

MINIMAL IDEAL $\alpha\psi$ SUBMAXIMAL IN MINIMAL STRUCTURE SPACES

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ABSTRACT

In this paper we introduce the notion of $mI\alpha\psi$ locally closed set, $mI\alpha\psi$ closed and $mI\alpha\psi$ locally m^ closed sets in ideal minimal spaces and investigate some of their properties. Further we study the $mI\alpha\psi$ submaximal space and derive some of their properties.*

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1. INTRODUCTION

O.Njastad [13] introduced the concept of α closed sets in topological spaces. The notion of ψ closed sets are introduced by M.K.R.S.Veerakumar [17]. R.Devi *et al.*, [5] introduced the concept of $\alpha\psi$ closed sets in topological spaces. The notion of minimal structures and minimal spaces as a generalization of topology and topological spaces were introduced in [11, 12]. Some other results about minimal spaces can be seen in [1, 2, 15]. Ideal in topological spaces were introduced by Kuratowski [10] and Vaidyanathaswamy [18]. Jankovic and Hamlet [9] gave the notion on I-open sets in ideal topological spaces. O.B.Ozbakir and E.D.Yildirim [14] have defined the minimal local function A^*_m in ideal minimal spaces (X, M, I) . In Maki [11] *et al.*, introduced the notion of minimal structure and minimal spaces as a generalization of topological spaces on a given non empty set. A subfamily M of the power set $P(X)$ of a non empty set X is a minimal structure, if $\phi, X \in M$ and (X, M) is called a minimal spaces. A subset A of X is said to be m open [11] if $A \in X$, then $m - \text{int}(A) = \cup \{U : U \subset A, U \in M\}$. A minimal spaces (X, M) has the property [U] and property [I] if "the arbitrary union of m open sets is m open" and "the any finite intersection of m open sets is m open" [15] respectively. Hewit[8] introduced the submaximality in general topology. Arhangel'skii *et al.*, [1] a systematic formulation on submaximality in topology. The concept of ideal submaximal space was investigated by Erdal Akici *et al.*, [7]. Parimala *et al.* [16] studied the concept of minimal locally closed set in minimal structure spaces. R.Chitra [4] introduced a new concept of m^* extremely disconnected ideal minimal space. M.Parimala *et al.*, [17] studied submaximal in terms of ideal minimal spaces. In this paper, we introduce the notion of $mI\alpha\psi$ locally m^* closed, $mI\alpha\psi$ submaximal space and we discussed about their properties and relationships.

2. PRELIMINARIES

Definition 2.1: [14] Let (X, M) be a minimal spaces with an ideal I on X and $(.)^*_m : P(X) \rightarrow P(X)$ be a set operator. For a subset $A \subset X$, $A^*_m(I, M) = \{x \in X : U_m \cap A / \in I \text{ for every } U_m \in U_m(x)\}$ is called the minimal local function of A . Throughout this paper $A^*_m(I, M)$ is denoted by simply A^*_m .

Lemma 2.2: [14] Let (X, M) be a minimal structure space with I, I' ideals on X and $A \subset X$ and $B \subset X$. Then

- (a) $A^*_m \cup B^*_m \subset (A \cup B)^*_m$
- (b) $A^*_m = m - \text{cl}(A^*_m) \subset m - \text{cl}(A)$
- (c) $(A^*_m)^*_m \subset A^*_m$
- (d) $A \subset B \Rightarrow A^*_m \subset B^*_m$
- (e) $I \subset I' \Rightarrow A^*_m(I') \subset A^*_m(I)$

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Lemma 2.3: [14] The set operator $m-cl^*$ satisfies the following conditions:

- (1) $A \subset m-cl^*(A)$
- (2) $m-cl^*(\phi) = \phi$ and $mcl^*(X) = X$
- (3) If $A \subset B$ then $m-cl^*(A) \subset m-cl^*(B)$
- (4) $m-cl^*(A) \cup m-cl^*(B) \subset m-cl^*(A \cap B)$

Lemma 2.4: [14] If (X, M) has property [I], then $mcl^*(m-cl^*(A)) = m-cl^*(A)$ and $m-cl^*(A) \cup m-cl^*(B) = m-cl^*(A \cup B)$.

