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# NEW NOTION OF GENERALIZED BINARY CLOSED SETS IN BINARY TOPOLOGICAL SPACE

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***ABSTRACT***

*In this paper we define and study generalized binary semi closed sets using binary semi open sets in binary topological spaces. Furthermore, binary semi $T_{1/2}$-space, binary semi $T_{1/2}{}^{*}$-space and binary semi $T_{1/2}{}^{**}$-space are introduced and their properties are investigated.*

***Keywords:*** *Binary topology, binary closed sets, binary semi open sets, generalized binary semi closed sets.*

## 1. INTRODUCTION

In 2011, authors S.Nithyanantha Jothi and P.Thangavelu [1] introduced binary topology. Recently generalized binary closed sets are introduced and some of their basic properties are discussed by S.Nithyanantha Jothi *et al.* in [4]. In 2014, S.Nithyanantha Jothi [2] introduced binary semi open sets in binary topological spaces and studied their characteristics. In this paper, a new class of sets called generalized binary semi closed sets in binary topological spaces are introduced via binary semi open sets and some of their basic properties are investigated. Furthermore, binary semi $T_{1/2}$-space, binary semi $T_{1/2}{}^{*}$-space and binary semi $T_{1/2}{}^{**}$-space are introduced and their properties are discussed.

## 2. PRELIMINARIES

In this section, definitions referred by us which will be required throughout the present work are presented.

**Definition 2.1:** [1] A binary topology from X to Y is a binary structure from $\mathcal{M} \subseteq \wp(X) \times \wp(Y)$ that satisfies the following axioms

(i) $(\emptyset, \emptyset)$ and $(X,Y) \in \mathcal{M}$.

(ii) $(A_1 \cap A_2, B_1 \cap B_2) \in \mathcal{M}$ whenever $(A_1, B_1) \in \mathcal{M}$ and $(A_2, B_2) \in \mathcal{M}$.

(iii) If $\{(A_\alpha, B_\alpha) : \alpha \in \Delta\}$ is a family of members of $\mathcal{M}$, then $(\cup_{\alpha\in\Delta} A_\alpha, \cup_{\alpha\in\Delta} B_\alpha) \in \mathcal{M}$.

**Definition 2.2:** [4] If $\mathcal{M}$ is a binary topology from X to Y, then the triplet $(X, Y, \mathcal{M})$ is a binary topological space and the members of $\mathcal{M}$ are called the binary open subsets of the binary topological space $(X,Y, \mathcal{M})$. The elements of $X \times Y$ are called the binary points of the binary topological space $(X, Y, \mathcal{M})$.

If Y=X, then $\mathcal{M}$ is called a binary topology on X in which case we write $(X, \mathcal{M})$ as a binary topological space.

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**Definition 2.3:** [1] Let (X, Y, $\mathcal{M}$) be a binary topological space (A, B) ⊆ (X, Y ). Let $(A, B)^{1*} = \cap \{A_\alpha :$ $(A_\alpha, B_\alpha)$ is binary closed} and (A, B) ⊆ $(A_\alpha, B_\alpha)$ and $(A, B)^{2*} = \cap \{B_\alpha : (A_\alpha, B_\alpha)$ is binary closed} and (A,B)⊆ $(A_\alpha, B_\alpha)$. Then $((A, B)^{1*}, (A, B)^{2*})$ is binary closed and (A, B) ⊆ $((A, B)^{1*}, (A, B)^{2*})$.

**Definition 2.4:** [2] The ordered pair $((A, B)^{1*}, (A, B)^{2*})$ is called the binary closure of (A, B) denoted by b-cl(A, B) in the binary space (X,Y, $\mathcal{M}$) where (A, B) ⊆ (X, Y).

**Definition 2.5:** [2] Let (X, Y, $\mathcal{M}$) be a binary topological space and A⊆ X, B ⊆ Y. Then (A, B) is a binary closed in (X, Y, $\mathcal{M}$) if $(X \setminus A, Y \setminus B) \in \mathcal{M}$.

**Definition 2.6:** [4] Let (X, Y, $\mathcal{M}$) be a binary topological space. Let (A, B) ∈ $\wp(X) \times \wp(Y)$.Then (A, B) is called generalized binary closed if b- cl(A,B)⊆(U,V) whenever (A, B) ⊆ (U, V) and (U, V) is binary open in (X,Y, $\mathcal{M}$).

