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FIXED POINT THEOREMS FOR MAPPINGS IN HILBERT SPACE

S. K. NIGAM* AND SHAILENDRA KUMAR DWIVEDI**

*,**Department of Mathematics, Govt. PG. College Satna - (M.P.), 485001, INDIA.

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ABSTACT

In this paper we have tried to establish fixed point theorems for mappings in Hilbert space, which generalized the results of many authors.

Keyword: *Hilbert space, Contraction mapping, fixed point theorem.*

AMS subject classification No*. 47H10, 54 H25.*

INTRODUCTION

Ciric [3] introduced the notion of generalized contraction mapping and proved fixed point theorems, Dass and Gupta [4] introduced fixed point theorems in reflexive Banach spaces. The study of properties and applications of fixed points of various types of contractive mappings were obtained among others by Banach [1], R. Kannan[6], S.K. Chatterjee [2] Naimpally and Singh [7] & Rhoads [8].

The most of fixed point theorems in metric spaces satisfying different contraction condition may be extended to the abstract spaces like Hilbert, Banach and Locally convex spaces with some modifications. In this paper fixed point theorem for mapping in Hilbert space has been proved which generalized the results of T.Veerapandi and M.Mariappan [9].We know that Banach space is Hilbert Space if and only if its norm satisfies the parallelogram law i.e. for every x, y $\in X$

$$\|x+y\|^2+\|x-y\|^2=2\|x\|^2+2\|y\|^2$$

This implies

$$\|x+y\|^2\leq 2\|x\|^2+2\|y\|^2$$

We prove the result concerning the existence of fixed point for mappings satisfying the contractive condition of the type.

$$\|Tx-Ty\|^2\leq a\|x-Tx\|\|y-Ty\|+b[\|x-Tx\|\|y-Tx\|+\|y-Ty\|\|x-Ty\|]+c\|x-Ty\|\|y-Tx\| \\ +d[\|x-Tx\|^2+\|y-Ty\|^2]+e[\|x-Ty\|^2+\|y-Tx\|^2]+f\|x-y\|^2$$

Where, $0\leq a+2b+c+2d+4e+f<1$ & $\forall x,y\in C, x\neq y$.

OUR MAIN RESULTS

Theorem 1: Let C be a closed subset of Hilbert space X and let $T: C\to C$ be a self mapping on C satisfying:

$$\|Tx-Ty\|^2\leq a\|x-Tx\|\|y-Ty\|+b[\|x-Tx\|\|y-Tx\|+\|y-Ty\|\|x-Ty\|]+c\|x-Ty\|\|y-Tx\| \\ +d[\|x-Tx\|^2+\|y-Ty\|^2]+e[\|x-Ty\|^2+\|y-Tx\|^2]+f\|x-y\|^2$$

Where, $0\leq a+2b+c+2d+4e+f<1$ & $\forall x,y\in C, x\neq y$. Then T has a unique fixed point.

Proof: Let $x_0\in C$ be any arbitrary element in. Define the sequence $\{x_n\}$ in C as follows:

Let $x_{n+1}=Tx_n$, for $n=0,1,2,3,\ldots..$

*Corresponding Author: Shailendra Kumar Dwivedi***
,Department of Mathematics, Govt. PG. College Satna - (M.P.), 485001, INDIA.*

