International Journal of Mathematical Archive-9(12), 2018, 41-49
Available online through www.ijma.info ISSN 2229 – 5046

ON F-LEAP INDICES AND F-LEAP POLYNOMIALS OF SOME GRAPHS

V. R. KULLI

Department of Mathematics, Gulbarga University, Gulbarga 585106, India.

(Received On: 17-11-18; Revised & Accepted On: 10-12-18)

ABSTRACT

We introduce the F-leap and F_1-leap indices of a graph. In this paper, the F-leap and F_1-leap indices and their polynomials of wheel graphs, gear graphs, helm graphs, flower graphs and sunflower graphs are determined.

Keywords: *F-leap index, F_1-leap index, wheel, helm graph, flower graph.*

Mathematics Subject Classification: *05C07, 05C12, 05C76.*

1. INTRODUCTION

We consider only finite, connected, undirected graphs without multiple edges and loops. Let G be a graph with a vertex set $V(G)$ and an edge set $E(G)$. Let $d(v)$ be the number of vertices adjacent to v. The distance $d(u, v)$ between any two vertices u and v of G is the number of edges in a shortest path connecting these two vertices u and v. For a positive integer k and a vertex v in G, the open neighborhood of v in G is defined as $N_k(v/G) = \{u \in V(G) : d(u, v) = k\}$. The k-distance degree of a vertex v in G is the number of k neighbors of v in G, and it is denoted by $d_k(v)$, see [1]. Any undefined term here may be found in [2].

In [1], the first leap Zagreb index was introduced based on the second vertex degrees. The first leap Zagreb index of a graph G is defined as

$$LM_1(G) = \sum_{u\in V(G)} d_2^2(u).$$

Considering the first leap Zagreb index, we introduce the first leap Zagreb polynomial of a graph G and it is defined as

$$LM_1(G, x) = \sum_{u\in V(G)} x^{d_2^2(u)}. \tag{1}$$

Very recently, some other leap indices were proposed and studied such as leap hyper-Zagreb indices, [3], augmented leap index [4], sum connectivity leap index and geometric-arithmetic leap index [5], minus leap index and square leap index [6].

The F-index was studied by Furtula and Gutman in [7] and it is defined as

$$F(G) = \sum_{u\in V(G)} d(u)^3 = \sum_{uv\in E(G)} \left[d(u)^2 + d(v)^2\right].$$

The F-index was also studied in [8, 9, 10, 11, 12, 13].

Motivated by the definition of the F-index and its applications, we introduce the F-leap index and F_1-leap index of a graph as follows:

The F-leap index of a graph G is defined as

$$FL(G) = \sum_{u\in V(G)} d_2^3(u). \tag{2}$$

Corresponding Author: V. R. Kulli
Department of Mathematics, Gulbarga University, Gulbarga 585106, India.

Considering the F-leap index, we propose the F-leap polynomial of a graph G as

$$FL(G,x)=\sum_{u\in V(G)} x^{d_2^3(u)}. \tag{3}$$

The F_1-leap index of a graph G is defined as

$$F_1L(G)=\sum_{uv\in E(G)} \left[d_2^2(u)+d_2^2(v)\right] \tag{4}$$

Considering the F_1-leap index, we propose the F_1-leap polynomial of a graph G as

$$F_1L(G,x)=\sum_{uv\in E(G)} x^{\left[d_2^2(u)+d_2^2(v)\right]} \tag{5}$$

Recently, some different type of polynomials were studied in [14, 15, 16, 17, 18, 19, 20, 21, 22].

In this paper, we consider wheel graphs and some related graphs, see [23]. We determine the F-leap and F_1-leap indices and their polynomials of wheel graphs and some related graphs.

2. RESULTS FOR WHEELS

The wheel W_n is the join of C_n and K_1. Clearly W_n has n+1 vertices and 2n edges. The vertex K_1 is called apex and the vertices of C_n are called rim vertices. The graph W_n is presented in Figure 1.

