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RESTRICTIONS OF PRE A*-ALGEBRA FUNCTIONS

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ABSTRACT

In this paper restriction of Pre A-algebra function has been derived. Shannon expansion of Pre A*-algebra function is explained with an example. Theorems related to the restriction have been proved.*

***Key words**: Restriction of Pre A*-algebra function, Shannon expansion*

1. INTRODUCTION

In 1994, P. Koteswara Rao [1] first introduced the concept of A*-algebra $\left(A,\wedge,\vee,*,(-)^{\sim}\,(\text{-})_{\pi},0,1,2\right)$.

In 2000, J.Venkateswara Rao[2] introduced the concept Pre A*-algebra $\left(A,\wedge,\vee,(-)^{\sim}\right)$ analogous to C-algebra as a reduct of A*- algebra. In [4] ternary operation on Pre-A* algebra have been proved and studied the properties. J.Venkateswara Rao [5] analyze the properties of PreA*-function. He defined implicants of Pre A*-algebra function[6].

2. PRELIMINARIES

Definition 2.1 [4]: An algebra $(A,\wedge,\vee,(-)^{\sim})$ where A is non-empty set with 1, $\wedge$, $\vee$ are binary operations and $(-)^{\sim}$ is a unary operation satisfying

(a) $x^{\sim\sim}=x, \quad \forall x \in A$

(b) $x \wedge x = x, \quad \forall x \in A$

(c) $x \wedge y = y \wedge x, \quad \forall x, y \in A$

(d) $(x \wedge y)^{\sim} = x^{\sim} \vee y^{\sim}, \quad \forall x, y \in A$

(e) $x \wedge (y \wedge z) = (x \wedge y) \wedge z, \quad \forall x, y, z \in A$

(f) $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z), \quad \forall x, y, z \in A$

(g) $x \wedge y = x \wedge (x^{\sim} \vee y), \quad \forall x, y \in A$.

is called a Pre A*-algebra

Example 2.1[4]: 3 = {0, 1, 2} with operations $\wedge,\vee,(-)^{\sim}$ defined below is a Pre A*-algebra.

$\wedge$	0	1	2
0	0	0	2
1	0	1	2
2	2	2	2

$\vee$	0	1	2
0	0	1	2
1	1	1	2
2	2	2	2

x	$x^{\sim}$
0	1
1	0
2	2

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Lemma 2.2 [4]: Every Pre A*-algebra with 1 satisfies the following laws

(a) $x \vee 1 = x \vee x^{\sim}$ (b) $x \wedge 0 = x \wedge x^{\sim}$

Lemma 2.3 [4]: Every Pre A*-algebra with 1 satisfies the following laws.

(a) $x \wedge (x^{\sim} \vee x) \quad x \forall (x^{\sim} \wedge x) = x$

(b) $(x \vee x^{\sim}) \wedge \mathrm{y} = (\mathrm{x} \wedge \mathrm{y}) \vee (\mathrm{x}^{\sim} \wedge \mathrm{y})$

(c) $(x \vee y) \wedge \mathrm{z} = (x \wedge \mathrm{z}) \vee (x^{\sim} \wedge \mathrm{y} \wedge \mathrm{z})$

Definition 2.4 [4]: Let A be a Pre A*-algebra. An element $x \in A$ is called central element of A if $x \vee x^{\sim}=1$ and the set { $x \in A / x \vee x^{\sim}=1$ } of all central elements of A is called the centre of A and it is denoted by $B(A)$.

Theorem 2.5 [4]: Let A be a Pre A*-algebra with 1, then $B(A)$ is a Boolean algebra with the induced operations $\wedge, \vee, (\text{-})^{\sim}$

Theorem 2.6 [4]: Let A is a Pre A*-algebra with 1. Then A has trivial centre if and only if $A = \overline{A_o}$, for some Pre A*-algebra A_o.

