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NEW APPROACH TO SOLVE UNBALANCED TRANSPORTATION PROBLEMS USING VOGELS APPROXIMATION METHOD

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ABSTRACT

Transportation problem (TP) is considered a vitally important aspect that has been studied in a wide range of operations including research domains. As such, it has been used in simulation of several real life problems. Thus, optimizing TP of variables has been remarkably significant to various disciplines. This paper suggests a heuristic approach in order to balance the unbalanced TP and improve the Vogel's Approximation Method (VAM) in order to get improved (sometimes) initial solution of unbalanced TP in comparison to usual VAM.

Key words: *Transportation Problem, Vogel's Approximation Method, Discounts and Schemes, Overtime.*

1.1 INTRODUCTION

TP is a type of Linear Programming Problem (LPP) that may be solved by using simplex technique called transportation method. It includes major application in solving problems involving several product sources and several destinations of products, this type of problem is frequently called the TP. The two common objectives of such problems are either
(1) Minimize the cost of shipping m units to n destinations (or)
(2) Maximize the profit of shipping m units to n destinations.

The aim of this study is to determine the minimum transportation cost in an easy and efficient manner.

TP can also be formulated as LPP that can be solved using either dual simplex or Big M method. Sometimes this can also be solved using interior approach method. However it is difficult to get the solution using all this method. There are many methods for solving TP. Vogel's method gives approximate solution while MODI and Stepping Stone (SS) method are considered as a standard technique for obtaining to optimal solution. Since decade these two methods are being used for solving TP. Goyal [1984] improving VAM for the Unbalanced TP, Ramakrishnan [1988] discussed some improvement to Goyal's Modified VAM for Unbalanced TP. Moreover Sultan [1988], Sultan and Goyal [1988] studied initial Goyal [1984] basic feasible solution and resolution of degeneracy in TP.

There are various types of transportation models and the simplest of them was first presented by Hitchcock [1941]. It was further developed by Koopmans [1949] and Dantzig [1951]. Several extensions of transportation model and methods have been subsequently developed. TP is based on supply and demand of commodities transported from several sources to the different destinations. The sources from which we need to transport refer the supply while the destination where commodities arrive referred the demand. It has been seen that on many occasion, the decision problem can also be formatting as TP. In general we try to minimize total transportation cost for the commodities transporting from source to destination.

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Adlakha and Kowalski [2009] suggested a systematic analysis for allocating loads to obtain an alternate optimal solution. However, the study on alternate optimal solutions is clearly limited in the literature of transportation with the exception of Sudhakar *et al.*, [2012] who suggested a new approach for finding an optimal solution for TPs. Girmay and Sharma [2013] suggested a heuristic approach in order to balance the unbalanced TP and improve the VAM. Few researchers have tried to give their alternate method for overcoming major obstacles over MODI and SS method.

Goyal [1984] suggested that to assume the largest unit cost of transportation to and from a dummy row or column, present in the given cost matrix rather than assuming to be zero as usual in VAM. He claimed that by this modification, the allocation of units to dummy row or column is automatically given least priority and in addition to this the row or column penalty costs are considered for each interaction. He justified his suggestion by comparing the solution of a numerical problem with VAM and Shimshak [1981]. While Shimshak [1981] suggested ignoring the penalty cost involved with the dummy row or column. So that to give least priority to the allocation of units in dummy row or column. With this suggestion Shimshak [1981] obtained initial solution by VAM.

1.2 TYPES OF TP

There are two types of TP namely Balanced TP and Unbalanced TP.

1.2.1 BALANCED TP

A TP is said to be balanced TP if total number of supply is same as total number of demand.

1.2.2 UNBALANCED TP

A TP is unbalanced if the sum of all available quantities is not equal to the sum of requirements or vice-versa. In regular approach, to balance the unbalanced TP either a dummy row or dummy column is introduced. If total availability is more than the total requirement then a dummy column (destination) is introduced with the requirement to overcome the difference between total availability and total requirement. Cost for dummy row or column is set equal to zero. Such problem is usually solved by VAM to find an initial solution. This paper suggests an algorithm which gives improved initial solution than VAM.

2. SUGGESTION

This paper suggests a method to balance an unbalanced TP. In this present method dummy row or dummy column is not needed in order to balance the unbalanced TP. To find the basic initial solution, increase the demand or supply to balance the unbalanced TP. That is, if the sum of supply is X (say) and the sum of demand is Y (say), then

(i) Suppose total supply is greater than total demand then increase the demand X-Y=C to the column which has the minimum cost cell.
(ii) Suppose total demand is greater than total supply then increase the demand X-Y=D to the row which has the minimum cost cell.

3. EXISTING METHOD FOR FINDING AN INITIAL BASIC FEASIBLE SOLUTION [IBFS]

A set of non-negative allocations which satisfies the row and column restrictions is known as IBFS. This is an initial solution of the problem and is also known as a starting solution of TP. The IBFS may or may not be optimal. By improving upon the IBFS we obtain an optimal solution.

