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A PRODUCTION INVENTORY SYSTEM WITH DIFFERENT RATES OF PRODUCTION AND RETRIALS

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ABSTRACT

This paper deals with a production inventory system with retrial of customers under (s,S) policy. The time between additions of two successive items by production to the inventory is exponentially distributed. When the inventory level lies between 0 and s, items are produced at higher rate. The higher production rate will reduce customers' loss in the absence of inventory. Arrival of customers is according to a Poisson process and service times are exponentially distributed. An arriving customer who finds the server busy or inventory level zero, proceeds to an orbit of infinite capacity and retry from there. Inter-retrial times follow an exponential distribution. Some important system performance measures related to the model are defined and analyzed numerically. A suitable cost function is constructed and its optimum values corresponding to different parameters are calculated graphically. The optimum (s,S) pair is also obtained.

Key words: *Production Inventory, Retrial, Matrix Analytic Method, Different Production Rates.*

2010 AMS Classification: *60K25, 90B05, 91B70.*

1. INTRODUCTION

Research on queuing-inventory systems has got much attention of researchers nowadays. Investigations are being carried out on queuing-inventory systems attached with production of items. Different notions such as retrial of the customers, impatience of the customers, interruptions of the service as well as the production process are being studied. These investigations have applications in all manufacturing industries. Krishnamoorthy and Jose [5] analyzed and compared three production inventory systems with positive service time and retrial of customers by assuming all the underlying distributions to be exponential. They obtained that the model with buffer size equal to the inventoried items is the best profitable model for practical purposes. Benjaafar *et al*. [2] studied a production-inventory system with customer impatience. The patience time was random and varies from one customer to another. They formulated the problem as a Markov decision process and described the optimal policy by a production base-stock level and an admission threshold.

Krishnamoorthy and Viswanath [6] studied a (s,S) production inventory system with positive service time. They obtained an explicit product form solution for the steady state probability vector, by assuming that no customer joins the queue when the inventory level is zero. They also expressed the expected length of a production cycle explicitly. The optimal values of S and s were calculated analytically. Yu and Dong [13] analyzed a production inventory problem which included customers, one retailer, and one manufacturer. Production rate of the manufacturer was assumed to be a finite constant. The order arrival times from customers followed a general distribution. The optimal solution to the problem was obtained numerically.

Rashid *et al*. [10] analyzed a production-inventory system by considering demand and production time as stochastic parameters and calculated long-run inventory costs. They also extended the model for multi-item inventory systems. They obtained a heuristic algorithm and illustrated it with a case study in Electroestil Company. Beak and Moon [1] studied an (s,S) production–inventory system. The customers arrival and production process were assumed to be according to Poisson processes. They analyzed the model using a regenerative process. They obtained that the queue size and inventory level processes were independent in steady state. They proposed cost models using mean performance measures.

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Otten *et al.* [9] considered a two-echelon production-inventory system. A continuous review base stock policy was used to control inventory replenishment. They derived stationary distributions of joint queue length and inventory processes in explicit product form and cost analysis was performed. Salini and Jose [11] considered a production inventory system with positive service time and retrial of customers. They assumed different rates of service depending on the inventory level and obtained the optimum service reduction value. Various system performance measures were derived and a cost function was constructed.

Zare *et al.* [14] studied a two-echelon production–inventory system where the inventory replenished according to (R, Q) policy. The time taken to transport inventory between the two echelons were generally distributed. The optimal reorder point at the distribution center was calculated. The optimal base-stock level in the warehouse and the batch order size at the distribution center were approximated. Chan *et al.* [3] investigated a production-inventory model for deteriorating items in which non-stop production is considered. The proposed model included production rate as one of the decision variables and considered deterioration during deliveries. They optimized the cost for the system in which some of the cost parameters were production rate dependent.

The rest of the paper is organized as follows. In section 2, Mathematical modeling and analysis of the model is presented which includes stability and performance measures of the system. In section 3, Numerical Results and its interpretations are provided. Cost analysis is described in section 4. In section 5, we incorporate concluding remarks.

The following notations are used in the proposed model
S: Maximum inventory level
s: Inventory level at which production starts
$I(t)$: Inventory level at time t.
$N(t)$: Number of customers in the orbit at time t.

$$C(t) = \begin{cases} 0, \text{if the server is idle} \\ 1, \text{if the server is busy} \end{cases}$$

$$J(t) = \begin{cases} 0, \text{if the production is in OFF mode} \\ 1, \text{if the production is in ON mode} \end{cases}$$

$\boldsymbol{e}$: $(1, 1, \dots 1)'$ a column vector of 1's of appropriate order.