Now,

$$\begin{aligned}
\|x_2 - x_1\|^2 &= \|Tx_1 - Tx_0\|^2 \\
&\le a\|x_1 - Tx_1\|\|x_0 - Tx_0\| + b[\|x_1 - Tx_1\|\|x_0 - Tx_1\| + \|x_0 - Tx_0\|\|x_1 - Tx_0\|] + c\|x_1 - Tx_0\|\|x_0 - Tx_1\| \\
&\qquad + d[\|x_1 - Tx_1\|^2 + \|x_0 - Tx_0\|^2] + e[\|x_1 - Tx_0\|^2 + \|x_0 - Tx_1\|^2] + f\|x_1 - x_0\|^2 \\
&= a\|x_1 - x_2\|\|x_0 - x_1\| + b[\|x_1 - x_2\|\|x_0 - x_2\| + \|x_0 - x_1\|\|x_1 - x_1\|] + c\|x_1 - x_1\|\|x_0 - x_2\| \\
&\qquad + d[\|x_1 - x_2\|^2 + \|x_0 - x_1\|^2] + e[\|x_1 - x_1\|^2 + \|x_0 - x_2\|^2] + f\|x_1 - x_0\|^2 \\
&= a\|x_1 - x_2\|\|x_0 - x_1\| + b[\|x_1 - x_2\|\|x_0 - x_2\|] + d[\|x_1 - x_2\|^2 + \|x_0 - x_1\|^2] + e[\|x_0 - x_2\|^2] \\
&\qquad + f\|x_1 - x_0\|^2 \\
&\le a\|x_1 - x_2\|\|x_0 - x_1\| + b\|x_1 - x_2\|\|x_0 - x_1\| + b\|x_1 - x_2\|^2 + d[\|x_1 - x_2\|^2 + \|x_0 - x_1\|^2] \\
&\qquad + e[\|x_0 - x_1\|^2 + \|x_1 - x_2\|^2 + 2\|x_0 - x_1\|\|x_1 - x_2\|] + f\|x_1 - x_0\|^2 \\
&= a\|x_2 - x_1\|\|x_1 - x_0\| + b\|x_2 - x_1\|\|x_1 - x_0\| + b\|x_2 - x_1\|^2 + d[\|x_2 - x_1\|^2 + \|x_1 - x_0\|^2] \\
&\qquad + e[\|x_1 - x_0\|^2 + \|x_2 - x_1\|^2 + 2\|x_1 - x_0\|\|x_2 - x_1\|] + f\|x_1 - x_0\|^2
\end{aligned}$$

Hence

$$(1 - b - d - e)\|x_2 - x_1\|^2 \le (a + b + 2e)\|x_2 - x_1\|\|x_1 - x_0\| + (d + e + f)\,\|x_1 - x_0\|^2$$

From this we have,

$$\|x_2 - x_1\|^2 \le \frac{(a + b + 2e)}{(1 - b - d - e)}\|x_2 - x_1\|\|x_1 - x_0\| + \frac{(d + e + f)}{(1 - b - d - e)}\,\|x_1 - x_0\|^2$$

Divide both sides by $\|x_1 - x_0\|^2$ we get,

$$\frac{\|x_2 - x_1\|^2}{\|x_1 - x_0\|^2} \le \frac{(a + b + 2e)}{(1 - b - d - e)}\frac{\|x_2 - x_1\|}{\|x_1 - x_0\|} + \frac{(d + e + f)}{(b - d - e)}$$

Here substitute

$$\gamma = \frac{\|x_2 - x_1\|}{\|x_1 - x_0\|}\ ; \qquad \alpha = \frac{(a + b + 2e)}{(1 - b - d - e)}\ ; \qquad \beta = \frac{(d + e + f)}{(b - d - e)}$$

Now we get that $\gamma^2 \le \gamma\alpha + \beta$ & $\alpha + \beta = \frac{(a+b+d+3e+f)}{(1-b-d-e)} < 1$

Since $a + 2b + 2d + 4e + f < 1$

This gives that $\gamma^2 - \gamma\alpha - \beta \le 0$ & $\alpha + \beta < 1$

And so $\gamma \le \frac{1}{2}(\alpha + \sqrt{\alpha^2 + 4\beta}) = r < 1$ $[\because \alpha + \beta < 1]$