3.1 VOGEL'S APPROXIMATION METHOD

This method usually produces better IBFS. The solution procedure of this method is described step by step in below.

1. Balance the TP using dummy row and dummy column with zero transportation cost.
2. Find the difference between the smallest and second smallest elements along every row and column. This difference is known as penalty. Enter the column penalties below the corresponding columns and row penalties to the right of the corresponding rows.
3. Select the highest penalty cost and observe the row or column along which this appears. If a tie occurs, choose any one of them randomly.
4. Identify the cost cell C_{ij} for allocation which has the least cost in the selected row/column. Make allocation $x_{ij} = min\left(s_i, d_j\right)$ to the cell (i, j).
5. No further consideration is required for the row or column which is satisfied. If both the row and column are satisfied at a time, delete only one of the two, and the remaining row or column is assigned by a zero supply (or demand).
6. Calculate fresh penalty costs for the remaining sub-matrix as in Step-2 and allocate following the procedure of Steps 3, 4 and 5. Continue the process until all rows and columns are satisfied.
7. Finally calculate the total transportation cost which is the sum of the product of cost and corresponding allocated value.

3.2 PROPOSED METHOD

1. Balance the TP without using dummy row and dummy column. That is, if total supply is greater than total demand then select the column which has minimum cost and increase the demand of the corresponding column by total supply minus total demand.
2. Find the difference between the smallest and second smallest elements along every row and column. This difference is known as penalty. Enter the column penalties below the corresponding columns and row penalties to the right of the corresponding rows.
3. Select the highest penalty cost and observe the row or column along which this appears. In case of tie, select the cell where maximum allocation can be allocated.
4. Identify the cost cell C_{ij} for allocation which has the least cost in the selected row/column. Make allocation $x_{ij} = min\left(s_i, d_j\right)$ to the cell (i, j).
5. No further consideration is required for the row or column which is satisfied. If both the row and column are satisfied at a time, delete only one of the two, and the remaining row or column is assigned by a zero supply (or demand).
6. Calculate fresh penalty costs for the remaining sub-matrix as in Step-2 and allocate following the procedure of Steps 3, 4 and 5. Continue the process until all rows and columns are satisfied.
7. Finally calculate the total transportation cost which is the sum of the product of cost and corresponding allocated value.

4. NUMERICAL EXAMPLES

Example 4.1: Consider the following transportation

	D1	D2	D3	Supply
O1	4	8	8	76
O2	16	24	16	82
03	8	16	21	77
demand	72	102	41	

Table-1.1

Here, total supply =235, total demand =215

That is, $\text{Total supply} \neq \text{Total demand}$

Therefore, the given TP is unbalanced.

4.1.1 INITIAL SOLUTION BY VAM

To find initial solution for the given problem introduce the dummy column with zero cost and requirement is equal to 20.

	D1	D2	D3	D4	Capacity
O1	4	8**(35)**	8**(41)**	0	76
O2	16	24**(62)**	16	0**(20)**	82
03	8**(72)**	16**(5)**	21	0	77
Requirement	72	102	41	20	

Table-1.2

Initial transportation cost is equal to

$$8 \times 35 + 8 \times 41 + 24 \times 62 + 0 \times 20 + 8 \times 72 + 16 \times 5 = 2752$$

4.1.1 INITIAL SOLUTION BY PRESENT ALGORITHM

Here total supply =235, total demand =215, so we increase the availability by 20

Here the minimum C_{ij} value is 4 corresponds to first column so increases the requirement of the first column by 20 and apply the proposed method

	D1	D2	D3	Capacity
O1	4	8	8	76
O2	16	24	16	82
03	8	16	21	77
Requirement	92	102	41	

Table-1.3

	D1	D2	D3	Capacity
O1	4	8 **(76)**	8	76
O2	16**(15)**	24**(26)**	16**(41)**	82
03	8**(77)**	16	21	77
Requirement	92	102	41	

Table-1.4

Initial transportation cost is equal to

$$8\times\mathbf{76}+16\times\mathbf{15}+24\times\mathbf{26}+16\times\mathbf{41}+8\times\mathbf{77}=2744$$

Example 4.2: Consider the following transportation

	D1	D2	D3	D4	D5	Capacity
O1	5	8	6	6	3	800
O2	4	7	7	6	5	500
03	8	4	6	6	4	900
Requirement	400	400	500	400	800	

Table-1.5

Here $\sum a_i = 2200$ and $\sum b_i = 2500$

That is, $\text{Total supply} \neq \text{Total demand}$

Therefore, the given TP is unbalanced.

4.2.1 INITIAL SOLUTION BY VAM

To find initial solution for the given problem introduce the dummy row with zero cost and capacity is equal to 300.