2. MATHEMATICAL MODELING AND ANALYSIS

We consider a production inventory system with retrial of customers under (s, S) policy. When the inventory level reduces to s, production starts and stops when the inventory level reaches back to S. The time between additions of two successive items by production to the inventory is exponentially distributed. The production rate is $\alpha\beta$, where $\alpha \in [1, k]$, k (finite) and greater than 1, when production starts; but the rate is β, when level crosses above s (i.e., for the level from $s+1$ to S). Arrival of customers is according to a Poisson process with rate λ and service times are exponentially distributed with parameter μ. An arriving customer who finds the server busy or inventory level zero, proceeds to an orbit with probability γ and is lost from the system with probability $(1-\gamma)$. A retrial customer in the orbit who finds the server busy or inventory level zero, returns to the orbit with probability δ and is lost from the system with probability $(1-\delta)$. The time between retrials follow an exponential distribution with linear rate $i\theta$ when there are i customers in the orbit.

Let $I(t)$ be the inventory level and $N(t)$ be the number of customers in the orbit at time t. Let $C(t)$ be the sever status at time t, which is equal to 1 if the server is busy and 0 if the sever is idle. Let $J(t)$ be the production status which is equal to 1 if the production is in ON mode and 0 if the production is in OFF mode. Now $\{X(t), t \geq 0\}$, where $X(t) = (N(t), C(t), J(t), I(t))$ is a level dependent quasi birth-death process on the state space $\{(i, k, 0, j); i \geq 0;\ k = 0,1;\ s+1 \leq j \leq S\} U \{(i, 0, 1, j);\ i \geq 0;\ 0 \leq j \leq S-1\} U\ \{(i, 1, 1, j); i \geq 0;\ 1 \leq j \leq S-1\}$ The infinitesimal generator Q, of the process is a block tri-diagonal matrix and it has the following form

$$Q = \begin{bmatrix} A_{1,0} & A_0 & & & \\ A_{2,1} & A_{1,1} & A_0 & & \\ & A_{2,2} & A_{1,2} & A_0 & \\ & & A_{2,3} & A_{1,3} & A_0 \\ & & & \ddots & \ddots & \ddots \end{bmatrix}$$

where the blocks A_0, $A_{1,i}$ $(i \geq 0)$ and $A_{2,i}$ $(i \geq 1)$ are square matrices, each of order $(4S - 2s - 1)$; they are given by

$$A_0 = \begin{array}{c} \underline{0,0} \\ \underline{0,1} \\ \underline{1,0} \\ \underline{1,1} \end{array} \begin{bmatrix} 0 & & & \\ & (\lambda\gamma)C_1 & & \\ & & (\lambda\gamma)I_{S-s} & \\ & & & (\lambda\gamma)I_{S-1} \end{bmatrix}$$

$$A_{1,i} = \begin{array}{c} \underline{0,0} \\ \underline{0,1} \\ \underline{1,0} \\ \underline{1,1} \end{array} \begin{bmatrix} (-(\lambda+i\theta))I_{S-s} & 0 & (\lambda)I_{S-s} & 0 \\ C2 & C3 & 0 & (\lambda)C_4 \\ C5 & C6 & (-(\lambda\gamma+\mu+i\theta(1-\delta)))I_{S-s} & 0 \\ 0 & C7 & C8 & C9 \end{bmatrix}$$

$$A_{2,i} = \begin{array}{c} \underline{0,0} \\ \underline{0,1} \\ \underline{1,0} \\ \underline{1,1} \end{array} \begin{bmatrix} 0 & 0 & (i\theta)I_{S-s} & 0 \\ 0 & (i\theta(1-\delta))C_1 & 0 & (i\theta)C_4 \\ 0 & 0 & (i\theta(1-\delta))I_{S-s} & 0 \\ 0 & 0 & 0 & (i\theta(1-\delta))I_{S-1} \end{bmatrix}$$

where $\underline{u,v}$ denotes the entry corresponding to the variations of inventory level j (phase) for a fixed i, the number of customers in the orbit (level). Here u stands for server status and v, for production status. $(p,q)^{th}$ element of the matrices contained in A_0, $A_{1,i}$ and $A_{2,i}$ are given by

$$[C_1]_{pq} = \begin{cases} 1, & p = q = 1 \\ 0, & otherwise \end{cases}$$

$$[C_2]_{pq} = \begin{cases} \beta, & p = S, \ q = S - s \\ 0, & otherwise \end{cases}$$