Lemma 2.5: [14] If (X, M) has property [U], and $I = \phi$ then $A^*_m = m-cl(A)$. In this case $m-cl^*(A) = m-cl(A)$

Lemma 2.6: [14] Let (X, M, I) be an ideal minimal spaces and $A \subset X$. If A is m^* dense in itself, then $A^*_m = mcl(A^*_m) = mcl(A) = mcl^*(A)$.

Definition 2.7: Let (X, M) be a minimal space is called

- (1) m^* dense in itself [14] if $A \subset A^*_m$.
- (2) m^* closed [14] if $A^*_m \subset A$.
- (3) $m\psi$ closed [6] if $mcl(A) \subseteq U$ whenever $A \subseteq U$ and U is m open.
- (4) $m\psi$ closed [6] if $mcl(A) \subseteq U$ whenever $A \subseteq U$ and U is m open.
- (5) $m\pi\psi$ closed [6] if $mcl(A) \subseteq U$ whenever $A \subseteq U$ and U is $m\pi$ open.
- (6) $m\pi\psi$ closed [6] if $mcl(A) \subseteq U$ whenever $A \subseteq U$ and U is $m\pi$ open.

Definition 2.8: Let (X, M, I) be an ideal minimal spaces is called

- (1) mI closed [14] if $A \subset m-int(A^*_m)$
- (2) mI_ψ closed [14] if $A^*_m \subseteq U$ whenever $A \subseteq U$ and U is m open

Lemma 2.9: [4] m^* dense sets are pre mI open sets.

Definition 2.10: [4] A subsets H of Y is defined as a m^* dense set if $mcl^*(H) = Y$.

Lemma 2.11: [17] Let Y be an ideal minimal space, if a subset H is a pre mI open set, then H can be expressed as $H = K \cap L$, where K is m open set and L is m^* dense set.

3. MINIMAL IDEAL $\alpha\psi$ SUBMAXIMAL IN MINIMAL STUCTURE SPACES

Definition 3.1: Let H be a subset of $mI\alpha\psi$ locally m^* closed set if there exist $mI\alpha\psi$ open set S and m^* closed set T then $H = S \cap T$.

Definition 3.2: A subset H of an ideal minimal space (Y, M, I) is $mI_{\alpha\psi}$ closed if $mcl^* \subseteq V$ whenever $H \subseteq V$ and V is $m\alpha\psi$ open.

Theorem 3.3: Let G be a m closed set then G is $mI_{\alpha\psi}$ closed set.

Proof: Let us take V be a $m\alpha\psi$ open and $G \subseteq V$. Here G is m closed set in Y , then $mcl(G) = G$, which implies $mcl(G) \subseteq V$. Therefore $mcl^*(G) \subseteq mcl(G) \subseteq V$. Since $mcl^*(G) = G \cup G^*_m$ and $G \subseteq V$, therefore $G^*_m \subseteq V$, where V is $m\alpha\psi$ open. Hence G is $mI_{\alpha\psi}$ closed set.

Theorem 3.4: Let us consider a ideal minimal space (Y, M, I) and $A \subset Y$ is m^* dense in itself then the following are equivalent.

- (i) H is $mI\alpha\psi$ locally m^* closed set.
- (ii) $H = S \cap mcl^*(H)$ where S is $mI_{\alpha\psi}$ open set.
- (iii) $mcl^*(H) - H = H^*_m - H$ and also $mI_{\alpha\psi}$ closed set.
- (iv) $H \cup (Y - mcl^*(H)) = H \cup (Y - H^*_m)$ and also $mI_{\alpha\psi}$ open sets.
- (v) $H \subset \text{mint}(H \cup (Y - H^*_m))$.

Proof:

(i) \Rightarrow (ii): If H is $mI_{\alpha\psi}$ locally m^* closed set, then $H = S \cap T$ where S is $mI_{\alpha\psi}$ open and T is m^* closed, that is $T^*_m \subset T$. Since $H = S \cap T$, we have $H \subset S$ and $H \subset mcl^*(H)$. Hence $H \subset S \cap mcl^*(H)$. Since T is m^* closed and $H \subset T$, $mcl^*(H) \subset mcl^*(T)$, which implies $S \cap mcl^*(H) \subset S \cap mcl^*(T) \subset S \cap T = H$. Then $H = S \cap mcl^*(H)$.