	D1	D2	D3	D4	D5	Capacity
O1	5	8	6**(200)**	6**(300)**	3**(300)**	800
O2	4**(400)**	7	7	6**(100)**	5	500
O3	8	4**(400)**	6	6	4**(500)**	900
O4	0	0	0**(300)**	0	0	300
Requirement	400	400	500	400	800	

Table-1.6

Using VAM initial solution is equal to

$$6\times\mathbf{200}+6\times\mathbf{300}+3\times\mathbf{300}+4\times\mathbf{400}+6\times\mathbf{100}+4\times\mathbf{400}+4\times\mathbf{500}+0\times\mathbf{300}=9700$$

4.2.2 INITIAL SOLUTION BY PRESENT ALGORITHM

Here, $\sum a_i = 2200$ and $\sum b_i = 2500$, so we increase the capacity by 300

Here the minimum Cij value is 3 so increases the capacity by 300 in the first row and then apply the proposed method

	D1	D2	D3	D4	D5	Capacity
O1	5	8	6	6	3	1100
O2	4	7	7	6	5	500
03	8	4	6	6	4	900
Requirement	400	400	500	400	800	

Table-1.7

	D1	D2	D3	D4	D5	Capacity
O1	5	8	6	6**(300)**	3**(800)**	1100
O2	4**(400)**	7	7	6**(100)**	5	500
03	8	4**(400)**	6**(500)**	6	4	900
Requirement	400	400	500	400	800	

Table-1.8

Initial solution is equal to

$$6\times\mathbf{300}+3\times\mathbf{800}+4\times\mathbf{400}+6\times\mathbf{100}+4\times\mathbf{400}+6\times\mathbf{500}=11000$$

5. DISCOUNTS AND SCHEMES

Festival season is a good time for the retailers to appeal to customers with discounts and schemes. The consumers too look forward to avail the discounts on offer. The festival season is considered auspicious for shopping too which adds to its attraction.
Festival season discounts comes in two ways

(i) The cash discount where the retailer cuts the price by a significant amount and sells cheaper
(ii) The retailer provides a loan to buy goods at an easy EMI. Typically the interest rate is zero.

Cash discount is easy to understand and is offered by all retailers, big or small. Buy one, get one free is one such discount. 50% off on your purchase is another type of cash discount. Buy one get 50% off on second item is another type. These types are easy to understand. The discount is also immediate. Hence this is most attractive to consumers.

Loan discount, EMI facilities, and zero interest rates are generally offered by big retailers because offering such discounts need an infrastructure with banks and credit card companies. Discount with zero interest rate, easy EMI, pay by credit card and pay back to credit card in instalments are types of discount where cash flow is not immediate.

5.1 OVERTIME

Overtime refers to the time worked in excess of one’s regular working hours. They are doing/working overtime to get the job done on time. Everyone is on overtime means being paid extra for working after the usual time in that weekend.

6. CONCLUSION

When supply exceeds the actual demand sale may be increased by special offers. If the demand exceeds the supply, then the required demand is balanced by working over time

7. REFERENCES

1. F. L. Hitchcock, The distribution of a Product from Several Sources to Numerous Localities, Journal of Mathematics and Physics, 20, (1941), 224-230.
2. T. C. Koopmans, Optimum Utilization of the Transportation System, Econometrical, 17, (1947), 3-4.
3. G. B. Dantzig, Application of the Simplex Method to a TP, Activity Analysis of Production and Allocation, (T.C. Koopmans ed.), New York: John Wiley and Sons, (1951), 359-373.
4. S.K. Goyal, Improving VAM for unbalanced TP, Journal of Operational Research Society, Vol., 35 No.12 (1984), 113-114.
5. Nigus Girmay and Tripti Sharma, Balance an unbalanced TP by a heuristic approach, International Journal of Mathematics and its Applications (2013)
6. Veena Adlakha and Krzysztof Kowalski, Alternate Solutions Analysis For TPs, Journal of Business & Economics Research – November, Vol 7, (2009)
7. Sudhakar VJ, Arunnsankar N, Karpagam T, A new approach for find an Optimal Solution for TPs, European Journal of Scientific Research 68 254-257, (2012).
8. Rekha vivek joshi, Optimization Techniques for TPs of Three Variables, Journal of mathematics, 2013, 2278 – 5728.
9. A.S.Soomro; G.A.Tularam and G.M.Bhayo, A Comparative Study of Initial Basic Feasible Solution Methods for TP, Mathematical theory and modelling, 2014, 2224 – 5804.
10. M.A.Hakim, An Alternative Method to find Initial Basic Feasible Solution of a TP, Annals of pure and applied mathematics, 2012, 2279 – 087X.
11. D.G. Shimshak, J.A. Kashlik and T.D. Barclay, A modification of vogels approximation method through the use of heuristics, Can. J. Opl. Inf. Processing, 19(1981), 259-263.
12. C.S. Ramakrishnan An Imporvement to Goyal’s modified VAM for the unbalanced transportation problem, J. Opl. Res. Soc. 39, (1988) 609-610.
13. A.Sultan and S.K.Goyal, Resolution of degeneracy in transportation problems, Journal of the Operational Research Society, 39 (1988) 411-413.

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