$$[C_3]_{pq} = \begin{cases} -\big(\lambda\gamma + \alpha\beta + i\theta(1-\delta)\big), \ p = q = 1 \\ -(\lambda + \alpha\beta + i\theta), \ 2 \le p \le s, \ q = p \\ -(\lambda + \beta + i\theta), s+1 \le p \le S, \ q = p \\ \alpha\beta, 1 \le p \le s, \ q = p+1 \\ \beta, s+1 \le p \le S-1, \ q = p+1 \\ 0, \ otherwise \end{cases}$$

$$[C_4]_{pq} = \begin{cases} 1, 2 \le p \le S, \ q = p - 1 \\ 0, otherwise \end{cases}$$

$$[C_5]_{pq} = \begin{cases} \mu, & 2 \le p \le S - s, \ q = p - 1 \\ 0, & otherwise \end{cases}$$

$$[C_6]_{pq} = \begin{cases} \mu, & p = 1, \ q = s + 1 \\ 0, & otherwise \end{cases}$$

$$[C_7]_{pq} = \begin{cases} \mu, & 1 \le p \le S - 1, \ q = p \\ 0, & otherwise \end{cases}$$

$$[C_8]_{pq} = \begin{cases} \beta, & p = S - 1, q = S - s \\ 0, & otherwise \end{cases}$$

$$[C_9]_{pq} = \begin{cases} -\big(\lambda\gamma + \alpha\beta + \mu + i\theta(1-\delta)\big), 1 \le p \le s-1, q = p \\ -\big(\lambda\gamma + \beta + \mu + i\theta(1-\delta)\big), s \le p \le S-1, q = p \\ \alpha\beta, \quad 1 \le p \le s-1, \ q = p+1 \\ \beta, s \le p \le S-2, \ q = p+1 \\ 0, \ otherwise \end{cases}$$

In order to modify the infinitesimal generator Q to the following form where $A_{1,i} = A_1 and A_{2,i} = A_2$ for $i \ge N$, Neuts– Rao [8] truncation method is used.

$$Q = \begin{bmatrix} A_{1,0} & A_0 & & & & & & \\ A_{2,1} & A_{1,1} & A_0 & & & & & \\ & A_{2,2} & A_{1,2} & A_0 & & & & \\ & & \ddots & \ddots & \ddots & & & \\ & & & A_{2,N-1} & A_{1,N-1} & A_0 & & \\ & & & & A_2 & A_1 & A_0 & \\ & & & & & A_2 & A_1 & A_0 \end{bmatrix}$$

2.1 System Stability

In order to find the stability of the system, we take the Lyapunov test function (Falin and Templeton [4]): $\varphi(r) = i$, if r is a state in the level i

The mean drift y_r for any r belonging to the level $i \geq 1$ is given by

$$y_r = \sum_{p \neq r} q_{rp}(\varphi(p) - \varphi(r))$$
$$= \sum_u q_{ru}\big(\varphi(u) - \varphi(r)\big) + \sum_v q_{rv}\big(\varphi(v) - \varphi(r)\big) + \sum_w q_{rw}\big(\varphi(w) - \varphi(r)\big)$$

where u, v, w vary over the states belonging to the levels $(i-1), i$ and $(i+1)$ respectively. Then by the definition of φ, $\varphi(u) = i - 1$, $\varphi(v) = i$ and $\varphi(w) = i + 1$ so that

$$y_r = -\sum_u q_{ru} + \sum_w q_{rw}$$
$$= \begin{cases} -i\theta, \text{ if the server is idle and inventory level is positive} \\ -i\theta(1-\delta) + \lambda\gamma, \quad otherwise \end{cases}$$

Since $(1 - \delta) > 0$, for any $\varepsilon > 0$, we can find N' large enough that $y_r < -\varepsilon$ for any r belonging to the level $i \geq N'$. Hence, the system under consideration is stable by Tweedi's [12] result.

2.2 Rate matrix Rand Truncation level N

Rate matrix R is evaluated using Iterative method. We denote the sequence of R by $\{R(N)\}$ and is defined by $R_0(N) = 0$ and $R_{n+1}(N) = (-R^2(N)A_2(N) - A_0(N))A_1^{-1}(N)$. As in Neuts [7], Elsner's algorithm is used to evaluate the spectral radius $\eta(N)$ of $R(N)$. N must be chosen such that $|\eta(N) - \eta(N+1)| < \varepsilon$, where ε is an arbitrarily small value.