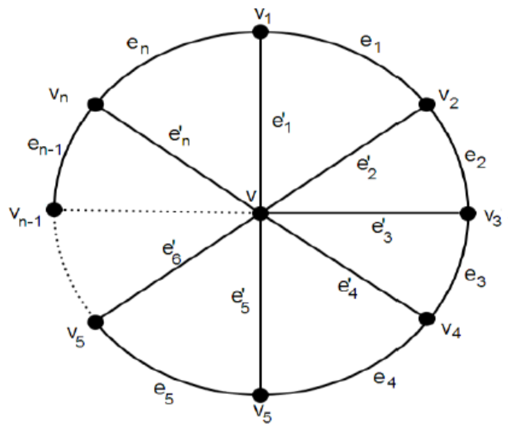

Figure-1: Wheel W_n

Lemma 1: Let W_n be a wheel with n+1 vertices, $n\geq 3$. Then there are two types of the 2-distance degree of vertices as given below:

$V_1 = \{u \in V(W_n) \mid d_2(u) = 0\}$, $\quad |V_1| = 1$.
$V_2 = \{u \in V(W_n) \mid d_2(u) = n-3\}$, $\quad |V_2| = n$.

Lemma 2: Let W_n be a wheel with n+1 vertices, $n\geq 3$. Then there are two types of the 2-distance degree of edges as follows:

$E_1 = \{uv \in E(W_n) \mid d_2(u) = 0, d_2(v) = n-3\}$, $\quad |E_1| = n$.
$E_2 = \{uv \in E(W_n) \mid d_2(u) = d_2(v) = n-3\}$, $\quad |E_2| = n$.

Theorem 3: Let W_n be a wheel with n+1 vertices, $n\geq 3$. Then the F-leap index of W_n is

$$FL(W_n)=n(n-3)^3.$$

Proof: From equation (2) and by Lemma 1, we deduce

$$FL(W_n)=\sum_{u\in V(W_n)} d_2^3(u)=\sum_{u\in V_1} d_2^3(u)+\sum_{u\in V_2} d_2^3(u)$$
$$=1\times 0+n(n-3)^3=n(n-3)^3.$$

Theorem 4: Let W_n be a wheel with n+1 vertices, n≥3. Then

(a) $LM_1(W_n, x) = x^0 + nx^{(n-3)^2}$.

(b) $FL(W_n, x) = x^0 + nx^{(n-3)^3}$.

Proof:
(a) From equation (1) and by Lemma 1, we have

$$LM_1(W_n, x) = \sum_{u \in V(W_n)} x^{d_2^2(u)} = \sum_{u \in V_1} x^{d_2^2(u)} + \sum_{u \in V_2} x^{d_2^2(u)}$$

$$= x^0 + nx^{(n-3)^2}.$$

(b) From equation (3) and by Lemma 1, we obtain

$$FL(W_n, x) = \sum_{u \in V(W_n)} x^{d_2^3(u)} = \sum_{u \in V_1} x^{d_2^3(u)} + \sum_{u \in V_2} x^{d_2^3(u)}$$

$$= x^0 + nx^{(n-3)^3}.$$

Theorem 3: Let W_n be a wheel with n+1 vertices, n≥3. Then

(a) $F_1L(W_n) = 3n(n-3)^2$

(b) $F_1L(W_n, x) = nx^{(n-3)^2} + nx^{2(n-3)^2}$.

Proof:
(a) From equation (4) and Lemma 2, we deduce

$$F_1L(W_n) = \sum_{uv \in E(W_n)} \left[d_2^2(u) + d_2^2(v)\right]$$

$$= n\left[0^2 + (n-3)^2\right] + n\left[(n-3)^2 + (n-3)^2\right] = 3n(n-3)^2.$$

(b) From equation (5) and by Lemma 2, we derive

$$F_1L(W_n, x) = \sum_{uv \in E(W_n)} x^{\left[d_2^2(u) + d_2^2(v)\right]}$$

$$= nx^{0^2 + (n-3)^2} + nx^{(n-3)^2 + (n-3)^2} = nx^{(n-3)^2} + nx^{2(n-3)^2}$$

3. RESULTS FOR GEAR GRAPHS

A bipartite wheel graph is a graph obtained from W_n with n+1 vertices adding a vertex between each pair of adjacent rim vertices and this graph is denoted by G_n and also called as a gear graph. Clearly, $|V(G_n)| = 2n+1$ and $|E(G_n)| = 3n$. A gear graph G_n is depicted in Figure 2.