(ii) \Rightarrow (iii): Let $mcl^*(H) - H = H^*_m - H = H^*_m \cap (Y - H) = H^*_m \cap (Y - (S \cap mcl^*(H))) = H^*_m \cap (Y - S)$. Then $H^*_m \cap (Y - S) \subset U$, which implies $(Y - S) \subset ((Y - H^*_m) \cup U)$. Since S is $mI_{\alpha\psi}$ open set $(Y - S)$ is $mI_{\alpha\psi}$ closed set. Therefore by Definition [3.2], we have $mcl^*(Y - S) \subset ((Y - H^*_m) \cup S)$. Also $H^*_m \cap mcl^*(Y - S) \subset U$.

Now $H_m^* \cap (Y - S) \subset H^* m$ and $H^* m \cap (Y - S) \subset (Y - S)$ by Lemma [2.4], $H^* m \cap (Y - S)^* m \subset (H^* m)^* m \subset H_m^*$ and $H_m^* \cap (Y - S)^* m \subset (Y - S)^* m \subset mcl^*(Y - S)$. Hence $(H_m^* \cap (Y - S))^* m \subset H_m^* \cap mcl^*(Y - S) \subset U$. Also $mcl^*(H) - H = H^* m \cap (Y - S)$. Hence we get $(mcl^*(H) - H)^* m \subset H_m^* \cap mcl^*(Y - S) \subset U$. That is $(mcl^*(H) - H)^* m \subset U$ and U is $m\alpha\psi$ open set. Therefore $mcl^*(H) - H)^* m \subset U$ whenever $mcl^*(H) - H \subset U$, U is $m\alpha\psi$ open set this implies that $mcl^*(H) - H = H_m^* - H$ is a $m\alpha\psi$ closed set.

(iii) \Rightarrow (iv): Since $mcl^*(H) - H$ is $m\alpha\psi$ closed. $(Y - (mcl^*(H) - H))$ is $m\alpha\psi$ open, which implies $H \cup (mcl^*(H) - H) \Rightarrow H \cup (Y - (H - \cup H_m^*))$ is $m\alpha\psi$ open and hence $H \cup (Y - H_m^*)$ is $m\alpha\psi$ open.

(iv) \Rightarrow (v): Since $H \subset (H \cup (Y - H_m^*))$, $mint(H) \subset mint(H \cup (Y - H_m^*))$. Therefore $H \subset mint(H) \subset mint(H \cup (Y - H_m^*))$ and hence $H \subset mint(H \cup (Y - H_m^*))$.

(v) \Rightarrow (i): By (v) we have $H \cup (Y - H_m^*) \subset mint(H \cup (Y - H_m^*))$. By (v), $H \cup (Y - H_m^*) = H \cup (Y - H_m^*)$ is $m\alpha\psi$ open set. Also $H \cup (Y - H_m^*) \cap mcl^*(H) = (H \cap mcl^*(H)) \cup (Y - mcl^*(H)) \cap mcl^*(H) = (H \cap (H \cup H_m^*)) \cup \phi = H \cup (H \cap H_m^*) \cup \phi = H$. Since H is m^* dense set. Also H is m^* dense set, $H \subset H_m^*$. Hence $mcl^*(H) = H^* m$. Therefore $(mcl^*(H))^* m = (H_m^*)^* m = H_m^* = mcl^*(H)$. That is $(mcl^*(H))^* m \subset mcl^*(H)$. Hence $mcl^*(H)$ is m^* closed set. That is $H = (H \cup (Y - H_m^*) \cap mcl^*(H))$ such that $H = (H \cup (Y - H_m^*))$ is $m\alpha\psi$ open and $mcl^*(H)$ is m^* closed. Therefore H is $m\alpha\psi$ locally m^* closed set.

Theorem 3.5: An ideal minimal space with property [I], then the conditions are equivalent,

- (i) Each subset Y is $m\alpha\psi$ locally m^* closed set.
- (ii) Each m^* dense set is $m\alpha\psi$ open set.

Proof:

(i) \Rightarrow (ii): This is proved by Theorem [3.4].