**Definition 2.7:** [3] Let (X, Y, $\mathcal{M}$) be a binary topological space. Let (A, B) ⊆ (X, Y). Then (A, B) is called binary regular open if (A, B) = b-int (b-cl(A, B)).

**Definition 2.8:** [3] Let (X, Y, $\mathcal{M}$) be a binary topological space. Let (A, B) ⊆ (X,Y). Then (A, B) is called generalized binary regular closed if b-cl (A, B) ⊆ (U, V) whenever (A, B) ⊆ (U, V) is binary regular open.

**Definition 2.9:** A binary topological space (X, Y, $\mathcal{M}$) is called binary-$T_{1/2}$ if every generalized binary closed set is binary closed.

**Definition 2.10:** [5] A subset A of a topological space (X, τ) is called $\hat{g}$ -closed if cl(A) ⊆ U whenever A ⊆ U and U is semi open in (X, τ).

**Definition 2.11:** Let (X,Y, $\mathcal{M}$) be a binary topological space. Let (A, B) ⊆ (X, Y). Then (A, B) is called binary semi open if there exists a binary open set (U, V) such that (U, V) ⊆ (A, B) ⊆ b-cl (U, V).

**3. GENERALIZED BINARY SEMI CLOSED SET**

In this section we introduce generalized binary semi closed sets and investigated some of their basic properties.

**Definition 3.1:** Let (X, Y, $\mathcal{M}$) be a binary topological space. Then (A, B) ⊆ (X, Y) is called generalized binary semi closed if b-cl(A, B) ⊆(U, V) whenever (A, B) ⊆(U, V) and (U, V) is binary semi open.

**Example 3.2:** Let X={0, 1}, Y={a, b, c} and $\mathcal{M}$ = {(∅, ∅), ({0}, {a}), ({1}, {b}), (X, {a, b}), (X, Y )} is a binary topology from X to Y in (X, Y, $\mathcal{M}$). In this, ({0}, {b, c}) is generalized binary semi closed set, since b-cl ({0}, {b, c}) = (X, Y) ⊆ (X, Y ) where (X,Y) is binary semi open.

**Theorem 3.3:** Every binary closed set in a binary topological space is generalized binary semi closed.

**Proof:** Let (X, Y, $\mathcal{M}$) be a binary topological space and (A, B) be binary closed in (X, Y, $\mathcal{M}$). Let (A, B)⊆ (U, V) where (U, V) is binary semi open in (X, Y, $\mathcal{M}$). Since (A, B) is binary closed, b-cl(A, B) =(A, B) ⊆ (U, V) and hence (A, B) is generalized binary semi closed.

**Remark 3.4:** The converse of the above theorem is not true as seen from the following example.

**Example 3.5:** Let X={a, b}, Y={1, 2, 3} and $\mathcal{M}$ = {(∅, ∅), ({a}, {1}), ({b}, {2}), (X, {1,2}), (X, Y)} is a binary topology from X to Y. Also {(∅, ∅), ({b}, {2, 3}), ({a}, ({1, 3}), (∅, {3}), (X, Y)} are the binary closed sets in (X,Y, $\mathcal{M}$). Now, consider ({a, b}, {1, 3} ⊆ (X, Y) where (X, Y) is binary semi open. Then b-cl ({a, b}, {1,3}) = (X, Y) ⊆ (X, Y). Therefore, ({a, b},{1, 3}) is generalized binary semi closed set but not binary closed.

**Theorem 3.6:** Every generalized binary semi closed set in a binary topological space is generalized binary closed.

**Proof:** Let (A, B) be a generalized binary semi closed and (A, B)⊆(U, V) where (U, V) is binary open. By proposition 3.5 [2], (U, V) is binary semi open. Since (A, B) is generalized binary semi closed set and b-cl(A, B) ⊆ (U,V). Consequently (A, B) is generalized binary closed.

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**Remark 3.7:** The converse of the above theorem is not true as seen from the following example.