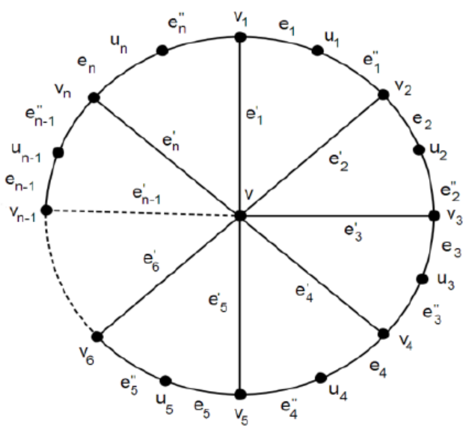

Figure-2: Gear graph G_n

Lemma 6: Let G_n be a gear graph with $2n$+1 vertices, n≥3. Then G_n has three types of the 2-distance degree of vertices as follows:

$V_1 = \{u \in V(G_n) \mid d_2(u) = n\}$, $|V_1| = n$.
$V_2 = \{u \in V(G_n) \mid d_2(u) = n - 1\}$, $|V_2| = n$.
$V_3 = \{u \in V(G_n) \mid d_2(u) = 3\}$, $|V_3| = n$.

Lemma 7: Let G_n be a gear graph with $3n$ edges, $n\geq3$. Then G_n has two types of the 2-distance degree of edges as follows:

$E_1 = \{u \in E(G_n) \mid d_2(u) = n, d_2(v) = n - 1\}, \quad |E_1| = n.$
$E_2 = \{u \in E(G_n) \mid d_2(u) = 3, , d_2(v) = n - 1\}, \quad |E_2| = 2n.$

Theorem 8: Let G_n be a gear graph with $2n+1$ vertices, $n\geq3$. Then the F-leap index of G_n is

$$FL(G_n) = n^4 - 2n^3 + 3n^2 + 26n.$$

Proof: By using equation (2) and by Lemma 6, we have

$$FL(G_n) = \sum_{u\in V(W_n)} d_2^3(u) = \sum_{u\in V_1} d_2^3(u) + \sum_{u\in V_2} d_2^3(u) + \sum_{u\in V_3} d_2^3(u)$$
$$= n^3 + n(n-1)^3 + n\times3^3 = n^4 - 2n^3 + 3n^2 + 26n.$$

Theorem 9: Let G_n be a gear graph with $2n+1$ vertices, $n\geq3$. Then

(a) $LM_1(G_n, x) = x^{n^2} + nx^{(n-1)^2} + nx^9.$

(b) $FL(G_n, x) = x^{n^3} + nx^{(n-1)^3} + nx^{27}.$

Proof:
(a) By using equation (1) and by Lemma 6, we obtain

$$LM_1(G_n, x) = \sum_{u\in V(G_n)} x^{d_2^2(u)} = \sum_{u\in V_1} x^{d_2^2(u)} + \sum_{u\in V_2} x^{d_2^2(u)} + \sum_{u\in V_3} x^{d_2^2(u)}$$
$$= x^{n^2} + nx^{(n-1)^2} + nx^9.$$

(b) By using equation (3) and by Lemma 6, we have

$$FL(G_n, x) = \sum_{u\in V(G_n)} x^{d_2^3(u)} = \sum_{u\in V_1} x^{d_2^3(u)} + \sum_{u\in V_2} x^{d_2^3(u)} + \sum_{u\in V_3} x^{d_2^3(u)}$$
$$= x^{n^3} + nx^{(n-1)^3} + nx^{27}.$$

Theorem 10: Let G_n be a gear graph with $3n$ edges, $n\geq3$. Then

(a) $F_1L(G_n) = 4n^3 - 6n^2 + 21n.$

(b) $F_1L(G_n, x) = nx^{2n^2-2n+1} + 2nx^{n^2-2n+1}.$

Proof:
(a) From equation (4) and Lemma 7, we deduce

$$F_1L(G_n) = \sum_{uv\in E(G_n)} \left[d_2^2(u) + d_2^2(v)\right]$$
$$= n\left[n^2 + (n-1)^2\right] + 2n\left[3^2 + (n-1)^2\right] = 4n^3 - 6n^2 + 21n.$$

(b) From equation (5) and by Lemma 7, we derive

$$F_1L(G_n, x) = \sum_{uv\in E(G_n)} x^{\left[d_2^2(u) + d_2^2(v)\right]}$$
$$= nx^{\left[n^2 + (n-1)^2\right]} + 2nx^{\left[3^2 + (n-1)^2\right]} = nx^{2n^2-2n+1} + 2nx^{n^2-2n+1}$$

4. RESULTS FOR HELM GRAPHS

The helm graph H_n is a graph obtained from W_n (with $n+1$ vertices) by attaching an end edge to each rim vertex of W_n. Clearly, $|V(H_n)| = 2n+1$ and $|E(H_n)| = 3n$. A graph H_n is shown in Figure 3.

