^*(T-zI)^*u, x) < 1\}$ [since, by definition of FHN]
$\geq \langle ((T-zI)^*u, (T-zI)^*u) \rangle$
$\therefore \quad \|(T-zI)u\|^2 \geq \|(T-zI)^*u\|^2$
$\Rightarrow \quad \|(T-zI)u\| \geq \|(T-zI)^*u\|$
$i.e. \|(T-zI)u\| \geq \|\overline{(T-zI)}u\|$
Thus $\|(T-zI)u\| \geq \|(T^* - \bar{z}I)u\|$

Theorem 3.3: Let T be a FHN – operator on the Hilbert space H. Then $Tu = \lambda u \Rightarrow T^*u = \bar{\lambda}u.$

Proof: Let u be an eigen vector of T corresponding to the eigen value λ.
$$\Rightarrow Tu = \lambda u$$

Now $\sup\{x \in R: F(Tu, Tu, x) < 1\} = \sup\{x \in R: F(u, T^*Tu, x) < 1\}$
$= \sup\{x \in R: F(u, TT^*u, x) < 1\}$ [since, T is Fuzzy Normal operator]
$= \sup\{x \in R: F(T^*u, T^*u, x) < 1\}$

Since $T-\lambda I$ is fuzzy hyponormal, therefore $u \in H$
$$\sup\{x \in R: F\big((T-\lambda I)u, (T-\lambda I)u, x\big) < 1\} \geq \sup\{x \in R: F((T-\lambda I)^*u, (T-\lambda I)^*u, x) < 1\}$$
Since $Tu = \lambda u$
$\Rightarrow \; Tu = \lambda I u \quad \Rightarrow \; Tu - \lambda I u = 0 \quad \Rightarrow (T-\lambda I)u = 0$
$\therefore \; T - \lambda I = 0.$
$\sup\{x \in R: F\big((T-\lambda I)u, (T-\lambda I)u, x\big) < 1\} = 0 \quad \forall\, u \in H$ (3.1)

From (2.19) $T = 0 \;\; iff \;\; \sup\{x \in R: F(Tu, Tu, x) < 1\} = 0$
$\Rightarrow \sup\{x \in R: F((T-\lambda I)^*u, (T-\lambda I)^*u, x) < 1\} = 0 \;\; \forall\, u \in H$

From (3.1) $(T-\lambda I)^* = 0$ then for each $u \in H$
$(T-\lambda I)^*u = 0$
$\Rightarrow \big(T^* - \bar{\lambda} I\big)u = 0$
$\Rightarrow T^*u - \bar{\lambda} I u = 0$
$\Rightarrow T^*u = \bar{\lambda}u$
$\therefore$ u is an eigen vector of T^* corresponding to eigen value $\bar{\lambda}$.

Theorem 3.4: Let T be fuzzy hyponormal iff $\|T^*u\| \leq \|Tu\|$for all $u \in H$.

Proof: Since T is a fuzzy hyponormal, $T^*T - TT^* \geq 0$
$\Rightarrow T^*T \geq TT^*$
i.e. $TT^* \leq T^*T$
Let $\|T^*u\| \leq \|Tu\|$
$\Leftrightarrow \|T^*u\|^2 \leq \|Tu\|^2$
$\Leftrightarrow \langle T^*u, T^*u \rangle \leq \langle Tu, Tu \rangle$
$\Leftrightarrow \sup\{x \in R: F(T^*u, T^*u, x) < 1\} \leq \sup\{x \in R: F(Tu, Tu, x) < 1\}$
$\Leftrightarrow \sup\{x \in R: F(TT^*u, u, x) < 1\} \leq \sup\{x \in R: F(T^*Tu, u, x) < 1\}$
$\Leftrightarrow \langle TT^*u, u \rangle \leq \langle T^*Tu, u \rangle$
$\Leftrightarrow \langle (TT^* - T^*T)u, u \rangle \leq 0$
$\Leftrightarrow TT^* - T^*T \leq 0$
$\Leftrightarrow TT^* \leq T^*T$
i.e. $T^*T - TT^* \geq 0$

Theorem 3.5: Let T be FHN on H. Then $\|T\| = R_{sp}(T)$ (the spectral radius of T).