**Example 3.8:** Let X={0, 1}, Y={a, b, c} and $\mathcal{M}$={(∅, ∅), (∅, {a}), (∅, {b}), (∅, {a, b}), ({0}, ∅), ({1}, ∅), ({0}, {a}), ({0}, {b}), ({1}, {a}), ({1}, {b}), ({0}, {a, b}), ({1}, {a, b}), ({0}, Y), ({1}, Y), (X, ∅), (∅, Y), (X, {a}), (X, {b}), (X, {a, b}), (X, Y)} is a binary topology from X to Y. Now, consider ({0, 1}, {a, b}) ⊆ (X, Y) where (X, Y) is binary open. Then b-cl ({0, 1}, {a, b}) = (X,Y) ⊆ (X,Y). Therefore, ({0, 1}, {a, b}) is generalized binary closed set but not generalized binary semi closed, since ({0, 1}, {a, b}) ⊆ ({0, 1}, {a, b}) and (X, Y) where ({0, 1},{a, b}) and (X, Y) is a binary semi open and b-cl ({0,1},{a, b}) =(X,Y) ⊄ ({0,1},{a, b}).

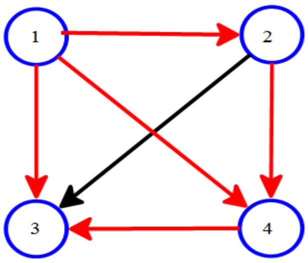
**Theorem 3.9:** Every generalized binary semi closed set in a binary topological space is generalized binary regular closed.

**Proof:** Let (A, B) be a generalized binary semi closed and (A, B) ⊆ (U, V) where (U, V) is binary regular open. By proposition 3.2 [3], (U, V) is binary open and by proposition 3.5 [2], (U, V) is binary semi open. Since (A, B) is generalized binary semi closed, b-cl(A, B) ⊆ (U, V).Consequently (A, B) is generalized binary regular closed.

**Remark 3 .10:** The converse of the above theorem is not true as seen from the following example.

**Example 3.11:** Let X={a, b}, Y={1, 2, 3}, $\mathcal{M}$ ={(∅, ∅), (∅, {1}), (∅, {2}), (∅, {1, 2}), ({a}, ∅), ({b}, ∅), ({a}, {1}), ({a}, {2}), ({b}, {2}), ({a}, {1, 2}), ({b}, {1}), ({b}, {1, 2}), (X, {1}), (X,{1, 2}), (X, {2}), (X, ∅), (∅, Y), ({b}, Y), (X,Y)} is a binary topology from X to Y. Now, consider ({b}, {1, 2}) ⊆ (X, Y) where (X, Y) is binary regular open. Then b-cl({b}, {1, 2}) = ({b}, Y) ⊆ (X,Y). Therefore ({b}, {1, 2}) is generalized binary regular closed set but not generalized binary semi closed. Since ({b}, {1, 2}) ⊆ (X,{1, 2}) where (X, {1, 2}) is binary semi open and b-cl({b}, {1, 2}) = ({b}, Y) ⊄ (X, {1, 2}).

**Remark 3.12:** From the above relations we get the following diagram where 1 → 2 represents 1 implies 2 but not conversely.




(1): Binary closed (2): Generalized binary semi closed

(3): Generalized binary regular closed (4): Generalized binary closed

## 4. CHARACTERIZATIONS OF GENERALIZED BINARY SEMI CLOSED SET

**Theorem 4.1:** Let (A, B) be a generalized binary semi closed set in a binary topological space (X, Y, $\mathcal{M}$). Suppose (C, D)⊆ ($(A, B)^{1*}$ \A, $(A, B)^{2*}$ \B) where (C, D) is binary semi closed. Then (C, D)=(∅, ∅).

**Proof:** Let (C, D) ⊆ ($(A, B)^{1*}$ \A, $(A, B)^{2*}$ \B) where (C, D) is binary semi closed. Then (C,D) ⊆ ($(A,B)^{1*}$ ,$(A,B)^{2*}$ ) .Also C ⊆ X-A which implies A ⊆ X-C . Similarly, B ⊆ Y-D. Now, (A, B) ⊆ (X\C,Y\D) where (X\C, Y\D) is binary semi open. Since (A, B) is generalized binary semi closed. b-cl(A, B)⊆(X\C, Y\D) and so ($(A, B)^{1*}$ , $(A,B)^{2*}$ ) ⊆ (X\C, Y\D). Hence (C, D) ⊆ (X \ $(A, B)^{1*}$ ,Y\ $(A, B)^{2*}$ ). Now, (C, D) ⊆ ($(A, B)^{1*}$ ∩ (X \ $(A, B)^{1*}$ ), $(A, B)^{2*}$ ∩ (Y\ $(A, B)^{2*}$ )). Hence (C, D) ⊆ (∅, ∅) which implies (C, D) =(∅, ∅).