Lemma 2.7 [4]: Let A be a Pre A*-algebra with 1 ,

(a) If $y \in B(A)$ then $x \wedge x^{\sim} \wedge \mathrm{y} = x \wedge x^{\sim}, \forall x \in \mathrm{A}$

(b) If $x, y \in B(\mathrm{A})$ then $x \wedge (x \vee y) = x \vee (x \wedge y) = x$

Lemma 2.8 [4]: Let A be a Pre A*algebra with 1, 0 and let $x, y \in A$

(a) If $x \vee y = 0,$ then $x = y = 0$ (b) If $x \vee y = 1,$ then $x \vee x^{\sim} = 1$

Theorem 2.9 [4]: Let A be a Pre A*-algebra with 1 and $x, y \in A$, if $x \wedge y = 0$, $x \vee y = 1,$ then $y = x^{\sim}$

Definition 2.10[7]: A Pre A*-algebra function is said to be in disjunctive normal form in n variables $x_1, x_2, x_3, \ldots\ldots\ldots\ldots x_n$ if it can be written as join of terms of the type $f_1(x_1) \wedge f_2(x_2) \wedge \ldots\ldots\ldots f_n(x_n)$ where $f_i(x_i) = x_i \ \ or \ \ x_i^{\sim} \quad \forall i = 1 \ \ to \ \ n$ and no two terms are same. $f_1(x_1) \wedge f_2(x_2) \wedge \ldots\ldots\ldots f_n(x_n)$ are called minterms or minimal polynomials.

Thus a minterm in n variables is a product of n literals in which each variable is represented by the variable itself or its complement.

Definition 2.11[7]: If a DNF contains all the possible minterms then it is complete DNF.

Definition 2.12[7]: A Pre A*-algebra function is said to be in conjunctive normal form in n variables $x_1, x_2, x_3, \ldots\ldots\ldots\ldots x_n$ if it can be written as meet of terms of the type $f_1(x_1) \vee f_2(x_2) \vee \ldots\ldots\ldots f_n(x_n)$ where $f_i(x_i) = x_i \ \ or \ \ x_i^{\sim} \quad \forall i = 1 \ \ to \ \ n$ and no two terms are same. $f_1(x_1) \vee f_2(x_2) \vee \ldots\ldots\ldots f_n(x_n)$ are called maxterms or maximal polynomials

3. RESTRICTION OF PRE A*-ALGEBRA FUNCTION

If X_1 is any subset of X, the restriction of function is the function $f_{|X_1}$ from X_1 to Y.

If $f_{|X_1}$ is the restriction of f, then f is the extension of $f_{|X_1}$.Informally, a restriction of a function f is the result of trimming its domain.

Definition 3.1: Let f be a Pre A*-function on A^n and let $k \in \{1, 2, \ldots\ldots\ldots n\}$. We denote by $f_{|\alpha_k=2}$, $f_{|\alpha_k=1}$, and $f_{|x_k=0}$ respectively, the Pre A*-function defined as follows:

for every $(\alpha_1, \alpha_2 \ldots\ldots \alpha_{k-1}, \alpha_{k+1} \ldots \alpha_n) \in A^{n-1}$

$$f_{|x_k=2}(\alpha_1, \alpha_2 \ldots \alpha_{k-1}, \alpha_{k+1} \ldots \alpha_n) = f(2)$$

$f_{|x_k=1}(\alpha_1,\alpha_2....\alpha_{k-1},\alpha_{k+1}...\alpha_n) = f(\alpha_1,\alpha_2....\alpha_{k-1},1,\alpha_{k+1}...\alpha_n)$

$f_{|x_k=0}(\alpha_1,\alpha_2....\alpha_{k-1},\alpha_{k+1}...\alpha_n) = f(\alpha_1,\alpha_2....\alpha_{k-1},0,\alpha_{k+1}..\alpha_n)$

$f_{|\alpha_k=2}$ is the restriction of f to $f(2)$

$f_{|\alpha_k=1}$ is the restriction of f to $f(\alpha_1,\alpha_2....\alpha_{k-1},1,\alpha_{k+1}...\alpha_n)$ in which $\alpha_k = 1$

$f_{|\alpha_k=0}$ is the restriction of f to $f(\alpha_1,\alpha_2....\alpha_{k-1},0,\alpha_{k+1}...\alpha_n)$ in which $\alpha_k = 0$

Even though $f_{|\alpha_k=2}, f_{|\alpha_k=1}$, and $f_{|\alpha_k=0}$ are by definition, functions of (n-1) variables, it is considered as functions on A^n rather then A^{n-1} for every $(\alpha_1,\alpha_2.......\alpha_n) \in A^n$,we simply let