Therefore $\|x_2 - x_1\| \le r\|x_1 - x_0\|$

Continuing in this way we get

$$\begin{aligned}
\|x_{n+1} - x_n\| &\le r\|x_n - x_{n-1}\| \qquad , \text{for } n = 1,2,3, \dots\dots\dots \\
&\le r^n\|x_1 - x_0\| \to 0 \quad \text{as} \quad n \to \infty
\end{aligned}$$

Hence $\{x_n\}$ is a Cauchy sequence in C. Since C is complete, $\{x_n\}$ converges to a point x in C. Therefore $\lim_{n\to\infty} x_n = x$

Suppose $x \neq Tx$

Then,

$$\begin{aligned}
\|x - Tx\|^2 &= \|x - x_{n+1} + x_{n+1} - Tx\|^2 \\
&\le [\|x - x_{n+1}\| + \|x_{n+1} - Tx\|]^2 \\
&= \|x - x_{n+1}\|^2 + 2\|x - x_{n+1}\|\|x_{n+1} - Tx\| + \|x_{n+1} - Tx\|^2 \\
&= \|x - x_{n+1}\|^2 + 2\|x - x_{n+1}\|\|x_{n+1} - Tx\| + \|Tx_n - Tx\|^2 \\
&\le \|x - x_{n+1}\|^2 + 2\|x - x_{n+1}\|\|x_{n+1} - Tx\| + a\|x_n - Tx_n\|\|x - Tx\| \\
&\qquad + b[\|x_n - Tx_n\|\|x - Tx_n\| + \|x - Tx\|\|x_n - Tx\|] + c\|x_n - Tx\|\|x - Tx_n\| \\
&\qquad + d[\|x_n - Tx_n\|^2 + \|x - Tx\|^2] + e[\|x_n - Tx\|^2 + \|x - Tx_n\|^2] + f\|x_n - x\|^2
\end{aligned}$$

That is

$$\begin{aligned}
\|x - Tx\|^2 \le\ & \|x - x_{n+1}\|^2 + 2\|x - x_{n+1}\|\|x_{n+1} - Tx\| + a\|x_n - x_{n+1}\|\|x - Tx\| \\
&+ b[\|x_n - x_{n+1}\|\|x - x_{n+1}\| + \|x - Tx\|\|x_n - Tx\|] + c\|x_n - Tx\|\|x - x_{n+1}\| \\
&+ d[\|x_n - x_{n+1}\|^2 + \|x - Tx\|^2] + e[\|x_n - Tx\|^2 + \|x - x_{n+1}\|^2] + f\|x_n - x\|^2
\end{aligned}$$

Allowing $n \to \infty$, we get that

$$\begin{aligned}
\|x - Tx\|^2 \to\ & \|x - x\|^2 + 2\|x - x\|\|x - Tx\| + a\|x - x\|\|x - Tx\| + b[\|x - x\|\|x - x\| + \|x - Tx\|\|x - Tx\|] \\
&+ c\|x - Tx\|\|x - x\| + d[\|x - x\|^2 + \|x - Tx\|^2] + e[\|x - Tx\|^2 + \|x - x\|^2] + f\|x - x\|^2
\end{aligned}$$

From this we have

$$\|x - Tx\|^2 \le (b + d + e)\|x - Tx\|^2$$

So that

$$\|x - Tx\|^2 < \|x - Tx\|^2$$

Since $b + d + e < 1$ which is contradiction.
Therefore $x = Tx$

In order to prove the uniqueness of fixed point, assume that x and y be two distinct fixed points of T. Then $Tx = x$ and $Ty = y$ and $x \neq y$.
Then we have,

$$\|Tx - Ty\|^2 \leq a\|x - Tx\|\|y - Ty\| + b[\|x - Tx\|\|y - Tx\| + \|y - Ty\|\|x - Ty\|] + c\|x - Ty\|\|y - Tx\| + d[\|x - Tx\|^2 + \|y - Ty\|^2] + e[\|x - Ty\|^2 + \|y - Tx\|^2] + f\|x - y\|^2$$