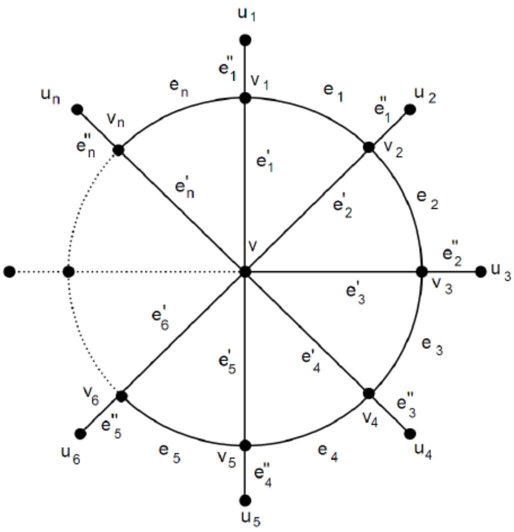

Figure-3: Helm graph H_n

Lemma 11: Let H_n be a helm graph with $2n+1$ vertices, $n \geq 3$. Then H_n has three types of the 2-distance degree of vertices as given below:

$V_1 = \{u \in V(H_n) \mid d_2(u) = n\}$, $\quad |V_1| = 1$.
$V_2 = \{u \in V(H_n) \mid d_2(u) = n-1\}$, $\quad |V_2| = n$.
$V_3 = \{u \in V(H_n) \mid d_2(u) = 3\}$, $\quad |V_3| = n$.

Lemma 12: Let H_n be a helm graph with $3n$ edges, $n \geq 3$. Then H_n has three types of the 2-distance degree of edges as follows:

$E_1 = \{uv \in E(H_n) \mid d_2(u) = n, d_2(v) = n-1\}$, $\quad |E_1| = n$.
$E_2 = \{uv \in E(H_n) \mid d_2(u) = 3, d_2(v) = n-1\}$, $\quad |E_2| = n$.
$E_3 = \{uv \in E(H_n) \mid d_2(u) = d_2(v) = n-1\}$, $\quad |E_3| = n$.

Theorem 13: Let H_n be a helm graph with $2n+1$ vertices, $n \geq 3$. Then the F-leap index of H_n is

$$FL(H_n) = n^4 - 2n^3 + 3n^2 + 26n.$$

Proof: By using equation (2) and by Lemma 11, we obtain

$$FL(H_n) = \sum_{u \in V(H_n)} d_2^3(u) = \sum_{u \in V_1} d_2^3(u) + \sum_{u \in V_2} d_2^3(u) + \sum_{u \in V_3} d_2^3(u)$$
$$= n^3 + n(n-1)^3 + n \times 3^3 = n^4 - 2n^3 + 3n^2 + 26n.$$

Theorem 14: Let H_n be a helm graph with $2n+1$ vertices, $n \geq 3$. Then

(a) $LM_1(H_n, x) = x^{n^2} + nx^{(n-1)^2} + nx^9.$

(b) $FL(H_n, x) = x^{n^3} + nx^{(n-1)^3} + nx^{27}.$

Proof:

(a) By using equation (1) and by Lemma 11, we have

$$LM_1(H_n, x) = \sum_{u \in V(H_n)} x^{d_2^2(u)} = \sum_{u \in V_1} x^{d_2^2(u)} + \sum_{u \in V_2} x^{d_2^2(u)} + \sum_{u \in V_3} x^{d_2^2(u)}$$
$$= x^{n^2} + nx^{(n-1)^2} + nx^9.$$

(b) From equation (3) and Lemma 11, we duce

$$FL(H_n, x) = \sum_{u \in V(H_n)} x^{d_2^3(u)} = \sum_{u \in V_1} x^{d_2^3(u)} + \sum_{u \in V_2} x^{d_2^3(u)} + \sum_{u \in V_3} x^{d_2^3(u)}$$
$$= x^{n^3} + nx^{(n-1)^3} + nx^{27}.$$

Theorem 15: Let H_n be a helm graph with $3n$ edges, $n\geq 3$. Then

(a) $F_1L(H_n) = 5n^3 - 8n^2 + 13n.$

(b) $F_1L(H_n, x) = nx^{2n^2-2n+1} + nx^{n^2-2n+10} + nx^{2(n^2-2n+1)}.$

Proof:

(a) From equation (4) and Lemma 12, we obtain

$$F_1L(H_n) = \sum_{uv\in E(H_n)} \left[d_2^2(u) + d_2^2(v)\right]$$
$$= n\left[n^2 + (n-1)^2\right] + n\left[3^2 + (n-1)^2\right] + n\left[(n-1)^2 + (n-1)^2\right]$$
$$= 5n^3 - 8n^2 + 13n.$$

(b) From equation (5) and by Lemma 12, we have

$$F_1L(H_n, x) = \sum_{uv\in E(H_n)} x^{\left[d_2^2(u)+d_2^2(v)\right]}$$
$$= nx^{\left[n^2+(n-1)^2\right]} + nx^{\left[3^2+(n-1)^2\right]} + nx^{\left[(n-1)^2+(n-1)^2\right]}$$
$$= nx^{2n^2-2n+1} + nx^{n^2-2n+10} + 2nx^{2(n^2-2n+1)}.$$

5. RESULTS FOR FLOWER GRAPHS

The graph Fl_n, is a flower graph obtained from a helm graph H_n by joining an end vertex to the apex of the helm graph. Then $|V(Fl_n)| = 2n+1$ and $|E(Fl_n)| = 4n$. A graph Fl_n is shown in Figure 4.

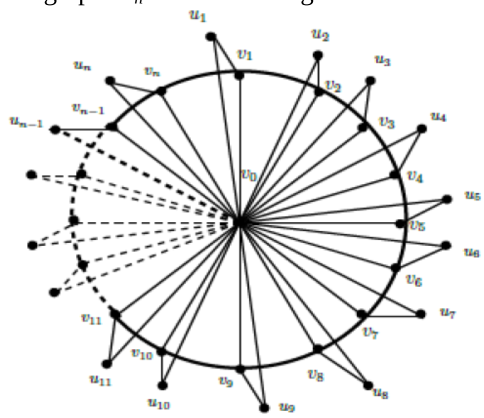

Figure-4: Flower graph Fl_n

Lemma 16: Let Fl_n be a flower graph with $2n+1$ vertices, $n\geq 3$. Then Fl_n has three types of the 2-distance degree of vertices as given below:

$V_1 = \{u \in E(Fl_n) \mid d_2(u) = 0\}$, $\quad |V_1| = 1.$
$V_2 = \{u \in E(Fl_n) \mid d_2(u) = n - 5\}$, $\quad |V_2| = n.$
$V_3 = \{u \in E(Fl_n) \mid d_2(u) = n - 2\}$, $\quad |V_3| = n.$

Lemma 17: Let Fl_n be a flower graph with $4n$ edges, $n\geq 3$. Then Fl_n has four types of the 2-distance degree of edges as follows:

$E_1 = \{uv \in E(Fl_n) \mid d_2(u) = 0, d_2(v) = n - 5\}$, $|E_1| = n.$
$E_2 = \{uv \in E(Fl_n) \mid d_2(u) = 0, d_2(v) = n - 2\}$, $|E_2| = n.$
$E_3 = \{uv \in E(Fl_n) \mid d_2(u) = n - 5, d_2(v) = n - 2\}$,$|E_3| = n.$
$E_4 = \{uv \in E(Fl_n) \mid d_2(u) = d_2(v) = n - 5\}$, $\quad |E_4| = n.$

Theorem 18: Let Fl_n be a flower graph with $2n+1$ vertices, $n\geq 3$. Then the F-leap index of Fl_n is

$$FL(Fl_n) = 2n^4 - 21n^3 + 87n^2 - 133n.$$

Proof: From equation (2) and by Lemma 16, we have

$$FL(Fl_n) = \sum_{u\in V(Fl_n)} d_2^3(u) = \sum_{u\in V_1} d_2^3(u) + \sum_{u\in V_2} d_2^3(u) + \sum_{u\in V_3} d_2^3(u)$$
$$= 0 + n(n-5)^3 + n(n-2)^3 = 2n^4 - 21n^3 + 87n^2 - 133n.$$

Theorem 19: Let Fl_n be a flower graph with $2n+1$ vertices, $n \geq 3$. Then

(a) $LM_1(Fl_n, x) = x^0 + nx^{(n-5)^2} + nx^{(n-2)^2}$.

(b) $FL(Fl_n, x) = x^0 + nx^{(n-5)^3} + nx^{(n-2)^3}$.