$f_{|x_k=2}(\alpha_1,\alpha_2...\alpha_{k-1},\alpha_{k+1}...\alpha_n) = f(2)$

$f_{|x_k=1}(\alpha_1,\alpha_2...\alpha_{k-1},\alpha_{k+1}...\alpha_n) = f(\alpha_1,\alpha_2...\alpha_{k-1},1,\alpha_{k+1}...\alpha_n)$

$f_{|x_k=0}(\alpha_1,\alpha_2....\alpha_{k-1},\alpha_{k+1}...\alpha_n) = f(\alpha_1,\alpha_2....\alpha_{k-1},0,\alpha_{k+1}..\alpha_n)$

Theorem 3.2: Let f be a Pre A*-algebra function on A^n. Let ψ be a representation of f and let $k \in \{1,2,........n\}$ Then the expression obtained by substituting the constant 0 or 1 or 2 for every occurrence of x_k in ψ represents $f_{|x_k=0}$ or $f_{|x_k=1}$ or $f_{|x_k=2}$.

Proof: This is an immediate consequence of above definition.

Example 3.3: Consider Pre A*-function

$f = (\alpha \wedge \beta) \vee (\alpha \wedge \gamma) \vee (\beta \wedge \gamma)$

We derive the following expressions for $f_{|\alpha_k=2}, f_{|\alpha_k=1}$, and $f_{|\alpha_k=0}$

$f = (\alpha \wedge \beta) \vee (\alpha \wedge \gamma) \vee (\beta \wedge \gamma)$

$f_{|\alpha=2} = (2 \wedge \beta) \vee (2 \wedge \gamma) \vee (\beta \wedge \gamma) = 2 \vee 2 \vee (\beta \wedge \gamma) = 2$

$f_{|\alpha=1} = (1 \wedge \beta) \vee (1 \wedge \gamma) \vee (\beta \wedge \gamma) = \beta \vee \gamma \vee (\beta \wedge \gamma) = (\beta \vee \gamma)$

$f_{|\alpha=0} = (0 \wedge \beta) \vee (0 \wedge \gamma) \vee (\beta \wedge \gamma) = 0 \vee 0 \vee (\beta \wedge \gamma) = (\beta \wedge \gamma)$

Theorem 3.4: Let f be a Pre A*-function on A^n and let $k \in \{1,2,...n\}$.

Then $f(\alpha_1,\alpha_2,......\alpha_n) = \alpha_k f_{|\alpha_{k=2}} \vee \alpha_k^{\sim} f_{|\alpha_{k=2}} \vee \alpha_k f_{|\alpha_{k=1}} \vee \alpha_k^{\sim} f_{|\alpha_{k=0}}$ for all $(\alpha_1,\alpha_2,......\alpha_n) \in A^n$.

Proof: This is immediate by substitute of the values $\alpha_k = 2, \alpha_k = 1,$ *or* $\alpha_k = 0$

$f(\alpha_1,\alpha_2,...2...\alpha_n) = 2 f_{|\alpha_{k=2}}$

$f(\alpha_1,\alpha_2,...1..\alpha_n) = 1 f_{|\alpha_k=1}$

$f(\alpha_1,\alpha_2,...0..\alpha_n) = 0^{\sim} f_{|\alpha_{k=0}}$

$f(\alpha_1,\alpha_2,......\alpha_n) = 2 f_{|\alpha_{k=2}} \vee 2 f_{|\alpha_{k=2}} \vee 1 f_{|\alpha_k=1} \vee 0^{\sim} f_{|\alpha_{k=0}}$

Example 3.5: Consider the function $f = (\alpha \wedge \beta) \vee (\alpha \wedge \gamma) \vee (\alpha^{\sim} \wedge \beta) \vee (\beta \wedge \gamma^{\sim})$

The expansion of $f_{|\beta=1}$ with respect to α is $\alpha f_{|\beta=1\alpha=1} \vee \alpha^{\sim} f_{|\beta=1\alpha=0} \vee \alpha f_{|\beta=1\alpha=2} \vee \alpha^{\sim} f_{|\beta=1\alpha=2}$