And so

$$\begin{aligned}\|x - y\|^2 &\leq a\|x - x\|\|y - y\| + b[\|x - x\|\|y - x\| + \|y - y\|\|x - y\|] + c\|x - y\|\|y - x\| + d[\|x - x\|^2 + \|y - y\|^2] \\ &\quad + e[\|x - y\|^2 + \|y - x\|^2] + f\|x - y\|^2 \\ &= c\|x - y\|\|y - x\| + e[\|x - y\|^2 + \|y - x\|^2] + f\|x - y\|^2 \\ &= c\|x - y\|\|x - y\| + e[\|x - y\|^2 + \|x - y\|^2] + f\|x - y\|^2 \\ &= c\|x - y\|^2 + 2e\|x - y\|^2 + f\|x - y\|^2 \\ &= (c + 2e + f)\|x - y\|^2\end{aligned}$$

$$\|x - y\|^2 < \|x - y\|^2$$

Since $c + 2e + f < 1$, this is contradiction.
Therefore $x = y$
Thus T has a unique fixed point.

Taking $a = b = c = d = e = 0$ & $f = h^2$ we get the following theorem as Corollary:

Corollary 1.1: Let C be convex subset of Hilbert space X and $T: C \to C$ be mapping such that $\|Tx - Ty\| \leq h\|x - y\|$ where $0 \leq h < 1$, $\forall x, y \in C$. Then T has a unique fixed point.

Taking $b = c = e = f = 0$, $a = \frac{h^2}{2}$ & $d = \frac{h^2}{4}$ we get the following theorem as Corollary:

Corollary 1.2: Let C be convex subset of Hilbert space X and $T: C \to C$ be mapping such that

$$\|Tx - Ty\| \leq \frac{h}{2}[\|x - Tx\| + \|y - Ty\|]$$

Where $0 \leq h < 1$, $\forall x, y \in C$. Then T has a unique fixed point.

Taking $a = b = d = f = 0$, $c = \frac{h^2}{2}$ & $e = \frac{h^2}{4}$ we get the following theorem as Corollary:

Corollary 1.3: Let C be convex subset of Hilbert space X and $T: C \to C$ be mapping such that

$$\|Tx - Ty\| \leq \frac{h}{2}[\|x - Ty\| + \|y - Tx\|]$$

Where $0 \leq h < 1$, $\forall x, y \in C$. Then T has a unique fixed point.

Taking $b = 0$, $a = c = \frac{h^2}{2}$ & $d = e = f = \frac{h^2}{4}$ we get the following theorem as Corollary:

Corollary 1.4: Let C be convex subset of Hilbert space X and $T: C \to C$ be mapping such that

$$\|Tx - Ty\| \leq \frac{h}{2}[\|x - y\| + \|x - Tx\| + \|y - Ty\| + \|x - Ty\| + \|y - Tx\|]$$

where $0 \leq h < 1$, $\forall x, y \in C$. Then T has a unique fixed point.

Theorem 2: Let C be a closed subset of Hilbert space X and let $T, S: C \to C$ be a self mappings on C satisfying:

$$\|Tx - Sy\|^2 \leq a\|x - Tx\|\|y - Sy\| + b[\|x - Tx\|\|y - Tx\| + \|y - Sy\|\|x - Sy\|] + c\|x - Sy\|\|y - Tx\| + d[\|x - Tx\|^2 + \|y - Sy\|^2] + e[\|x - Sy\|^2 + \|y - Tx\|^2] + f\|x - y\|^2$$

Where, $0 \leq a + 2b + c + 2d + 4e + f < 1$ & $\forall x, y \in C, x \neq y$. Then T, S have a unique fixed point.