Proof:

(a) By using equation (1) and by Lemma 16, we obtain

$$LM_1(Fl_n, x) = \sum_{u \in V(Fl_n)} x^{d_2^2(u)} = \sum_{u \in V_1} x^{d_2^2(u)} + \sum_{u \in V_2} x^{d_2^2(u)} + \sum_{u \in V_3} x^{d_2^2(u)}$$

$$= x^0 + nx^{(n-5)^2} + nx^{(n-2)^2}.$$

(b) From equation (3) and Lemma 16, we deduce

$$FL(Fl_n, x) = \sum_{u \in V(Fl_n)} x^{d_2^3(u)} = \sum_{u \in V_1} x^{d_2^3(u)} + \sum_{u \in V_2} x^{d_2^3(u)} + \sum_{u \in V_3} x^{d_2^3(u)}$$

$$= x^0 + nx^{(n-5)^3} + nx^{(x-2)^3}.$$

Theorem 20: Let Fl_n be a flower graph with $4n$ edges, $n \geq 3$. Then

(a) $F_1L(Fl_n) = 6n^3 - 48n^2 + 108n.$

(b) $F_1L(Fl_n, x) = nx^{n^2-10n+25} + nx^{n^2-4n+4} + nx^{2n^2-14n+29} + nx^{2n^2-20n+50}$.

Proof:

(a) From equation (4) and Lemma 17, we deduce

$$F_1L(Fl_n) = \sum_{uv \in E(Fl_n)} \left[d_2^2(u) + d_2^2(v)\right]$$

$$= n\left[0^2 + (n-5)^2\right] + n\left[0^2 + (n-2)^2\right] + n\left[(n-5)^2 + (n-2)^2\right]$$

$$+ n\left[(n-5)^2 + (n-5)^2\right] = 6n^3 - 48n^2 + 108n.$$

(b) From equation (5) and by Lemma 17, we derive

$$F_1L(Fl_n, x) = \sum_{uv \in E(Fl_n)} x^{\left[d_2^2(u) + d_2^2(v)\right]}$$

$$= nx^{\left[0^2 - (n-5)^2\right]} + nx^{\left[0^2 + (n-2)^2\right]} + nx^{\left[(n-5)^2 + (n-1)^2\right]} + nx^{\left[(n-5)^2 + (n-5)^2\right]}$$

$$= nx^{n^2-10n+25} + nx^{n^2-4n+4} + nx^{2n^2-14n+29} + nx^{2n^2-20n+50}.$$

6. RESULTS FOR SUNFLOWER GRAPHS

The graph Sf_n, is a sunflower graph which is obtained from the flower graph Fl_n by attaching n end edges to the apex vertex. Then we have $|V(Sf_n)| = 3n+1$ and $|E(Sf_n)| = 5n$. A graph Sf_n is presented in Figure 5.

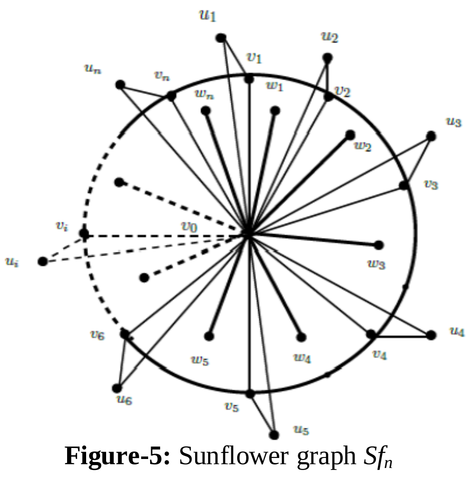

Figure-5: Sunflower graph Sf_n

Lemma 21: Let Sf_n be a sunflower graph with $3n+1$ vertices, $n \geq 3$. Then Sf_n has four types of the 2-distance degree of vertices as follows:

$V_1 = \{u \in E(Sf_n) \mid d_2(u) = 0\}$, $\quad |V_1| = 1$.
$V_2 = \{u \in E(Sf_n) \mid d_2(u) = 3n - 4\}$, $\quad |V_2| = n$.
$V_3 = \{u \in E(Sf_n) \mid d_2(u) = 3n - 2\}$, $\quad |V_3| = n$.
$V_4 = \{u \in E(Sf_n) \mid d_2(u) = 3n - 1\}$, $\quad |V_4| = n$.