Proof: For $u \in H$, $\|u\| = 1$,
$\|Tu\|^2 = \langle Tu, Tu \rangle$
$= \sup\{x \in R: F(Tu, Tu, x) < 1\}, u \in H$
$= \sup\{x \in R: F(T^*Tu, u, x) < 1\}, u \in H$
$= \langle T^*Tu, u \rangle$
$\leq \|T^*Tu\|$
$\therefore \|Tu\|^2 \leq \|T^2u\|$

But then $\|T\|^2 \leq \|T^2\| \leq \|T\|^2$ which implies that $\|T\|^2 = \|T^2\|$.

Now $\|T^n u\|^2 = \langle T^n u, T^n u\rangle$
$= \langle T^n u, TT^{n-1}u\rangle$
$= \langle T^*T^n u, T^{n-1}u\rangle$
$\leq \|T^*T^n u\|\|T^{n-1}u\|$
$\leq \|T^*TT^{n-1}u\|\|T^{n-1}u\|$
$\leq \|T^{n-1}u\|\|T^{n-1}u\|$

Then $\|T^n\|^2 \leq \|T^{n-1}\|\|T^{n-1}\|$

And combining this with the equality, a simple induction argument yields,
$\|T^n\| = \|T\|^n$ for n=1,2,3, …

Since $R_{sp}(T) = \lim_{n\to\infty} \|T^n\|^{\frac{1}{n}}$
$R_{sp}(T) = \lim_{n\to\infty} \|T\|$
i.e. $R_{sp}(T) = \|T\|$ where $R_{sp}(T)$ means the spectral radius of T.

Theorem 3.6: Let $T \in FB(H)$ be a fuzzy hyponormal operator with $Tu_1 = \lambda_1 u_1$, $Tu_2 = \lambda_2 u_2$ and $\lambda_1 \neq \lambda_2$ then $\langle u_1, u_2\rangle = 0$.

Proof: Since T be a fuzzy hyponormal operator with $Tu_1 = \lambda_1 u_1, Tu_2 = \lambda_2 u_2$ and $\lambda_1 \neq \lambda_2$ then by theorem (3.3) $T^*u_1 = \overline{\lambda}_1 u_1$ and $T^*u_2 = \overline{\lambda_2} u_2$.

Let $\lambda_1\langle u_1, u_2\rangle = \langle \lambda_1 u_1, u_2\rangle$
$= \sup\{x \in R: F(\lambda_1 u_1, u_2, x) < 1\}$
$= \sup\{x \in R: F(Tu_1, u_2, x) < 1\}$
$= \sup\{x \in R: F(u_1, T^*u_2, x) < 1\}$
$= \sup\{x \in R: F\left(u_1, \overline{\lambda_2} u_2, x\right) < 1\}$
$= \sup\{x \in R: F(\lambda_2 u_1, u_2, x) < 1\}$
$= \langle \lambda_2 u_1, u_2\rangle$
$= \lambda_2\langle u_1, u_2\rangle$
Hence if $\lambda_1 \neq \lambda_2$ then $\langle u_1, u_2\rangle = 0$. i.e. $u_1 \perp u_2$.

Theorem 3.7: Let $(H, F, *)$ be a FH-space with IP:$\langle u, v\rangle = \sup\{x \in R: F(u, v, x) < 1\}\,\forall u, v \in H$ and let $T \in FB(H)$ is a fuzzy hyponormal operator on H with
M$\subset H\ invariant\ under\ T\ and\ let\ T_M\ ba\ fuzzy\ hyponormal\ then\ M\ reduces\ T$

Proof: Let u$\in M\ the\ eigen\ space\ of\ T, and\ corresponding\ eigen\ value\ be\ \lambda. so\ that\ Tu = \lambda u$. Since T is fuzzy normal then by theorem 2.8 eigen values for T^* . that is $T^*u = \overline{\lambda} u$, $u \in H$.