Proof: Let $x_0 \in C$ be any arbitrary element in C. Define the sequence $\{x_n\}$ in C as follows:
Let $x_{2n+1} = Tx_n$ & $x_{2n+2} = Sx_{n+1}$, for $n = 0,1,2,3, \ldots$

Now,

$$\begin{aligned}\|x_2 - x_1\|^2 &= \|Sx_1 - Tx_0\|^2 \\ &\leq a\|x_1 - Sx_1\|\|x_0 - Tx_0\| + b[\|x_1 - Sx_1\|\|x_0 - Sx_1\| + \|x_0 - Tx_0\|\|x_1 - Tx_0\|] \\ &\quad + c\|x_1 - Tx_0\|\|x_0 - Sx_1\| + d[\|x_1 - Sx_1\|^2 + \|x_0 - Tx_0\|^2] \\ &\quad + e[\|x_1 - Tx_0\|^2 + \|x_0 - Sx_1\|^2] + f\|x_1 - x_0\|^2 \\ &= a\|x_1 - x_2\|\|x_0 - x_1\| + b[\|x_1 - x_2\|\|x_0 - x_2\| + \|x_0 - x_1\|\|x_1 - x_1\|] + c\|x_1 - x_1\|\|x_0 - x_2\| \\ &\quad + d[\|x_1 - x_2\|^2 + \|x_0 - x_1\|^2] + e[\|x_1 - x_1\|^2 + \|x_0 - x_2\|^2] + f\|x_1 - x_0\|^2\end{aligned}$$

$$= a\|x_1 - x_2\|\|x_0 - x_1\| + b[\|x_1 - x_2\|\|x_0 - x_2\|] \quad + d[\|x_1 - x_2\|^2 + \|x_0 - x_1\|^2] + e[\|x_0 - x_2\|^2]$$
$$+ f\|x_1 - x_0\|^2$$
$$\leq a\|x_1 - x_2\|\|x_0 - x_1\| + b\|x_1 - x_2\|\|x_0 - x_1\| + b\|x_1 - x_2\|^2$$
$$+ d[\|x_1 - x_2\|^2 + \|x_0 - x_1\|^2] + e[\|x_0 - x_1\|^2 + \|x_1 - x_2\|^2 + 2\|x_0 - x_1\|\|x_1 - x_2\|]$$
$$+ f\|x_1 - x_0\|^2$$
$$= a\|x_2 - x_1\|\|x_1 - x_0\| + b\|x_2 - x_1\|\|x_1 - x_0\| + b\|x_2 - x_1\|^2 + d[\|x_2 - x_1\|^2 + \|x_1 - x_0\|^2]$$
$$+ e[\|x_1 - x_0\|^2 + \|x_2 - x_1\|^2 + 2\|x_1 - x_0\|\|x_2 - x_1\|] + f\|x_1 - x_0\|^2$$

Hence

$$(1 - b - d - e)\|x_2 - x_1\|^2 \leq (a + b + 2e)\|x_2 - x_1\|\|x_1 - x_0\| + (d + e + f)\,\|x_1 - x_0\|^2$$

From this we have,

$$\|x_2 - x_1\|^2 \leq \frac{(a + b + 2e)}{(1 - b - d - e)}\|x_2 - x_1\|\|x_1 - x_0\| + \frac{(d + e + f)}{(1 - b - d - e)}\,\|x_1 - x_0\|^2$$

Divide both sides by $\|x_1 - x_0\|^2$ we get,

$$\frac{\|x_2 - x_1\|^2}{\|x_1 - x_0\|^2} \leq \frac{(a + b + 2e)}{(1 - b - d - e)}\frac{\|x_2 - x_1\|}{\|x_1 - x_0\|} + \frac{(d + e + f)}{(b - d - e)}$$

Here substitute

$$\gamma = \frac{\|x_2 - x_1\|}{\|x_1 - x_0\|}, \qquad \alpha = \frac{(a + b + 2e)}{(1 - b - d - e)}, \quad \beta = \frac{(d + e + f)}{(b - d - e)}$$

Now we get that, $\gamma^2 \leq \gamma\alpha + \beta$ & $\alpha + \beta = \frac{(a+b+d+3e+f)}{(1-b-d-e)} < 1$