Lemma 22: Let Sf_n be a sunflower graph with $5n$ edges, $n \geq 3$. Then Sf_n has five types of the 2-distance degree of edges as given below:

$E_1 = \{uv \in E(Sf_n) \mid d_2(u) = 0, d_2(v) = 3n - 4\}$, $\quad |E_1| = n$.
$E_2 = \{uv \in E(Sf_n) \mid d_2(u) = 0, d_2(v) = 3n - 2\}$, $\quad |E_2| = n$.
$E_3 = \{uv \in E(Sf_n) \mid d_2(u) = 0, d_2(v) = 3n - 1\}$, $\quad |E_3| = n$.
$E_4 = \{uv \in E(Sf_n) \mid d_2(u) = d_2(v) = 3n - 4\}$, $\quad |E_4| = n$.
$E_5 = \{uv \in E(Sf_n) \mid d_2(u) = 3n - 4, d_2(v) = 3n - 2\}$, $\quad |E_5| = n$.

Theorem 23: Let Sf_n be a sunflower graph with $3n+1$ vertices, $n \geq 3$. Then the F-leap index of Sf_n is

$$FL(Sf_n) = 81n^4 - 189n^3 + 189n^2 - 73n.$$

Proof: From equation (2) and by Lemma 21, we have

$$FL(Sf_n) = \sum_{u \in V(Sf_n)} d_2^3(u) = \sum_{u \in V_1} d_2^3(u) + \sum_{u \in V_2} d_2^3(u) + \sum_{u \in V_3} d_2^3(u) + \sum_{u \in V_4} d_2^3(u)$$
$$= 0 + n(3n-4)^3 + n(3n-2)^3 + n(3n-1)^3$$
$$= 81n^4 - 189n^3 + 189n^2 - 73n.$$

Theorem 24: Let Sf_n be a sunflower graph with $3n+1$ vertices, $n \geq 3$. Then

(a) $LM_1(Sl_n, x) = x^0 + nx^{(3n-4)^2} + nx^{(3n-2)^2} + nx^{(3n-1)^2}$.

(b) $FL(Sf_n, x) = x^0 + nx^{(3n-4)^3} + nx^{(3n-2)^3} + nx^{(3n-1)^3}$.

Proof:

(a) By using equation (1) and by Lemma 21, we derive

$$LM_1(Sf_n, x) = \sum_{u \in V(Sf_n)} x^{d_2^2(u)} = \sum_{u \in V_1} x^{d_2^2(u)} + \sum_{u \in V_2} x^{d_2^2(u)} + \sum_{u \in V_3} x^{d_2^2(u)} + \sum_{u \in V_4} x^{d_2^2(u)}$$
$$= x^0 + nx^{(3n-4)^2} + nx^{(3n-2)^2} + nx^{(3n-1)^2}.$$

(b) From equation (3) and Lemma 21, we duce

$$FL(Sf_n, x) = \sum_{u \in V(Sf_n)} x^{d_2^3(u)} = \sum_{u \in V_1} x^{d_2^3(u)} + \sum_{u \in V_2} x^{d_2^3(u)} + \sum_{u \in V_3} x^{d_2^3(u)} + \sum_{u \in V_2} x^{d_2^3(u)}$$
$$= x^0 + nx^{(3n-4)^3} + nx^{(3n-2)^3} + nx^{(3n-1)^3}.$$

Theorem 25: Let Sf_n be a sunflower graph with $5n$ edges, $n \geq 3$. Then

(a) $F_1L(Sf_n) = 63n^3 - 120n^2 + 70n$.

(b) $F_1L(Sf_n, x) = nx^{(3n-4)^2} + nx^{(3n-2)^2} + nx^{(3n-1)^2} + nx^{2(3n-4)^2} + nx^{18n^2-30n+17}$.

Proof:

(a) From equation (4) and Lemma 22, we have

$$F_1L(Sf_n) = \sum_{uv \in E(Sf_n)} \left[d_2^2(u) + d_2^2(v)\right]$$
$$= n\left[0^2 + (3n-4)^2\right] + n\left[0^2 + (3n-2)^2\right] + n\left[0^2 + (3n-1)^2\right]$$
$$+ n\left[(3n-4)^2 + (3n-4)^2\right] + n\left[(3n-4)^2 + (3n-2)^2\right]$$
$$= 63n^3 - 120n^2 + 70n.$$