2.3 System Performance Measures

We partition the steady state probability vector $\mathbf{x} = (x_0, x_1, \ldots, x_{N-1}, x_N, \ldots)$ such that its $(i+1)^{th}$ component is given by

$x_i = \left(y_{i,0,0,s+1}, \ldots, y_{i,0,0,S}, y_{i,0,1,0}, \ldots, y_{i,0,1,S-1}, y_{i,1,0,s+1}, \ldots, y_{i,1,0,S}, y_{i,1,1,1}, \ldots, y_{i,1,1,S-1}\right)$. Then,

i. Expected Inventory level, EI, in the system is given by

$$EI = \sum_{i=0}^{\infty}\sum_{k=0}^{1}\sum_{j=s+1}^{S} j y_{i,k,0,j} + \sum_{i=0}^{\infty}\sum_{k=0}^{1}\sum_{j=1}^{S-1} j y_{i,k,1,j}$$

ii. Expected number of customers, EC, in the orbit is given by

$$EC = \left(\sum_{i=1}^{\infty} i x_i\right)\boldsymbol{e} = \left(\left(\sum_{i=1}^{N-1} i x_i\right) + x_N\,(N(I-R)^{-1} + R(I-R)^{-2})\right)\boldsymbol{e}$$

iii. Expected switching rate, ESR, is given by

$$ESR = \mu \sum_{i=0}^{\infty} y_{i,1,0,s+1}$$

iv. Expected number of departures, EDS, after completing service is

$$EDS = \mu \sum_{i=0}^{\infty}\sum_{j=s+1}^{S} y_{i,1,0,j} + \mu \sum_{i=0}^{\infty}\sum_{j=1}^{S-1} y_{i,1,1,j}$$

v. Expected number of external customers lost, EL_1, before entering the orbit per unit time is

$$EL_1 = (1-\gamma)\lambda \sum_{i=0}^{\infty}\left(y_{i,0,1,0} + \sum_{j=s+1}^{S} y_{i,1,0,j} + \sum_{j=1}^{S-1} y_{i,1,1,j}\right)$$

vi. Expected number of customers lost, EL_2, due to retrials per unit time

$$EL_2 = \theta(1-\delta) \sum_{i=1}^{\infty} i\left(y_{i,0,1,0} + \sum_{j=s+1}^{S} y_{i,1,0,j} + \sum_{j=1}^{S-1} y_{i,1,1,j}\right)$$

vii. Overall rate of retrials, ORR, is given by,

$$ORR = \theta\left(\sum_{i=1}^{\infty} i x_i\right)\boldsymbol{e}$$

viii. Successful rate of retrials, SRR, is given by,

$$SRR = \theta \sum_{i=0}^{\infty} i\left(\sum_{j=s+1}^{S} y_{i,0,0,j} + \sum_{j=1}^{S-1} y_{i,0,1,j}\right)$$

ix. Server busy probability, SBP, is given by,

$$SBP = \sum_{i=0}^{\infty}\sum_{j=s+1}^{S} y_{i,1,0,j} + \sum_{i=0}^{\infty}\sum_{j=1}^{S-1} y_{i,1,1,j}$$

3. NUMERICAL RESULTS AND INTERPRETATIONS

Here we analyze the nature of overall rate of retrials (ORR), successful rate of retrials (SRR) and server busy probability (SBP) with respect to the variations of different parameters in the model. Table 1 and Table 2 contain values of ORR, SRR and SBP with respect to variations of α and μ. When the production rate and service rate increase, the number of customers in the orbit decreases. Hence overall rate of retrials decreases and the successful rate of retrials increases. As the production rate increases, SBP increases and as the service rate increases, SBP decreases. Tables 3, 4 and 5 show the changes of ORR, SRR and SBP with respect to variations of γ, δ and λ respectively. In all these cases, as the values of γ, δ and λ increase, the number of customers in the orbit increases and hence the overall and successful rate of retrials and server busy probability increase. Table 6 shows that, as the retrial rate θ of customers in the orbit increases, the overall and successful rate of retrials increase. SBP also increases with retrial rate.

S=50,s=5,λ=1.5,γ=0.8,N=25,
θ=1.5, δ=0.7, β=2, μ=3.