(ii) \Rightarrow (i): Let $G \subset Y$. Let $U = G \cup (Y - mcl^*(G))$. Then $mcl^*(U) = mcl^*(G \cup (Y - mcl^*(G)))$. Then $mcl^*(U) = mcl^*(G \cup (Y - mcl^*(G))) = (G \cup (Y - mcl^*(G))) \cup G \cup (Y - mcl^*(G))^* m = (G \cup (Y - mcl^*(G)))^* m = (G \cup (Y - mcl^*(G))) \cup G^* m \cup (Y - mcl^*(G))^* m = mcl^*(G) \cup mcl^*(Y - mcl^*(G)) = mcl^*(G) \cup (Y - mcl^*(G)) = Y$. Then we have $mcl^*(U) = Y$. Then by definition of m^* dense set. By (ii) U is $m\alpha\psi$ open set. Hence by Theorem [3.4] G is $m\alpha\psi$ locally m^* closed set.

Definition 3.6: Let (Y, M, I) be an ideal minimal spaces is called $m\alpha\psi$ submaximal if each m^* dense subset of Y is $m\alpha\psi$ open.

Remarks 3.7:

- (i) Property [U] refers union of two $m\alpha\psi$ closed set is a $m\alpha\psi$ closed set.
- (ii) Property [I] refers any two intersection of $m\alpha\psi$ closed set is a $m\alpha\psi$ closed set.

Theorem 3.8: An ideal minimal space satisfying the property [I] then the following statements are equivalent.

- (i) An ideal minimal space is a $m\alpha\psi$ submaximal space.
- (ii) If H is a pre m I open set then H is a $m\alpha\psi$ open set.

Proof:

(i) \Rightarrow (ii): Let (Y, M, I) be a $m\alpha\psi$ submaximal space and $H \subset Y$ be a pre m I open set, then $H = K \cap L$, where K is a m open set and L is a m^* dense set. Here Y is $m\alpha\psi$ submaximal space and L is $m\alpha\psi$ open set by definition. Also K is $m\alpha\psi$ open set. Then by property [I], H is also $m\alpha\psi$ open.

(ii) \Rightarrow (i): Let H be m^* dense subset of Y , then by Lemma [2.4] H is pre m I open set. By assumption H is a $m\alpha\psi$ open set. Hence Y is $m\alpha\psi$ submaximal space.

Theorem 3.9: Let (Y, M, I) be an ideal minimal space if the property [I] satisfies then the following statements are equivalent.

- (i) Y is $m\alpha\psi$ submaximal space.
- (ii) If $H \subseteq Y$, then H is $m\alpha\psi$ locally m^* closed set.
- (iii) Any m^* dense set and a $m\alpha\psi$ open subset of Y .

Proof:

(i) \Rightarrow (ii): Let (Y, M, I) be a $m\alpha\psi$ submaximal space by definition each m^* dense set is $m\alpha\psi$ open set by Theorem [3.5], every m^* dense and $m\alpha\psi$ open is $m\alpha\psi$ locally m^* closed set.

(ii) \Rightarrow (iii): Let H be m^* dense set by (ii), H is $m\alpha\psi$ locally m^* closed set. By Theorem [3.4], there exists $m\alpha\psi$ open set K such that $H = K \cap mcl^*(H)$. Now consider H is m^* dense set, then $mcl^*(H) = Y$.

Hence $H = K \cap mcl^*(H) = K \cap Y = K$ and K is $m\alpha\psi$ open set in Y .

(ii) \Rightarrow (i): Here A is m^* dense set by (iii) $H = K \cap L$, where K is $mI\alpha\psi$ open set and L is m^* dense set. Hence $mcl^*(L) = Y$. Since L is m^* closed set $mcl^*(L) = L = Y$. Therefore $H = K \cap L = K \cap Y = K$ and it is $mI\alpha\psi$ open set. Hence Y is $mI\alpha\psi$ submaximal space.

Theorem 3.10: An ideal minimal space (Y, M, I) , the following statements are equivalent.

- (i) (Y, M, I) is a $mI\alpha\psi$ submaximal space.
- (ii) If H is not $mI\alpha\psi$ open set, then $H - (mint(mcl^*(H))) \neq \phi$.

Proof

(i) \Rightarrow (ii): Contradiction, $H - (mint(mcl^*(H))) = \phi$. Hence $H \subset mint(mcl^*(H))$, which means H is pre m I open set. Let Y is a $mI\alpha\psi$ submaximal space, by Theorem [3.8] H is $mI\alpha\psi$ open which is contrary by the assumption. Hence $H - (mint(mcl^*(H))) = \phi$.