(b) From equation (5) and by Lemma 22, we obtain

$$F_1L(Sf_n, x) = \sum_{uv\in E(Sf_n)} x^{\left[d_2^2(u)+d_2^2(v)\right]}$$

$$= nx^{\left[0^2+(3n-4)^2\right]} + nx^{\left[0^2+(3n-2)^2\right]} + nx^{\left[0^2+(3n-1)^2\right]} + nx^{\left[(3n-4)^2+(3n-4)^2\right]} + nx^{\left[(3n-4)^2+(3n-1)^2\right]}$$

$$= nx^{(3n-4)^2} + nx^{(3n-2)^2} + nx^{(3n-1)^2} + nx^{2(3n-4)^2} + nx^{18n^2-30n+17}.$$

REFERENCES

1. A.M. Naji, N.D. Soner and I Gutman, On leap Zagreb indices of graphs, *Commun. Comb. Optim.* 2 (2017) 99-117.
2. V.R.Kulli, *College Graph Theory*, Vishwa International Publications, Gulbarga, India (2012).
3. V.R.Kulli, Leap hyper-Zagreb indices and their polynomials of certain graphs, *International Journal of Current Research in Life Sciences*, 7(10) (2018) 2783-2791.
4. V.R. Kulli, On augmented leap index and its polynomial of some wheel graphs, submitted.
5. V.R. Kulli, Sum connectivity leap index and geometric-arithmetic leap indices of certain windmill graphs, submitted.
6. V.R.Kulli, Minus leap and square leap indices and *their polynomials* of some special graphs, *International Research Journal of Pure Algebra*, 8(11) (2018) 54-60.
7. B. Furtula and I. Gutman, A forgotten topological index, *J. Math. Chem.* 53 (2015), 1184-1190.
8. N. De and S.M.A. Nayeem, Computing the *F*-index of nanostar dendrimers, *Pacific Science Review A: Natural Science and Engineering* (2016) DOI: http:/dx.doi.org/10.1016/j.psra.2016.06.001.
9. V.R.Kulli, Computation of F-reverse and modified reverse indices of some nanostructures, *Annals of Pure and Applied Mathematics*, 18(1) (2018) 37-43.
10. V.R.Kulli, Computing the F-ve-degree index and its polynomial of dominating oxide and regular triangulate oxide networks, *International Journal of Fuzzy Mathematical Archive*, 16(1) (2018) 1-6.
11. V.R. Kulli, Computing F-reverse index and F-reverse polynomial of certain networks, *International Journal of Mathematical Archive*, 8(8) (2018).
12. V.R. Kulli, Computing the F-Revan index and modified Revan indices of certain nanostructures, *Journal of Computer and Mathematical Sciences*, 9(10) (2018) 1326-1333.
13. V.R Kulli, Edge version of *F*-index, general sum connectivity index of certain nanotubes, *Annals of Pure and Applied Mathematics*, 14(3) (2017) 449-455.
14. V.R. Kulli, General Zagreb polynomials and F-polynomial of certain nanostructures, *International Journal of Mathematical Archive,* 8(10) (2017) 103-109.
15. V.R.Kulli, Certain topological indices and their polynomials of dendrimer nanostars, *Annals of Pure and Applied Mathematics* 14(2) (2017) 263-268.
16. V.R.Kulli, General fifth *M*-Zagreb indices and fifth *M*-Zagreb polynomials of PAMAM dendrimers, *International Journal of Fuzzy Mathematical Archive*, 13(1) (2017) 99-103.
17. V.R. Kulli, On augmented reverse index and its polynomial of certain nanostar dendrimers, *Journal of Engineering Sciences and Research Technology*, 7(8) (2018) 237-243.
18. V.R. Kulli, Reduced second Zagreb index and its polynomial of certain silicate networks, *Journal of Mathematics and Informatics*, 14 (2018) 11-16.
19. V.R. Kulli, On augmented Revan index and its polynomial of certain families of benzenoid systems, *International Journal of Mathematics and its Applications*, 6(4) (2018) 43-50.
20. V.R. Kulli, On the square ve-degree index and its polynomial of certain oxide networks, *Journal of Global Research in Mathematical Archives*, 5(10) (2018) 1-4.
21. V.R. Kulli, On KV indices and their polynomials of two families of dendrimers, *International Journal of Current Research in Life Sciences*, 7(9) (2018) 2739-2744.
22. V.R. Kulli, Computing square Revan index and its polynomial of certain benzenoid systems, *International Journal of Mathematics and its applications*, (2018).
23. P. Shiladhar, A.M. Naji and N.D. Soner, Leap Zagreb indices of some wheel related graphs, *Journal of Computer and Mathematical Sciences*, 9(3) (2018) 221-231.

Source of support: Nil, Conflict of interest: None Declared.