α	ORR	SRR	SBP
1.1	3.4487	1.1724	0.5906
1.2	3.4421	1.1744	0.5920
1.3	3.4371	1.1758	0.5929
1.4	3.4333	1.1768	0.5936
1.5	3.4305	1.1775	0.5941
1.6	3.4283	1.1780	0.5944
1.7	3.4266	1.1783	0.5947
1.8	3.4254	1.1785	0.5948
1.9	3.4244	1.1787	0.5950

Table-1: (Variations in α)

S=50,s=5,λ=1.6,γ=0.6,N=25,
θ=1.5, δ=0.7,β=2, α=1.4.

μ	ORR	SRR	SBP
2.1	3.2599	0.8989	0.6750
2.2	3.1914	0.9180	0.6622
2.3	3.1260	0.9358	0.6497
2.4	3.0636	0.9524	0.6375
2.5	3.0041	0.9679	0.6257
2.6	2.9474	0.9824	0.6142
2.7	2.8934	0.9959	0.6029
2.8	2.8420	1.0086	0.5920
2.9	2.7930	1.0204	0.5814

Table-2: (Variations in μ)

S=50,s=5,λ=1.5,N=25,θ=1.5,δ=0.7,
β=2, μ=3, α=1.4.

γ	ORR	SRR	SBP
0.1	1.5674	0.7699	0.5044
0.2	1.6511	0.7937	0.5096
0.3	1.7738	0.8264	0.5168
0.4	1.9595	0.8726	0.5269
0.5	2.2231	0.9337	0.5403
0.6	2.5654	1.0077	0.5565
0.7	2.9748	1.0904	0.5746
0.8	3.4333	1.1768	0.5936
0.9	3.9223	1.2629	0.6125

Table-3: (Variations in γ)

S=50,s=5,λ=1.5,γ=0.6,N=25,
θ=1.5, β=2, μ=3,α=1.4.

δ	ORR	SRR	SBP
0.1	1.8168	0.8388	0.5194
0.2	1.8577	0.8492	0.5217
0.3	1.9122	0.8629	0.5247
0.4	1.9880	0.8814	0.5288
0.5	2.0990	0.9075	0.5345
0.6	2.2722	0.9462	0.5430
0.7	2.5654	1.0077	0.5565
0.8	3.1202	1.1129	0.5795
0.9	4.3642	1.3115	0.6227

Table-4: (Variations in δ)

S=50,s=5, γ=0.6, N=25, θ=1.5,
δ=0.7,β=2,μ=3,α =1.4.

λ	ORR	SRR	SBP
1.1	2.0014	0.9387	0.4970
1.2	2.1168	0.9512	0.5118
1.3	2.2495	0.9671	0.5267
1.4	2.3993	0.9861	0.5416
1.5	2.5654	1.0077	0.5565
1.6	2.7463	1.0315	0.5711
1.7	2.9403	1.0569	0.5854
1.8	3.1454	1.0834	0.5994
1.9	3.3594	1.1105	0.6129

Table-5: (Variations in λ)

S=50,s=5,λ=1.6,γ=0.6, N=25,
δ=0.7, β=2, μ=2.5, α =1.4.

θ	ORR	SRR	SBP
1.1	2.7230	0.9099	0.6116
1.2	2.7950	0.9242	0.6150
1.3	2.8652	0.9385	0.6185
1.4	2.9346	0.9531	0.6221
1.5	3.0041	0.9679	0.6257
1.6	3.0743	0.9830	0.6294
1.7	3.1455	0.9983	0.6331
1.8	3.2179	1.0138	0.6369
1.9	3.2918	1.0294	0.6407

Table-6: (Variations in θ)

4. COST ANALYSIS

We define the expected total cost function as

$$ETC = (C + (S - s)c_1)\, ESR + c_2 EI + c_3 EC + c_4 EL_1 + c_5 EL_2 + c_6 EDS$$

where C is the fixed cost, c_1 is the procurement cost per unit, c_2 is the holding cost of inventory per unit per unit time, c_3 is the holding cost of customers per unit per unit time, c_4 is the cost due to loss of primary customers per unit per unit time, c_5 is the cost due to loss of retrial customers per unit per unit time, c_6 is the cost due to service per unit per unit time.