Since M is a subspace, $\overline{\lambda} u \in M \Rightarrow T^*u \in M$
$\Rightarrow M\ is\ invarient\ under T^*$, $but\ M\ is\ invarient\ under T$.

By a known theorem 2.20, M is reduces T.

Corollary 1: Let T be a fuzzy hyponormal on H and
M={ $x \in H$;. $Tu = \lambda u\}\ then\ M\ reduces\ T and T_M\ is\ fuzzy\ normal$.

Corollary 2; Let T be a fuzzy hyponormal on H and M$\subset H\ invarient\ under T\ \ then\ T_M\ is\ fuzzy\ hyponormal$.

Theorem 7: Let T be a fuzzy hyponormal on H and $\lambda_0\ be\ an\ isolated\ point\ in\ the\ spectrum\ of\ T\ \ then\ \lambda_0 \in \sigma_P(T) the\ point\ spectrum\ of\ T$.

Proof: By theorem 1, we may assume that λ_0=0, choose R>0 sufficiently small that 0 is the only point of $\sigma\ (T)$, $contained\ in\ or\ on\ the\ circle$ $|\lambda|$=R

Define E=$\int_{|\lambda|=R}(T - \lambda I)^{-1}\ d\lambda$ then E is a non zero projection which commutes with T.
Thus EH is invariant under T and T_{EH} is fuzzy hyponormal, also $\sigma(T_{EH}$)= $\sigma(T) \cap \{|z| = R\}$

Also $\sigma(T_{EH}$)={0}

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From the last corollary we may conclude that T_{EH} $is\ the\ zero\ transformation$.

It is clear that EH={ $u \in H; Tu = 0$} ⇒ EH reduces T.

Theorem 8: Let $(H, F,*)$ be a FH-space with IP:$\langle u, v\rangle = \sup\{x \in R: F(u, v, x) < 1\}\, \forall u, v \in H$ and let $T \in FB(H)$ is a fuzzy hyponormal operator on H with a single limit point in its spectrum then T is fuzzy hyponormal.

Proof: We may assume that by theorem 1, that the limit point is zero.

By theorem 7, there exist an isolated point in the $\lambda_1 \in \sigma(T) \ni |z_1|=\|T\|$

Let M_1={ $u \in H; Tu = \lambda_1 u$}
M_1 is not empty by theorem 7, and since M_1 reduces T. we conclude from theorem 6, that $T_{M_{1\perp}}$ does not have λ_1 in its spectrum. By corollary 2 that T_{M_1} is fuzzy normal.

We continue in this way, selecting points in $\sigma(T)$ ordered by absolute value , setting M_i={ $u \in H; Tu = \lambda_i u$} then $M_1 \oplus M_2 \oplus \ldots\ldots.\oplus M_n$ $reduces\ T\ and T_{M1\ \oplus M_2 \oplus\ \ldots\ldots\oplus M_n}$ $is\ fuzzy\ normal\ we\ observe\ that$ $T_{M1\ \oplus M_2 \oplus\ \ldots\ldots\oplus M_n\perp}$ is fuzzy hyponormal with its spectral radius equal to its fuzzy norm.

Thus, since 0 is the only point of $\sigma(T)$, the fuzzy hyponormal operator converge to T in the uniform operator topology. Hence T is fuzzy normal.

Cor: If T is an fuzzy hyponormal, completely continous operator then T is fuzzy normal

4. CONCLUSION

As the idea of fuzzy hyponormal operator in FH-space is relatively new and classic form of theorems play a role the prototype in our discussion of this paper. Some concepts and properties have been investigated about fuzzy hyponormal operator in fuzzy Hilbert space. The results of this paper will be helpful for researchers to develop fuzzy functional analysis.

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