Since $a + 2b + 2d + 4e + f < 1$

This gives that $\gamma^2 - \gamma\alpha - \beta \leq 0$ and $\alpha + \beta < 1$

From this we have, $\gamma \leq \frac{1}{2}(\alpha + \sqrt{\alpha^2 + 4\beta}) = r < 1$ $[\because \alpha + \beta < 1]$

Therefore $\|x_2 - x_1\| \leq r\|x_1 - x_0\|$

Continuing in this way, we get that

$$\|x_{n+1} - x_n\| \leq r\|x_n - x_{n-1}\| \qquad, \text{for } n = 1,2,3, \dots \dots \dots \dots$$
$$\leq r^n\|x_1 - x_0\| \to 0 \quad \text{As } n \to \infty.$$

Hence $\{x_n\}$ is a Cauchy sequence in C. Since C is complete, $\{x_n\}$ converges to a point x in C. Therefore $\lim_{n\to\infty} x_{2n+1} = \lim_{n\to\infty} x_{2n+2} = x$.

Suppose $x \neq Tx$ Then,

$$\|x - Tx\|^2 = \|x - x_{2n+2} + x_{2n+2} - Tx\|^2$$
$$\leq [\|x - x_{2n+2}\| + \|x_{2n+2} - Tx\|]^2$$
$$= \|x - x_{2n+2}\|^2 + 2\|x - x_{2n+2}\|\|x_{2n+2} - Tx\| + \|x_{2n+2} - Tx\|^2$$
$$= \|x - x_{2n+2}\|^2 + 2\|x - x_{2n+2}\|\|x_{2n+2} - Tx\| + \|Sx_{2n+1} - Tx\|^2$$
$$\leq \|x - x_{2n+2}\|^2 + 2\|x - x_{2n+2}\|\|x_{2n+2} - Tx\| + a\|x_{2n+1} - Sx_{2n+1}\|\|x - Tx\|$$
$$+ b[\|x_{2n+1} - Sx_{2n+1}\|\|x - Sx_{2n+1}\| + \|x - Tx\|\|x_{2n+1} - Tx\|]$$
$$+ c\|x_{2n+1} - Tx\|\|x - Sx_{2n+1}\| + d[\|x_{2n+1} - Sx_{2n+1}\|^2 + \|x - Tx\|^2]$$
$$+ e[\|x_{2n+1} - Tx\|^2 + \|x - Sx_{2n+1}\|^2] + f\|x_{2n+1} - x\|^2$$
$$= \|x - x_{2n+2}\|^2 + 2\|x - x_{2n+2}\|\|x_{2n+2} - Tx\| + a\|x_{2n+1} - x_{2n+2}\|\|x - Tx\|$$
$$+ b[\|x_{2n+1} - x_{2n+2}\|\|x - x_{2n+2}\| + \|x - Tx\|\|x_{2n+1} - Tx\|]$$
$$+ c\|x_{2n+1} - Tx\|\|x - x_{2n+2}\| + d[\|x_{2n+1} - x_{2n+2}\|^2 + \|x - Tx\|^2]$$
$$+ e[\|x_{2n+1} - Tx\|^2 + \|x - x_{2n+2}\|^2] + f\|x_{2n+1} - x\|^2$$

Allowing as $n \to \infty$, we get that

$$\|x - Tx\|^2 \to \|x - x\|^2 + 2\|x - x\|\|x - Tx\| + a\|x - x\|\|x - Tx\| + b[\|x - x\|\|x - x\| + \|x - Tx\|\|x - Tx\|]$$
$$+ c\|x - Tx\|\|x - x\| + d[\|x - x\|^2 + \|x - Tx\|^2] + e[\|x - Tx\|^2 + \|x - x\|^2] + f\|x - x\|^2$$

That is,

$$\|x - Tx\|^2 \leq b\|x - Tx\|^2 + d\|x - Tx\|^2 + e\|x - Tx\|^2$$

From this we have

$$\|x - Tx\|^2 \leq (b + d + e)\|x - Tx\|^2$$

So that

$$\|x - Tx\|^2 < \|x - Tx\|^2$$

Since $b + d + e < 1$ which is contradiction.