4.1 Graphical Illustrations and Interpretations

Here we calculate the expected total cost per unit time by varying the parameters one at a time keeping others fixed. The convexity of the cost function is obtained graphically. In fig. 1, the minimum expected total cost is obtained by varying the value of α. For given parameter values, minimum expected total cost is 66.6981 when $\alpha = 1.4$. By varying the value of μ, the minimum expected total cost is obtained in fig 2. The minimum value of ETC is 37.2199 at $\mu = 2.5$. In fig.3, the convexity of the cost function is obtained by varying the values of the parameter γ. The minimum value of ETC is 33.3562 at $\gamma = .6$. In fig.4, the convexity of the cost function is obtained by varying the values of the parameter δ. The minimum value of ETC is 35.6934 at $\delta = 0.7$. The convexities of the cost function by varying the values of the parameters λ and θ are obtained in fig.5 and fig.6 respectively. The minimum values of ETC are 34.4906 at $\lambda = 1.6$ and 30.1236 at $\theta = 1.4$.

$S = 50, s = 5, \lambda = 1.5, \gamma = 0.8, N = 25, \theta = 1.5, \beta = 2, \delta = 0.7, \mu = 3, C = 20,$
$c_1 = 1, c_2 = 1, c_3 = 10.8, c_4 = 20.8, c_5 = 20.8, c_6 = 1$

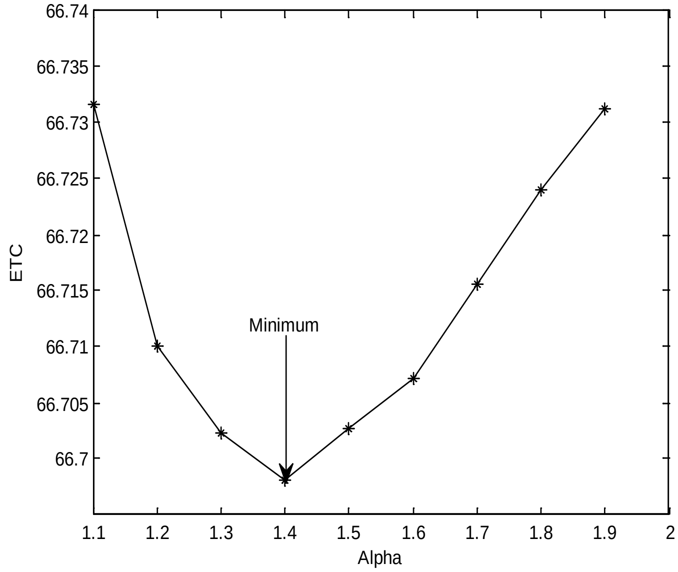

Figure-1: ETC versus α

S=50, s=5, λ=1.6, γ=0.6, N=25, θ=1.5, δ=0.7, β=2, α=1.4,
C=20, c_1=1, c_2=1, c_3=1, c_4=1, c_5=1, c_6=6.7

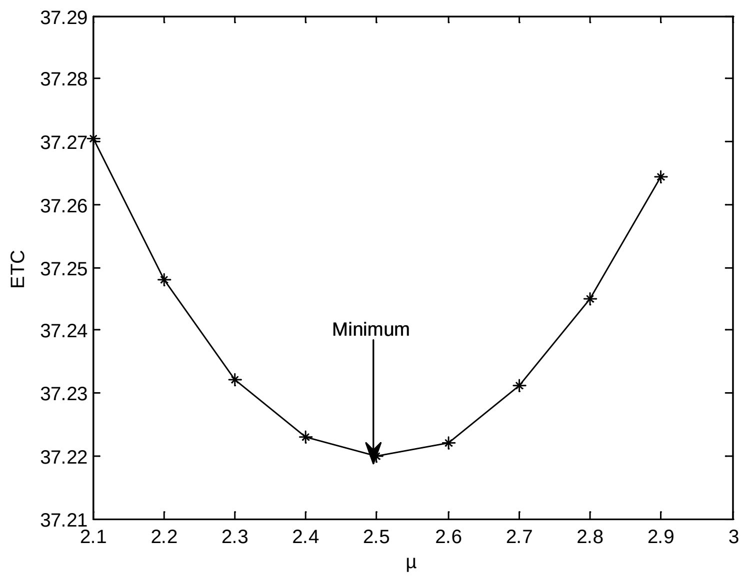

Figure-2: ETC versus μ

S=50, s=5, λ=1.5, α =1.4, N=25, θ=1.5, β=2, δ=0.7, μ=3,
C=20, c_1=1, c_2=1, c_3=2.8, c_4=1, c_5=1, c_6=2.4.