**Remark 4.2:** The converse of the above theorem is not true as seen from the following example.

**Example 4.3:** Let X={a, b}, Y={1, 2, 3}, $\mathcal{M}$ ={(∅, ∅), (∅, {1}), (∅, {2}), (∅, {1,2}), ({a}, ∅), ({b}, ∅), ({a}, {1}), ({b}, {1}), ({a}, {2}), ({b}, {2}), ({a}, {1,2}), ({b}, {1, 2}), (X, {1}), (X, {2}), (X, {1, 2}), (X, ∅), ({b}, Y), (∅, Y), (X, Y)} is a binary topology from X to Y. Let (A, B) = ({a}, {1}). In this, ($(A, B)^{1*}$ \A, $(A, B)^{2*}$ \B) = (({a}, {1}\ {a}, ({a}, {1})\{1})) = (({a}, {1}) ∩∅, ({a}, {1}) ∩ {2, 3})) = (∅, ∅)
which is binary closed but ({a}, {1}) is not generalized binary semi closed.

**Theorem 4.4:** Let (A, B) be binary semi open and generalized binary semi closed. Then (A, B) is binary closed.

**Proof:** Always (A, B)⊆(A, B). Since (A, B) is generalized binary semi closed, b-cl (A, B)⊆ (A, B). By proposition 3.7 [1] (A, B) ⊆ b-cl (A, B) and so b-cl(A, B) = (A, B) and by proposition 3.6 [1] (A, B) is binary closed.

**Remark 4.5:** Union of two generalized binary semi closed sets is not generalized binary semi closed.

**Example 4.6:** Let X= {0, 1}, Y={a, b, c}, $\mathcal{M}$={(∅, ∅), ({0}, {a}), ({1}, {b}), (X, {a, b}), (X, Y)} is a binary topology from X to Y in (X,Y, $\mathcal{M}$). Let A = ({1}, {b, c}) and B= ({1}, {a, c}), A∪B = ({1}, {b, c}) ∪ ({1}, {a, c}) = ({1}, {a, b, c}) is not a generalized binary semi closed whereas A & B are generalized binary semi closed.

**Theorem 4.7:** If (A, B) is a generalized binary semi closed set and (A, B) ⊆ (C, D) ⊆ b-cl(A,B), then (C,D) is generalized binary semi closed.

**Proof:** Let (C, D) ⊆ (U, V) where (U, V) is binary semi open. Now, (A, B) ⊆ (U, V). Since (A, B) is generalized binary semi closed, b-cl (A, B) ⊆ (U, V). By proposition 3.7 [1] b-cl(C,D) ⊆ b-cl(b-cl(A,B)) = b-cl(A,B) ⊆ (U,V). Consequently (C,D) is generalized binary semi closed.

**Theorem: 4.8:** Let (A, B) be a generalized binary semi closed set. Suppose that (C, D) is binary closed. If (A∩C, B∩D) ⊆ (U,V) where (U,V) is binary semi open, then $((A,B)^{1*} \cap C, (A,B)^{2*} \cap D) \subseteq (U,V)$.

**Proof:** Since (A∩C, B∩D) ⊆ (U, V), (A, B)⊆ (U, V) and (C, D)⊆(U, V).
Since (A, B) is generalized binary semi closed, b-cl(A,B)⊆(U,V) and hence $((A,B)^{1*}, (A,B)^{2*}) \subseteq (U,V)$. Consequently $((A, B)^{1*} \cap C, (A, B)^{2*} \cap D) \subseteq (U, V)$.

**Theorem 4.9:** If (A, B) is a generalized binary semi closed set and binary semi open and (U, V) is binary closed then (A∩U, B∩V) is generalized binary closed.

**Proof:** By theorem 4.4, (A, B) is binary closed. Since (U, V) is binary closed, by proposition 3.2 [1] (A, B)∩(U,V) = (A∩U, B∩V) is binary closed and hence (A∩U, B∩V) is generalized binary closed.

**Remark 4.10:** Let (A, B) be a generalized binary semi closed in (X, Y, $\mathcal{M}$) then A need not be $\hat{g}$ closed in (X, $\tau_X$) and B need not be $\hat{g}$ closed in (Y, $\tau_y$).