Therefore $x = Tx$

Similarly we can prove that $x = Sx$. Hence x is a common fixed point of T and S.

In order to prove the uniqueness of fixed point, let x and y be two fixed points of T and S, then $Tx = Sx = x$ & $Ty = Sy = y$.

Suppose $x \neq y$, then we have

$$\|Tx - Sy\|^2 \leq a\|x - Tx\|\|y - Sy\| + b[\|x - Tx\|\|y - Tx\| + \|y - Sy\|\|x - Sy\|] + c\|x - Sy\|\|y - Tx\| \\ + d[\|x - Tx\|^2 + \|y - Sy\|^2] + e[\|x - Sy\|^2 + \|y - Tx\|^2] + f\|x - y\|^2$$

And so

$$\begin{aligned}\|x - y\|^2 &\leq a\|x - x\|\|y - y\| + b[\|x - x\|\|y - x\| + \|y - y\|\|x - y\|] + c\|x - y\|\|y - x\| + d[\|x - x\|^2 + \|y - y\|^2] \\ &\quad + e[\|x - y\|^2 + \|y - x\|^2] + f\|x - y\|^2 \\ &= c\|x - y\|\|y - x\| + e[\|x - y\|^2 + \|y - x\|^2] + f\|x - y\|^2 \\ &= c\|x - y\|\|x - y\| + e[\|x - y\|^2 + \|x - y\|^2] + f\|x - y\|^2 \\ &= c\|x - y\|^2 + 2e\|x - y\|^2 + f\|x - y\|^2 \\ &= (c + 2e + f)\|x - y\|^2 \\ &< \|x - y\|^2\end{aligned}$$

Since $c + 2e + f < 1$
That is $\|x - y\|^2 < \|x - y\|^2$
This is a contradiction.
Therefore $x = y$
Thus T and S have a unique common fixed point.

Corollary 2.1: Let C be convex subset of Hilbert space X and $T, S: C \rightarrow C$ be mapping such that

$$\|Tx - Sy\|^2 \leq h\|x - y\|$$

where $0 \leq h < 1, \forall x, y \in C$. Then T and S have a unique fixed point.

REFERENCES

1. Banach, S. – Sur Les operations dans les ensembles abstraits et leur applications aux. Equations integraies, Fund. Math., 3 (1922), p. 133-181.
2. Chatterjee, S.K. – Fixed point theorem, Comte Read. Acad. Balgare Sc, 25 (1972), p. 727-730.
3. Ciric $L_{j.}$ B – Generalized contractions and fixed point theorem,Publ. Inst. Math. 12(26), (1971) 19-26.
4. Dass, B.K. and Gupta, S. – An extension of Banach contraction principle through rational Expression Indian Jour, Pure and Applied Math.6 (1975), 1455-1458.
5. Farkhunda, S. & Badshah, V.H. – Generalized contraction and common fixed point theorem in Hilbert space. J. Indian Acad. Math. Vol 23, No.2. (2001) 267-275.
6. Kannan, R. – Some results on fixed points. Bull. Cal. Math. Soc, 60 (1968), p. 71-76.
7. Naimpally S.A. and Sing, K.L. – Extensions of some fixed point theorems of Rhoades, J.Math. Anal.Appal. 96 (1983) 437-446.
8. Rhoades, B.E. –Extensions of some fixed point theorems of Ciric, Maiti and pal. Math Sem. Notes, Kobe univ. 6, (1978) 41-46.
9. Veerapandi T. and Mariappan M. – The mathematics education valume xxxix no-1 march 2005.

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