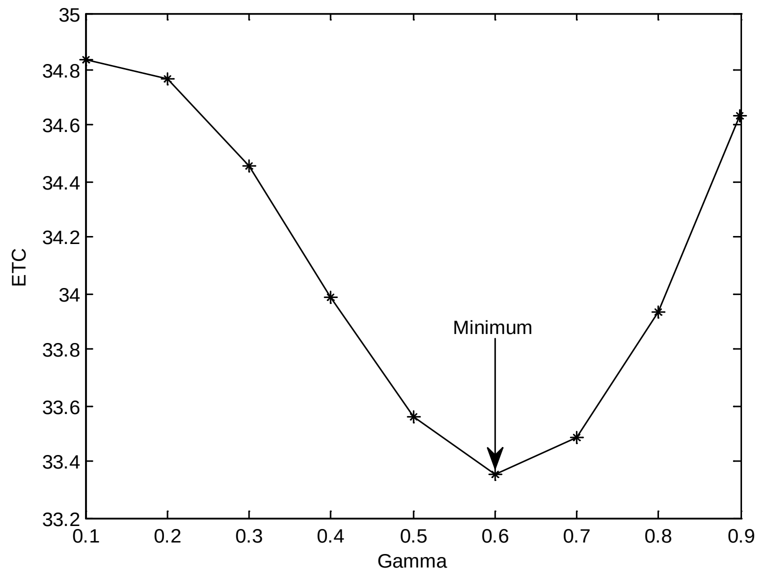

Figure-3: ETC versus γ

S=50, s=5, λ=1.5, α =1.4, N=25,θ=1.5, β=2, γ=0.6, μ=3,
C=20, c_1=1, c_2=1, c_3=2.8, c_4=1, c_5=1, c_6=3.8.

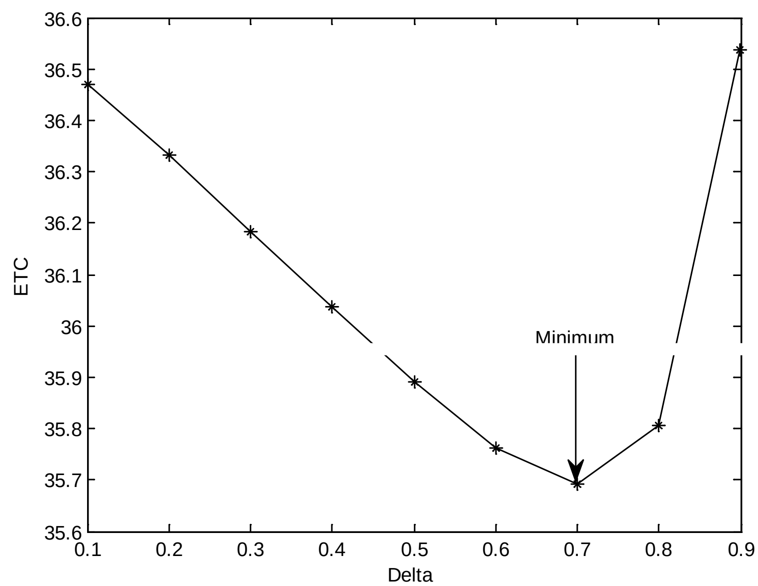

Figure-4: ETC versus δ

S=50,s=5,θ=1.5, α =1.4, N=25, β=2,γ=0.6, μ=3,δ=0.7,
C=20, c_1=1, c_2=1, c_3=3.3, c_4=1.6, c_5=1.6, c_6=2.3.

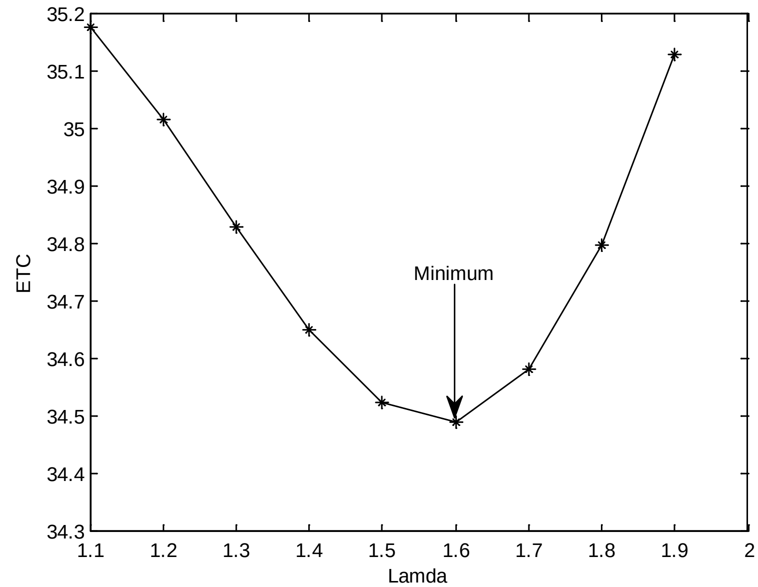

Figure-5: ETC versus λ

S=50, s =5, λ=1.6, α =1.4, N=25, β=2, γ=0.6, μ=2.5, δ=0.7,
C=20, c_1=1, c_2=1, c_3=1.91, c_4=1, c_5=1, c_6=1.