(ii) \Rightarrow (i): Assume H is pre m I open set but it is not $mI\alpha\psi$ open set, then by (ii) $H - mint(mcl^*(H)) \neq \phi$, which implies $H \neq mint(mcl^*(H))$. That is H is not pre m I open set which is contradiction by our assumption. Hence H is $mI\alpha\psi$ open set and hence by Theorem [3.8] Y is $mI\alpha\psi$ submaximal space.

Theorem 3.11: An ideal minimal space (Y, M, I) , if the property [I] satisfies, then the following statements are equivalent.

- (i) (Y, M, I) is a $mI\alpha\psi$ submaximal space.
- (ii) The $mI\alpha\psi$ open set $\gamma = \{K - H : K \text{ is } mI\alpha\psi \text{ open and } mint^*(H) = \phi\}$.

Proof:

(i) \Rightarrow (ii): Let Y be a $mI\alpha\psi$ submaximal space. Assume $\eta = \{K - H : K \text{ is } mI\alpha\psi \text{ open and } mint^*(H) = \phi\}$. To prove $\gamma = \eta$. Let us take an element $A \in \gamma$. Since $A = A - \phi$ and $mint^*(\phi) = \phi$, $A \in \eta$. Hence $\gamma \subset \eta$. Let $A \in \eta$, to prove A is $mI\alpha\psi$ open set. Since $A \in \eta$, A can be written as $A = K - H$ such that K is a $mI\alpha\psi$ open and $mint^*(H) = \phi$. Also $A = H - K = K \cap (Y - H)$. Since $mint^*(H) = \phi$, $Y - mint^*(Y - H) = mcl^*(H) = Y$. Hence $Y - H$ is m^* dense set. Since Y is $mI\alpha\psi$ submaximal $Y - H$ is m^* dense implies $(Y - H)$ is $mI\alpha\psi$ open set. Hence $A = K \cap (Y - H)$ is $mI\alpha\psi$ open set and so $\eta \subset \gamma$. Therefore $\eta = \gamma$.

(ii) \Rightarrow (i): Assume H is pre m I open set, by Lemma [2.4] H be the intersection of the sets S and T such that S is m open and T is m^* dense. Hence $mcl^*(T) = Y$ and so $mint^*(Y - T) = \phi$. That is $H = S \cap T = S - (Y - T)$ and $mint^*(Y - T) = \phi$. By Theorem [3.3] S is $mI\alpha\psi$ open set. Therefore $H \subset \gamma$ and hence H is $mI\alpha\psi$ open set. By assumption, the pre m I open set A is $mI\alpha\psi$ open. Hence by Theorem [3.8] Y is $mI\alpha\psi$ submaximal space.

Theorem 3.12: Let (Y, M, I) , satisfying the property [I], then the following statements are equivalent.

- (i) (Y, M, I) is a $mI\alpha\psi$ submaximal space.
- (ii) There exist a $mI\alpha\psi$ closed set $mcl^*(H) - H$ for each subset H of Y.

Proof:

(i) \Rightarrow (ii): Assume Y be $mI\alpha\psi$ submaximal and let $H \subset Y$. To prove $mcl^*(H) - H$ is $mI\alpha\psi$ closed set. It is enough to prove $Y - (mcl^*(H) - H)$ is $mI\alpha\psi$ open. That is to prove that $mcl^*(H \cup (Y - (mcl^*(H)))) = mcl^*(H) \cup mcl^*(Y - mcl^*(H)) = Y$. Hence $(Y - (mcl^*(H) - H))$ is a m^* dense set. Also since Y is $mI\alpha\psi$ submaximal space, by definition each m^* dense set is $mI\alpha\psi$ open set. Hence $(Y - (mcl^*(H) - H))$ is $mI\alpha\psi$ open set. Therefore $(mcl^*(H) - H)$ is $mI\alpha\psi$ closed set.

(ii) \Rightarrow (i): Let $(mcl^*(H) - H)$ is $mI\alpha\psi$ closed set and let $H \subset Y$ is m^* dense set in Y. Hence $mcl^*(H) = Y$ implies $(Y - H)$ is $mI\alpha\psi$ closed set. So H is $mI\alpha\psi$ open. Hence Y is $mI\alpha\psi$ submaximal space.

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