**Proof:** Let X={a, b, c}, Y={1, 2, 3} and $\mathcal{M}$= {(∅, ∅), ({a}, {1}), ({a, b}, {1, 2}, (X, Y)} is a binary topology from X to Y. $\tau_X$ ={∅, {a}, {a, b}, X} and $\tau_y$ ={∅, {1}, {1, 2}, Y}. Here ({a, c}, {2}) is generalized binary semi closed but {a, c} is not $\hat{g}$ closed in X and {2} is not $\hat{g}$ closed in Y.

## 5. VARIOUS SPACES ASSOCIATED WITH GENERALIZED BINARY SEMI CLOSED SETS

In this section, we introduce binary semi $T_{1/2}$-space, binary semi $T_{1/2}{}^{*}$-space and binary semi $T_{1/2}{}^{**}$-space and investigated their basic properties.

**Definition 5.1:** A binary topological space (X, Y, $\mathcal{M}$) is said to be

(i) binary semi $T_{1/2}$-space if every generalized binary semi closed set is binary closed.

(ii) binary semi $T_{1/2}{}^{*}$-space if every generalized binary closed set is generalized binary semi closed.

(iii) binary semi $T_{1/2}{}^{**}$ -space if every generalized binary regular closed set is generalized binary semi closed.

**Example 5.2:** Let X={a, b}, Y={1,2,3}, $\mathcal{M}$= {(∅, ∅), (∅, {1}), (∅, {2}), ({a}, ∅), ({b}, ∅), ({a}, {1}), ({b}, {1}), ({a}, {2}), ({b}, {2}), ({a}, {1, 2}), ({b}, {1, 2}), (X, {1}), (X, {1,2}), (X, {2}), (X, ∅), ({b}, Y), (∅, Y), (X, Y)} is a binary topology from X to Y. In this every generalized binary semi closed set is binary closed. Hence (X, Y, $\mathcal{M}$) is a binary semi $T_{1/2}$-space.

**Example 5.3:** Let X={a, b}, Y={1, 2}, $\mathcal{M}$ ={(∅, ∅), ({a}, {1}), ({b}, {2}), (X, {1}), (X, Y )} is a binary topology from X to Y in (X, Y, $\mathcal{M}$). In this, every generalized binary closed set is a generalized binary semi closed.

Hence (X, Y, $\mathcal{M}$) is a generalized binary semi $T_{1/2}{}^{*}$-space.

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**Example 5.4:** Let X={a, b, c}, Y={1, 2}, $\mathcal{M}$= {(∅, ∅), ({a}, ∅), ({b}, ∅), ({a, b}, ∅), ({a}, {1}), ({a}, {2}), ({b}, {1}), ({b}, {2}), ({a}, {1, 2}), ({b}, {1, 2}), ({a, b}, {1}), ({a, b}, {1, 2}), (X, ∅), (X, {1}), (X, {2}), (∅{1}), (∅, {2}), (∅, Y), (X, Y)} is a binary topology from X to Y in (X, Y, $\mathcal{M}$). In this, every generalized binary regular closed sets is generalized binary semi closed. Hence (X, Y, $\mathcal{M}$) is binary semi $T_{1/2}{}^{**}$-space.

**Theorem 5.5:** Every binary $T_{1/2}$-space is a binary semi $T_{1/2}$-space.

**Proof:** Let (X, Y, $\mathcal{M}$) be a binary $T_{1/2}$-space and (A, B) be a generalized binary semi closed set. By theorem 2.1.6, (A,B) is a generalized binary closed. Since (X, Y, $\mathcal{M}$) is a binary $T_{1/2}$-space, (A, B) is binary closed. Consequently (X, Y, $\mathcal{M}$) is a binary semi $T_{1/2}$-space.

**Remark 5.6:** The converse of the above theorem is not true as seen from the following example.

**Example 5.7:** Let X={a, b}, Y={1, 2, 3}, $\mathcal{M}$={(∅, ∅), ({∅, {1}), (∅, {2}), (∅, {1,2}), ({a}, ∅), ({b}, ∅), ({a}, {1}), ({b}, {1}), ({a}, {2}), ({b}, {2), ({a}, {1, 2}), ({b}, {1, 2}), (X, {1}), (X, {2}), (X, ∅), ({b}, Y), (∅, Y), (X, Y)} is a binary topology from X to Y. Hence (X, Y, $\mathcal{M}$) is a binary semi $T_{1/2}$–space but not a binary $T_{1/2}$–space, since ({a}, {2}) is generalized binary closed but not a binary closed.