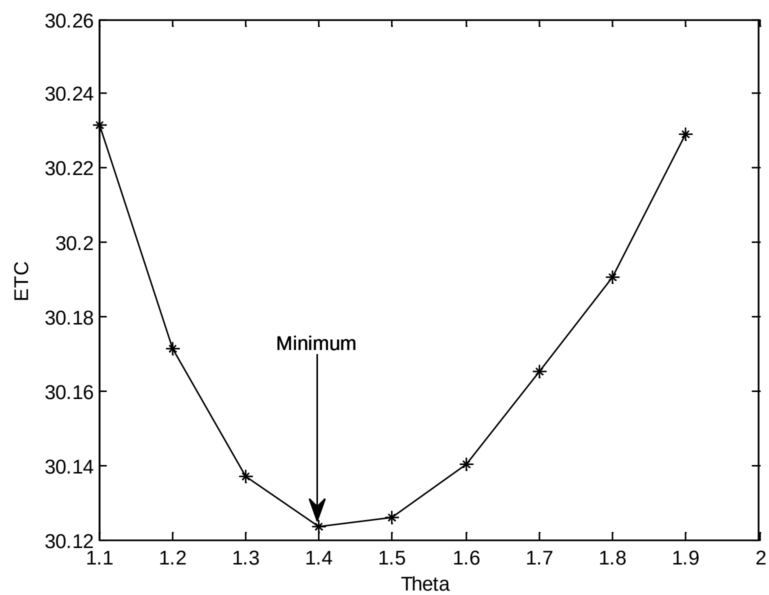

Figure-6: ETC versus θ

4.2 Optimization of (s, S) pair

Here we calculate the optimum value of expected total cost by varying the values of the maximum inventory level S and the inventory level s at which production starts. We find out the optimum (s, S) pair, by fixing the parameter values and cost values. The optimum value of s, for each value of S, is obtained as in Table 7. The optimum value of s is 7 when $S = 31, 32, 33, 34, 35$ and 36. The optimum (s, S) pair, which minimizes ETC, is $(\mathbf{7}, \mathbf{33})$ and the minimum value of ETC is $\mathbf{233.4968}$.

$$\lambda = 1.5, \alpha = 1.4, N = 25, \beta = 2, \gamma = 0.8, \mu = 3, \delta = 0.7, \theta = 1.5,$$
$$c_1 = 1, c_2 = 1, c_3 = 10.8, c_4 = 20.8, c_5 = 20.8, c_6 = 1$$

S / s	31	32	33	34	35	36
5	233.6654	233.6544	233.6470	233.6431	233.6422	233.6441
6	233.5345	233.5282	233.5253	233.5256	233.5287	233.5345
7	**233.5031**	**233.4982**	**233.4968**	**233.4984**	**233.5028**	**233.5097**
8	233.5243	233.5191	233.5173	233.5186	233.5228	233.5296
9	233.5721	233.5655	233.5625	233.5627	233.5659	233.5717
10	233.6326	233.6241	233.6194	233.6181	233.6199	233.6245

Table-7: Optimization of (s, S) pair

5. CONCLUDING REMARKS

In this paper, we considered a production inventory system with different production rates and retrial of customers. We derived some important measures of performances of the system in the steady state. We assumed the higher rate of production $\alpha\beta$, where $\alpha \in [1, k]$, when the inventory level reaches s. This will reduce customers' loss in the stock out period. A suitable cost function is constructed and the optimum value of the enhancing parameter α corresponding to the minimum expected total cost was obtained. The optimum values of other parameters corresponding to minimum expected total cost were also found. The optimum (s, S) pair was calculated. This model has many applications in manufacturing industries. For an example, in the case of a pharmaceutical company medicines can be considered as inventory. In some seasons, large amount of particular type of medicines are required for the treatment of patients. In such situations, the company has to increase its rate of production to satisfy the needs and when the inventory level crosses a particular level, the company keeps the usual production rate. The analyzed model can be extended by assuming Markovian arrival process and phase- type distribution instead of Poisson process and exponential distribution.

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