$f = (\alpha \wedge \beta) \vee (\alpha \wedge \gamma) \vee (\alpha^{\sim} \wedge \beta) \vee (\beta \wedge \gamma^{\sim})$

$$f_{|\beta=1\alpha=1} = (1\wedge 1)\vee(1\wedge\gamma)\vee(0\wedge 1)\vee(1\wedge\gamma^{\sim})$$
$$=1\vee\gamma\vee 0\vee\gamma^{\sim}=1$$
$$f_{|\beta=1\alpha=0} = (0\wedge 1)\vee(0\wedge\gamma)\vee(1\wedge 1)\vee(1\wedge\gamma^{\sim})$$
$$=0\vee 0\vee 1\vee\gamma^{\sim}=\gamma^{\sim}$$
$$f_{|\beta=1\alpha=2} = (2\wedge 1)\vee(2\wedge\gamma)\vee(2\wedge 1)\vee(1\wedge\gamma^{\sim})$$
$$=2\vee 2\vee 2\vee\gamma^{\sim}=2$$

The expansion of $f_{|\beta=1}$ with respect to α is

$$\alpha f_{|\beta=1\alpha=1}\vee\alpha^{\sim} f_{|\beta=1\alpha=0}\vee\alpha f_{|\beta=1\alpha=2}\vee\alpha^{\sim} f_{|\beta=1\alpha=2} = 1(1)\vee 0(\gamma^{\sim})\vee 2(2)\vee 2(2)=2$$

The expansion of $f_{|\beta=0}$ with respect to α is

$$f_{|\beta=0\alpha=1} = (1\wedge 0\ \vee)(1\wedge\gamma)\vee(0\wedge 0\ \vee)(0\wedge\gamma^{\sim})$$
$$=0\vee\gamma\vee 0\vee 0=\gamma$$
$$f_{|\beta=0\alpha=0} = (0\wedge 0)\vee(0\wedge\gamma)\vee(1\wedge 0)\vee(0\wedge\gamma^{\sim})$$
$$=0\vee 0\vee 0\vee 0=0$$
$$f_{|\beta=0\alpha=2} = (2\wedge 0)\vee(2\wedge\gamma)\vee(2\wedge 0)\vee(0\wedge\gamma^{\sim})$$
$$=2\vee 2\vee 2\vee 0=2$$

The expansion of $f_{|\beta=0}$ with respect to α is

$$\alpha f_{|\beta=0\alpha=1}\vee\alpha^{\sim} f_{|\beta=0\alpha=0}\vee\alpha f_{|\beta=0\alpha=2}\vee\alpha^{\sim} f_{|\beta=0\alpha=2} = 1(\gamma)\vee 1(0)\vee 2(2)=2$$

The expansion of $f_{|\beta=2}$ with respect to α is

$$f_{|\beta=2\alpha=1} = (1\wedge 2\ \vee)(1\wedge\gamma)\vee(0\wedge 2\ \vee)(2\wedge\gamma^{\sim})$$
$$=2\vee\gamma\vee 2\vee 2=2$$
$$f_{|\beta=2\alpha=0} = (0\wedge 2\ \vee)(0\wedge\gamma)\vee(1\wedge 2\ \vee)(2\wedge\gamma^{\sim})$$
$$=2\vee 0\vee 2\vee 2=2$$
$$f_{|\beta=2\alpha=2} = (2\wedge 2)\vee(2\wedge\gamma)\vee(2\wedge 2)\vee(2\wedge\gamma^{\sim})$$
$$=2\vee 2\vee 2\vee 2=2$$
$$\alpha f_{|\beta=1\alpha=1}\vee\alpha^{\sim} f_{|\beta=1\alpha=0}\vee\alpha f_{|\beta=1\alpha=2}\vee\alpha^{\sim} f_{|\beta=1\alpha=2} = 2$$

Similarly we can write the expansion for $\beta=2$ with respect to γ.

Note 3.6: The expansion $\alpha f_{|\beta=1\alpha=1}\vee\alpha^{\sim} f_{|\beta=1\alpha=0}\vee\alpha f_{|\beta=1\alpha=2}\vee\alpha^{\sim} f_{|\beta=1\alpha=2}$ is called as Shannon expansion. By applying this expansion to a function and its restriction becomes 0 or 1 or 2 or a literal.

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