**Theorem 5.8:** A space (X, Y, $\mathcal{M}$) is a binary semi $T_{1/2}{}^{*}$-space if it is a binary semi $T_{1/2}$ -space and a binary semi $T_{1/2}{}^{**}$-space.

**Proof:** Let (X, Y, $\mathcal{M}$) be a binary semi $T_{1/2}{}^{*}$-space. Let (A, B) be generalized binary closed set. Since (X, Y, $\mathcal{M}$) is a binary $T_{1/2}$-space, (A, B) is binary closed and hence (A, B) is generalized binary regular closed. Since (X, Y, $\mathcal{M}$) is a binary semi $T_{1/2}{}^{**}$-space, (A, B) is generalized binary semi closed. Hence (X, Y, $\mathcal{M}$) is binary semi $T_{1/2}{}^{*}$-space.

**Remark 5.9:** Binary $T_{1/2}$ -space and binary semi $T_{1/2}{}^{*}$-space are independent as seen from the following examples.

**Example 5.10:** Let X={a, b, c}, Y={1, 2} $\mathcal{M}$={(∅, ∅), (∅, {1}), (∅, {2}), ({a}, ∅), ({b}, ∅), ({a}, {1}), ({b}, {1}), ({a}, {2}), ({b}, {2}), ({a}, {1,2}), ({b}, {1, 2}), ({a, b}, {1}), ({a, b}, {2}), ({a, b}, {1, 2}), (X, ∅), (X, {1}), (∅, Y), (X, Y )} is a binary topology from X to Y. In this (X, Y, $\mathcal{M}$) is a binary $T_{1/2}$-space but not a binary semi $T_{1/2}{}^{*}$-space, since ({b, c},{1, 2}) is generalized binary closed but not generalized binary semi closed.

**Example 5.11:** Let X={0, 1}, Y={a, b, c}, $\mathcal{M}$ = {(∅, ∅), ({0}, {a}), {1}, {b}), (X, {a, b}), (X, Y )} is a binary topology from X to Y in (X, Y, $\mathcal{M}$). In this, (X, Y, $\mathcal{M}$) is a binary semi $T_{1/2}{}^{*}$- space but not a binary $T_{1/2}$-space, since ({1}, {a, c}) is generalized binary closed but not binary closed. Hence binary $T_{1/2}$-space and binary semi $T_{1/2}{}^{*}$-space are independent.

**Remark 5.15:** Binary $T_{1/2}{}^{*}$-space and binary semi $T_{1/2}{}^{**}$-space are independent as seen from the following examples.

**Example 5.16:** Let X={0, 1}, Y={a, b, c}, $\mathcal{M}$ = {(∅, ∅), ({0}, {a}), ({1}, {b}), (X, {a, b}), (X, Y)} is a binary topology from X to Y. In this (X, Y, $\mathcal{M}$) is a binary $T_{1/2}{}^{*}$-space but not a binary semi $T_{1/2}{}^{**}$-space, since ({0}, {b}) is generalized binary regular closed but not generalized binary semi closed.

**Example 5.17:** Let X = {a, b, c}, Y={1, 2}, $\mathcal{M}$={(∅, ∅), (∅, {1}), (∅, {2}), ({a}, ∅), ({b}, ∅), ({a}, {1}), ({b}, {1}), ({a}, {2}), ({b}, {2}), ({a}, {1, 2}), ({b}, {1, 2}), ({a, b}, {1}), ({a, b}, {2}), ({a, b}, {1, 2}), (X, ∅), (X, {1}), (∅, Y), (X, Y)} is a binary topology from X to Y. In this (X, Y, $\mathcal{M}$) is a binary $T_{1/2}{}^{**}$-space but not a binary semi $T_{1/2}{}^{*}$-space, since ({a}, {1}) is generalized binary closed but not generalized binary semi closed.

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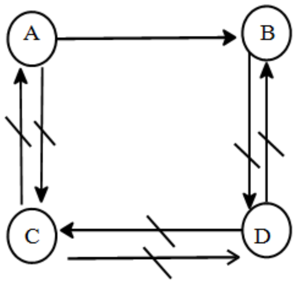
**Remark 5.18:** Binary semi $T_{1/2}$-space and binary $T_{1/2}{}^{**}$-space are independent as seen from the following example.

**Example 5.19:** Let $X=\{a, b, c\}$, $Y=\{1,2\}$, $\mathcal{M}=\{(\emptyset, \emptyset), (\emptyset, \{1\}), (\emptyset, \{2\}), (\{a\}, \emptyset), (\{b\}, \emptyset), (\{a\}, \{1\}), (\{b\}, \{1\}), (\{a\}, \{2\}), (\{b\}, \{2\}), (\{a\}, \{1,2\}), (\{b\}, \{1, 2\}), (\{a, b\}, \{1\}), (\{a, b\}, \{2\}), (\{a, b\}, \{1, 2\}), (X, \emptyset), (X, \{1\}), (\emptyset, Y), (X, Y)\}$ is a binary topology from X to Y. In this $(X, Y, \mathcal{M})$ is binary semi $T_{1/2}$-space but not a binary semi $T_{1/2}{}^{**}$-space, since $(\{a\}, \{1\})$ is generalized binary regular closed but not generalized binary semi closed.

**Remark 5.20:** From the above relations we get the following diagram where $A \to B$ represents A implies B but not conversely.




(A): Binary $T_{1/2}$-space (B): Binary semi $T_{1/2}$– space

(C): Binary semi $T_{1/2}{}^{*}$-space (D): Binary semi $T_{1/2}{}^{**}$- space

**Theorem 5.21:** Let $(X, Y, \mathcal{M})$ be a binary topological space and $(A, B) \subseteq (X, Y)$. If $(A, B)$ is generalized binary semi closed ,then for all $(x, y) \in$ b-cl(A, B),b-cl$(\{x\}, \{y\}) \cap (A,B) = \emptyset$.

**Proof:** Let $(x, y) \in$ b-cl (A, B). Suppose b-cl $(\{x\}, \{y\}) \cap (A, B) = \emptyset$, then $(A, B) \subseteq X$-(b-cl $(\{x\}, \{y\})$. Since (A, B) is generalized binary semi closed b-cl $(A, B) \subseteq X$-(b-cl $(\{x\}, \{y\})$ which is a contradiction and hence b-cl $(\{x\}, \{y\}) \cap (A, B) = \emptyset$.

**Theorem 5.22:** Let $(X, Y, \mathcal{M})$ be a binary topological space. Then,

(i) $(X, Y, \mathcal{M})$ is binary semi $T_{1/2}$-space

(ii) for each $(x, y) \in X\times Y$, $(\{x\}, \{y\})$ is either binary semi closed or binary open.

**Proof:**

**(i)⇒(ii):** Let $(X, Y, \mathcal{M})$ be a binary semi $T_{1/2}$-space and let $(x, y) \in X\times Y$. Suppose $(\{x\}, \{y\})$ is not binary semi closed, then (X, Y) is the only binary semi open set containing $(X\setminus\{x\}, Y\setminus\{y\})$ and so $(X\setminus\{x\}, Y\setminus\{y\})$ is generalized binary semi closed. Since $(X, Y, \mathcal{M})$ is binary semi $T_{1/2}$–space $(X\setminus\{x\}, Y\setminus\{y\})$ is binary closed.

**(ii) ⇒ (i):** Let (A, B) be a generalized binary semi closed set and $(\{x\}, \{y\}) \in$ b-cl (A, B). By (ii), $(\{x\}, \{y\})$ is either binary semi closed or binary open.

**Case-(i):** Suppose $(\{x\}, \{y\})$ is binary semi closed. If $(\{x\}, \{y\}) \not\in (A, B)$. Then b-cl $(A, B) \setminus (A, B)$ contains a non empty binary semi closed set $(\{x\}, \{y\})$ then $(\{x\}, \{y\}) \subset ((A, B)^{1*}\setminus A, (A, B)^{2*}\setminus B)$ which is a contradiction to theorem 4.1 and so $(\{x\}, \{y\}) \in (A,B)$ .

**Case- (ii):** Suppose $(\{x\}, \{y\})$ is binary open. Since $(\{x\}, \{y\}) \in$ b-cl (A, B). By theorem 4.1, b-cl $(\{x\}, \{y\}) \cap (A, B) = \emptyset$ and hence $(\{x\}, \{y\}) \in (A,B)$. Thus in both cases $(\{x\}, \{y\}) \in (A, B)$ which implies b-cl$(A, B) \subseteq (A, B)$ and hence b-cl$(A, B) = (A, B)$.

Consequently (A, B) is binary closed. Thus $(X, Y, \mathcal{M})$ is a binary $T_{1/2}$-space.

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